E. BERGSTRAND
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Some Recent Determinations of the Velocity of Light.

E. BERGSTRAND
Swedish Survey Office - Stockholm

In the last decade there has been a strong development of the methods for the determination of the velocity of light. Earlier the direct methods dominated. The new ones are mainly indirect. Here ESSEN or ESSEN and GORDON-SMITH are pioneers. In 1947 they used the cavity resonator for ultra short radio-waves. These waves can most expediently be conducted in silver tubes, the diameters of which are greater than half the wave length. If the length of the tube, limited by two silver end walls, is an even multiple of half the wave length, the reflection against the end walls erects a standing oscillation in the tube. The resonance frequency of this oscillation can be calculated from the dimensions of the cavity and the velocity by which the field-variations propagate within the cavity. The dimensions and the frequency are determined with great accuracy. The only thing left, the velocity, is then calculated on basis of the known relation. If the cavity is evacuated, we get directly the vacuum value of c.

Already the first determination [1] was carried out with rare precision and skill. The cavity was here fixed. In his apparatus No. 2 of 1950 ESSEN [2] inserted a movable plunger. Thus the length of the cavity could be variated and ESSEN could detect deviations from the theoretical relation between dimensions and frequency. The accuracy was now increased by 7 or 8 times to 1 km/s and since the first result was fully confirmed, the principles applied must have been correct.

In 1950 BOL [3] also used a cavity resonator. This instrument was of the fixed type. The value differed considerably from that of ESSEN. Since the error limits were given very narrow, 0.4 km/s, this determination has caused a lot of embarrassment. From the debate there seems to be evident that the influence of the skin-effect, here only calculated, was underestimated. With experience from ESSEN's experiments, DAHYOFF has increased BOL's skin-correction from

3 to 8 km/s. Thus only by applying different points of view, BOL's result can change the value of c by km/s. I have found it best to omit this determination.

FROOME [4] used radar waves in a different method, called micro-wave interferometry. By a transmitting horn an oscillator emits waves of little more than one cm towards a reflecting movable screen, some 10 m from the horn. By the screen the waves are reflected back into the horn. Here, in a wave guide, they interfere with waves comming directly from the transmitter. An attached receiver and detector yields minimum of current when the two wave systems differ by π in phase. This will occur to every half wave length of increase of the distance to the movable reflector. The reflector is displaced totally by 1.6 m through 250 minima. This proceeding is repeated systematically at different distances from the transmitting horn. Thus disturbing reflections from objects in the room will be eliminated. The wave front shape will also be considered. In this way we get the wave length. Knowing the frequency too, we get the velocity. The result of the measurements, carried out with great care, was almost identical with that of ESSEN. Next year, 1953, FROOME [5] had ready a new considerably improved instrument. Its characteristic is a marked symmetry of the arrangements. There are two transmitter-horns, and the distance between them is 12 m. They are directed towards each other. On the line of and between the horns is the movable receiver-detector. The two emitted wave systems from the horns interfere in the receiver and the detector shows minimum of current to each half wave length of displacement of the receiver unit. 162 minima or 1 m is passed at different distances between horns and receiver. The greatest uncertainty lies in insufficient information as to the shape of the wave fronts. For two measuring series this shape was deliberately made bad. In spite of this, due to a correct treating of the conditions, these series agree with the good one, used for the final result. This fact combined with Froome's experience from his first instrument give confidence in the value obtained.

As to the remaining indirect methods I will be brief. First we have ASLASKON's «Shoran» determinations [6, 7]. The constant c is obtained by adapting Shoran distances to those geodetically known. The underlying matter is extensive. The nature of the method, however, makes the error limits a little inconveniently wide without any prospect of further reduction below 2 km/s.

In 1952 RANK, RUTH, VANDER SLUIS [8] determined the constants in the formula for the wave-length of the rotation-vibration absorption bands of HCN. They measured in infrared. The wave-length of the ground state lies in the half-cm region, where the frequency could directly be compared to an ordinary frequency standard. The determination was repeated one year later [9] with improved instruments. Then the result was 14 km/s higher. The error limits of the first determination were later on re-estimated [10] and increased by three times to ± 19 km/s.

In 1955 PLYLER, BLAINE, CONNOR [11] carried out a measurement along similar lines. The result was 2 further km/s higher or now on level with ESSEN's value. The error limits were $\pm$ 4 km/s. The rotational band-spectrum method seems not yet to have stabilized and the main signification of this very interesting experiment seems still to lie within other domains.

