A Preliminary Geoid Chart of Australia

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The Department of National Development of the Commonwealth of Australia kindly made available to us a list of astrogeodetic deflections of the vertical at about 600 stations in Australia, referred to the Australian Geodetic Datum. The latter is defined by the Australian National Spheroid (a = 6378160m, f = 1/298·25) and the co-ordinates of Johnston Geodetic Station (25° 56′ 54″·5515 S., 133° 12′ 30″·0771 E., height 571·2m).(1)

According to A. G. Bomford's Report(2) the orientation of the chosen reference ellipsoid to the geoid had been carefully determined by analysing the deflections of the vertical at 275 stations, uniformly distributed throughout Australia including Cape York and Tasmania, but excluding New Guinea. Topographic-isostatic corrections to the deflections at 150 well-distributed stations had been computed and found to be quite small on the average(3). Thus we could expect a reasonably good fit and relatively small relief. The finished geoid map (figure 1) shows indeed a geoidal relief of less than 25m across the continent, with a high of about 18m in the southwest and a low of about —5m in the centre.

The Procedure.—The construction of the chart was carried out by two different procedures, and the results were combined in an adjustment. One procedure employed individual stations and their specific deflection values, the other was similar to the one used in North America(4). New Guinea and the Pacific Islands were treated differently. The choice of the various procedures was suggested by the specific layout of the given deflection stations.

Individual Stations.—When plotting the stations of the Australian mainland on a 1:10 mill. base map we saw a pattern of close spacing emerge, which could be utilized for huge loops of geoidal profiles along the major survey routes. The geoidal increments between neighbouring stations were computed on the Bendix G-15 computer with the well-known formula for $\triangle H$ in the following practical form:

$$\Delta H = \frac{1}{412530} (\xi'' \cdot s_m + \eta'' \cdot s_p),$$

where ξ, η are the deflections and s_m, s_p the distances in meters between the two stations along a meridian and the mean parallel respectively. The routes for the geoidal profiles were chosen after investigating loop closures. Eventually several huge geoidal loops with closures of a metre or less were accepted as a fixed framework for the interpolation across the blank spaces within the loops.

The final values for geoidal separation along these profiles were determined to conform with *zero metres at Johnston Geodetic Station,* where the spheroidal height equals the geoidal height of 571.2m by definition(1).