In the same year FLORMAN [12] used a radio-frequency interferometer. The distance was 1 500 m and the wave-length 1.7 m. Up to now the method is too new and the error sources seem hard to estimate or cancel. The limits were $\pm$ 3 km/s. As for the two last methods I think the risk of unknown systematic errors is not sufficiently minimized. The jump from infrared to radar region is very large and the diffraction and scattering of 2 m waves is considerable. Regarding these cases the best is to keep awaiting.

Alongside of these indirect methods there are by now a lot of results from my direct Kerr cell method. The apparatus, the geodimeter, is used in several countries for the reverse purpose of determining geodetic distances and is checked on stringed base lines. As by Shoran these checks have yielded values on c.

The geodimeter emits a light beam, the intensity of which is modulated by a crystal controlled high-frequency radio-transmitter. The corresponding radio-wave length is 30 m. The emitted light is reflected back by a distant mirror or reflecting system and is then absorbed by a photo-tube close to the emitter. The sensitivity of the tube is high-frequency controlled by the transmitter also. Consequently the amplitude of the photo-current variates in a periodical manner with the distance to the mirror or with the time for the light to cover the distance to and fro. Since the velocity of light equals that of radio waves the period of the photo-current variation corresponds to a change of 15 m of distance (the light has to go to and fro). A particular sort of compensation circuit makes the current to pass zero two times in each period. Thus, at every $7\frac{1}{2}$ m of increased distance to the mirror the photo-current is zero. Due to the crystal control the value $7\frac{1}{2}$ is constant to within two parts in ten millions. Now, this row of zero-points to every $7\frac{1}{2}$ m is the measuring scale for the distance measurements.

By the insertion of a longer or shorter delay of the radio transmitter control of the photo-tube, the zero-point row can be moved forth or back within 8 m. Thus, wherever the distant mirror stands, a zero-point can be adjusted to coincide with the mirror. Immediately after that, with unchanged delay, the distance to the first zero-point close to the geodimeter, is determined by aid of a built in calibrated continuously variable light path or light conductor of maximum 8 m of double length. The rest of the distance, that from the first zero-point up to the last one at the mirror, is an even multiple of the $7\frac{1}{2}$ m constant.

In the first geodimeters after hard transports, the calibration of the light conductor could change with the consequence of a false distance to the first

zero-point. The error always depended on a displacement of the emission point on the photo-cathode of light from the light conductor as compared to light from the distant mirror. A side way shift of this point caused a change in transit time of the photo-electrons through the tube and to the geodimeter this was equivalent to a change of the time for the light to cover the distance to and fro. By making the recorded distance less dependent of the transit time and of changes in the light conductor the resulting errors in this year are considerably reduced. Those errors can, however, also be cancelled or *compensated* by measuring the distance as a difference between the zero-point positions of two field-mirrors. Then the light conductor is eliminated. We can also do the compensation by a complementary measurement of the distance into two parts. The last case is equivalent to a renewed calibration of the light conductor in the field. If the distance is known from the length of a base line, we get direct information of the accurate 7 ½ m unit, that is of the 30 m « wave-length ». Knowing the frequency too, we obtain the velocity.

Well, in the following Table I of geodimeter results on different base lines I have selected the *compensated* determinations to get a reliable value on c.

TABLE I.

Base line		Distance (m)	Velocity (km/s)	Mean of base	Weight
Swedish [13-15]	I	6 910	299 793.02	3.05	10
	I	6 910	3.08	—	—
	II	5 400	3.17	2.80	8
	II	5 400	2.43	—	—
	III	7 320	3.37	3.37	7
Austr lian [16]	I	6 440	2.64	2.44	11
	I	6 440	2.05	—	—
	I	6 440	2.64	—	—
	II	9 660	2.41	2.46	14
	II	9 660	2.50	—	—
	III	6 440	3.61	3.61	3
	IV	11 100	3.21	3.21	6

Total mean: (299 792.85 ± 0.16) km/s

The two last measurements are *not* of the compensated type. They are, however, included in the table because they are carried out with the new improved instrument of this year, though the lack of compensation has rendered them half of weight. In this table the weights are proportionate to the product of distance and square root of the number of determination occasions. By the use of square root we have considered the errors in the base distance also.

If we regard the result from one base line, the error limits are $\pm$ 0.38 km/s. If we assume that one half of this error may come from the base and the second from the measurements, we get an accuracy of one part in a good million for each of these quantities, which seems likely.

In the following Table II are the uncompensated measurements with earlier models of the geodimeter.

TABLE II.

Base line		Distance (m)	Velocity (km/s)	Weight
Australian [16]	I	6 440	299 792.50	6
English [17]	I	11 260	2.40	11
	II	24 830	2.20	25
U.S.A. [18]	I	1 380	4.06	1
	II	12 800	4.27	13
	III	3 120	2.73	3
	IV	2 130	1.69	2
Weighted mean:			(299 792.75 $\pm$ 0.34) km/s	

The result from the Table II, with weights proportionate to the distances, may only serve as a comparison.