FIGURE I

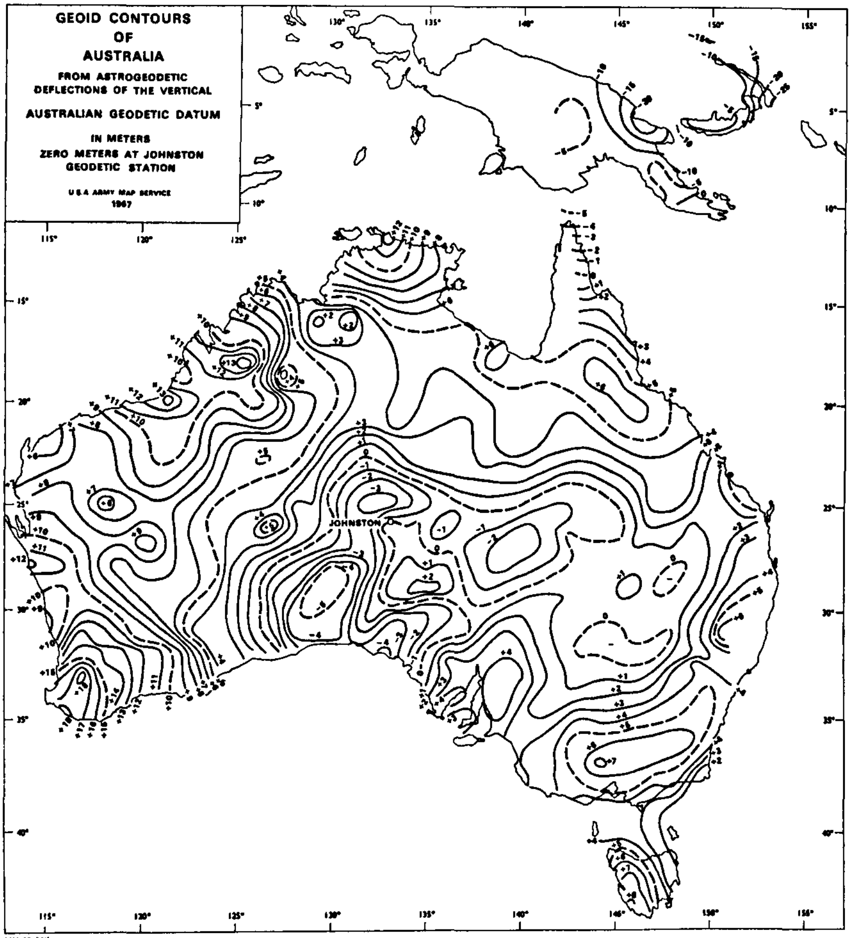

Fig. I Geoid Contours on the Australian Geodetic Datum, in meters

Deflection Charts.—As in the construction of the North American geoid map([4]) separate deflection charts for ξ and η were prepared showing isolines at one-second intervals. Along each meridian and parallel at full degrees of latitude and longitude the values of ξ and η respectively were read from the charts at 30-minute intervals by interpolation. From these values geoid profiles along the meridians and parallels were calculated by the Bendix computer and geoid separations in metres printed out at every degree intersection. The formulas used conform to the so-called projection method and are explained in detail in reference([4]).

For a meridian, in meters

$$N(m) = N_o(m) \cos S + \frac{R(m)}{206265} (\xi_o - \xi_o')'' \sin S$$
$$+ \frac{R(m)}{(206265)^2} \Delta l'' [\sin S \sum_{o}^{s} (\xi_i')'' \sin l_i + \cos S \sum_{o}^{s} (\xi_i')'' \cos l_i]$$

For a parallel, in meters

$$N(m) = N_o(m) \cos (L \cos \varphi_o) + \frac{R(m)}{206265} (\eta_o - \eta_o')'' \sin (L \cos \varphi_o)$$
$$+ \frac{R(m)}{(206265)^2} \Delta l'' \cos \varphi_o \left[\sin (L \cos \varphi_o) \sum_o^i (\eta_i')'' \sin (l_i \cos \varphi_o)\right.$$
$$\left. + \cos (L \cos \varphi_o) \sum_o^i (\eta_i')'' \cos (l_i \cos \varphi_o) \right]$$

where N = geoidal separation in meters.

ξ', η' = given "development" deflections.

ξ, η = "projection" deflections.

$S = L \cos \varphi$ = length of great circle arc increasing in the direction opposite to the positive direction of the deflection along the path.

l = variable co-ordinate along the path.

R = radius of the earth in meters.

Subscript $_o$ identifies the initial point on each path.

The well-surveyed 32nd parallel served as a backbone for the computation. The centrally located point at 32° S., 134° E. was determined first by hand computation from Johnston, where $N = 0$ meters by definition. Then the west and east parts of the 32nd parallel were computed and compared with the ΔH computation between the individual stations. The values at each full longitude degree were accepted as fixed initial points for all meridional profiles north and south.

Similarly the values computed for every full latitude degree on the 134th meridian were used as initial points for the east and west part of the parallels.

The Adjustment.—For each intersection of the i-th meridian with the j-th parallel the computer prints out two values, M_{ij} from the meridional profile and P_{ij} from the parallel profile, which will be different as a rule.

The final value for this point is $N_{ij} = M_{ij} + x_{ij}$, where x_{ij} is an unknown correction to be determined by adjustment. In general, each x_{ij} is influenced by four adjacent 1° sections, two on meridians and two on parallels. On a meridional section the computed increment $m_{ij} = M_{i,\,j+1} - M_{ij}$ will change to an assumed $a_{ij} = (M + x)_{i,\,j+1} - (M + x)_{ij}$. On a parallel section the computed increment $p_{i,\,j} = P_{i+1,\,j} - P_{ij}$ will change to an assumed $b_{ij} = (M + x)_{i+1,\,j} - (M + x)_{ij}$. The unknown corrections should be determined such that the computed increments be changed as little as possible; that is, the differences between assumed and computed increments should be minimized.

$$a_{ij} - m_{ij} = x_{i,\,j+1} - x_{ij} = v_{ij}.$$

$$b_{ij} - P_{ij} = (M - P)_{i+1,\,j} - (M - P)_{ij} + x_{i+1,\,j} - x_{i,\,j} = w_{ij}$$

A further restriction is imposed by the fixed framework of geoidal loops previously computed from individual deflection stations. Intersection points (i,j) with values N_{ij} taken or easily interpolated from that framework were plotted in red. These include the points along the