Now, what about the risk of systematic errors due to the common type of instrument and of measuring method? In the case of the compensated measurements there should only be three possibilities left, those of colour, of atmosphere and of base length. The effective and relatively narrow colour band is determined for each instrument according to the following proceeding: After the light has passed a calibrated variable monochromator, readings are made directly of the resulting photo-current. In the field, on account of the instrument's own filtering effect, the influence of atmospheric colour-absorption plays an insignificant role, above all at the short distances of the base lines. These lines also always are on flat country. Therefore, ordinary atmospheric conditions, temperature, pressure and humidity, are easy to determine and there is no risk of systematic errors from this domain. And from the base lines? HONKASALO [25] has shown in a careful investigation that no systematic errors are introduced by the string method during ordinary good measuring conditions.

It may now be convenient for a general summary. To get a final value on c there are, in a sense, two ways. We can take all existing values and apply weights with regard to the error limits. Or, we can select those measu-

rements weich, besides the error limits, sohw reason to be more reliable than the rest. From the latter point of view, as to the indirect methods, ESSEN's FROOME's and ASLAKSON's determinations inspire confidence on account of their calm and consistent development up to the final result. Thus including the extensive geodimeter measurements by several observers we arrive at the following summary listed in Table III.

TABLE III.

Method	Velocity (km/s)	Weight
Cavity resonator	299 792.50 ± 1.0	2
Micro-wave interferometry	3.00 ± 0.3	6
Shoran	3.40 ± 2.0	1
Geodimeter.	2.85 ± 0.2	8
Weighted mean.	(299 792.89 ± 0.11) km/s	

Both of ASLAKSON's determinations are included in the Shoran value because the method is identical, only the underlying stuff is different. All values

TABLE IV.

Year	Observer	Method	Distance (m)	Frequency (MHz)	Velocity (km/s)
1947	ESSEN, GORDON-SMITH	C. R.	0.1	3 000	299 792 ± 9
1947	JONES	Oboe	70 000	3 000	782 ± 25
1948	BERGSTRAND	K. C.	9 000	8	793 ± 2
1949	HOUSTOUN	V. Q.	40	100	775 ± 9
1950	ASLAKSON	Shoran	300 000	300	792.3 ± 2.4
1950	ESSEN	C. R.	0.1	10 000	792.5 ± 1
1950	BERGSTRAND	K. C.	60 00	8	793.1 ± 0.3
1950	BOL	C. R.	0.1	3 000	789.3 ± 0.4
1952	ASLAKSON	Shoran	300 000	300	794.2 ± 1.9
1952	FROOME	M. I.	2	24 000	792.6 ± 0.7
1951	RANK, RUTH and VANDER SLUIS	R. S.	—	44 000	776 ± 19
1953	RANK, SHEARER and VIGGINS	R. S.	—	44 000	789.8 ± 3
1953	FROOME	M. I.	1	24 000	793.0 ± 0.3
1955	PLYLER, BLAINE and CONNOR	R. S.	—	58 000	792 ± 4
1955	FLORMAN	R. I.	1 500	173	795.1 ± 3.1
1956	Summary of geodimeter	K. C.	7 000	10	792.9 ± 0.2

In the above table the methods are abbreviated by the following symbols: C. R. = cavity resonator, K. C. = Kerr cell, V. Q. = vibrating quartz, M. I. = microwave interferometry, R. S. = rotational spectrum, R. I. = radio-frequency interferometer.

in the Table agree harmonically according to their individual error limits. Admitting a remaining common systematic error of length of ample five parts in ten millions the square root limits are increased to $\pm$ 0.2 km/s. Thus the « best » value on the velocity should be

$$c = (299\,792.9 \pm 0.2) \text{ km/s} .$$

The results from rotational spectrum and from radar interferometry agree with this value. There is nothing revolutionary new in the above figures as compared to recent surveys of atomic constants as given by DuMond, Cohen a.o. or by Bearden and Thomsen. Interesting is however, the good concordance between the extended geodimeter results and those obtained by indirect methods.

I add a list of determinations in the past decade (Table IV).

At last, what of further precision? Froome has advised a still better determination with further shorter waves. As for the geodimeter method the limit mainly is the accuracy of the base. At present the most accurate length is the 864 m Väisälä interference-comparator base in Finland. The accuracy is better than one part in ten millions [25], expressed in cadmium-line waves [26]. To the geodimeter, impaired by cm-errors, the distance is too short. Using crystal modulated light of 300 MHz frequency, however, it seems possible to increase the present accuracy of c up to one part in five millions. A device for this purpose is tested in the laboratory [24].

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