FIGURE 2

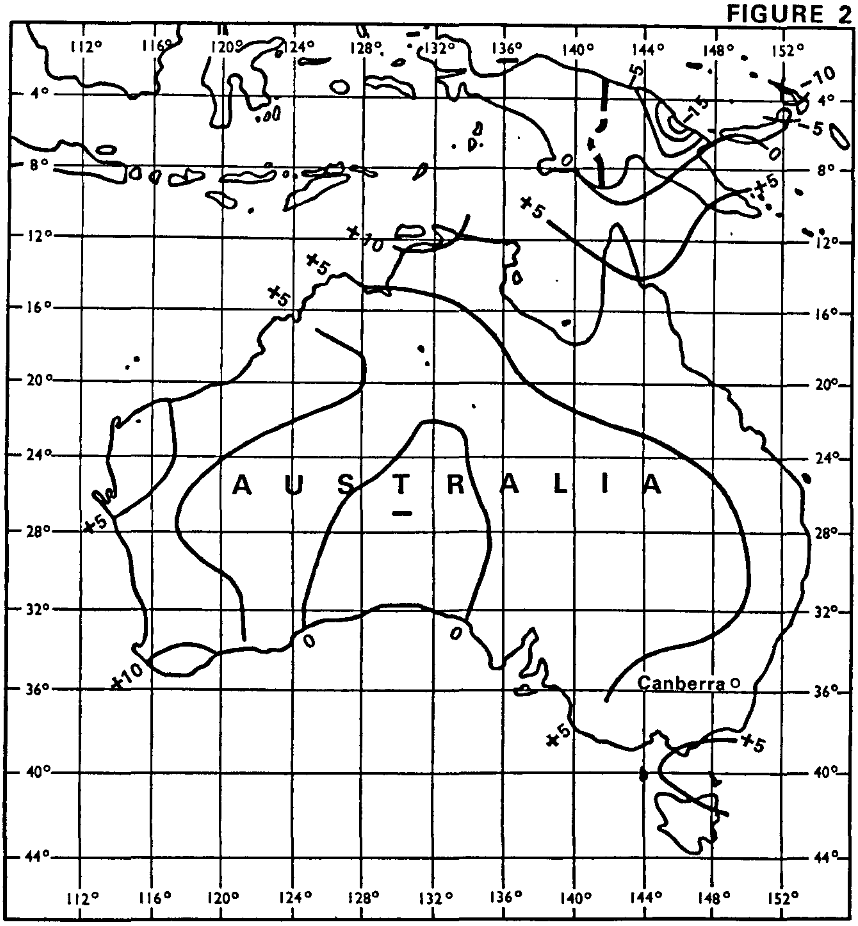

Fig. 2 Geoid Contours on the Australian National Spheroid in a Best Fitting Position in Meters

32nd parallel where $M_{i,\,32} = P_{i,\,32} = N_{i,\,32}$. Along the 134th meridian $M = P$ also, but most of these points do not lie on the fixed framework and thus will take a correction. They are plotted in green as are all other intersection points which are expected to receive a correction.

Meridional and parallel profiles to be included in the adjustment are terminated by two red points; they contain no other red points, but at least one green point. In a preliminary adjustment the difference in geoidal height at the endpoints of such a profile as computed from the fixed N values was compared to the difference computed along the profile, and the discrepancy distributed evenly among the green stations. All of these points were thus given a new set of M and P values which entered into the final overall adjustment. There were 1,127 equations representing 1° sections with at least one green endpoint. They contained 435 unknowns; 319 values were held fixed. The adjustment was programmed by Mr Philip Wyatt and carried out on the Honeywell-800 computer.

The final green values were plotted together with the fixed red values and contoured. A chart with the given deflections as vectors helped to check and guide the direction of the contour lines. The detailed geoid in the Woomera area, computed by A. G. Bomford[5] relative to station 60 for the Clarke 1858 ellipsoid Sydney origin was incorporated in its general slope after changing its reference to the Australian Geodetic Datum.

New Guinea and the adjacent islands.—The geoid contours in New Guinea were computed by loops of ΔH calculations and distribution of misclosures. The natural geoidal connection to the Australian mainland runs through the Cape York peninsula; a strengthening of that profile, preferably in a loop around the shore, would be desirable.

In New Britain and New Ireland the deflections given on the Australian Geodetic Datum were augmented for more detail by the deflections on the local Matupi Datum obtained in the shore-ship survey of 1955.

The general geoid features.—As mentioned earlier, the reference ellipsoid had been positioned in a favourable way so that the general good fit comes as no surprise. The geoidal contour lines have a gentle random character, with slightly higher values in the east and west than in the centre and north. Minimizing the geoidal separations for an optimum fit across the mainland, holding the zero separation at Johnston by definition, would lead to an insignificant tilt, expressed as a change of deflections at Johnston by —″·06 in the meridian and —″·56 in the prime vertical, for the convention of north and east positive. From a practical viewpoint there would be no advantage in such a change. Figure 2 shows the geoid contour lines after that tilt, and their extension to New Guinea and the Bismarck Archipelago, which were not included in the determination of that tilt. The general feature stands out more clearly: higher values in the east and west, and a lower region in the centre. However, a few more geoidal sections, especially a meridional section in the centre of the country, would be desirable.

The tremendous geoidal rise of around a hundred meters from southwest to northeast shown by the various satellite charts cannot possibly appear on a nearly "best-fitting" reference surface. A verification by other methods than satellite dynamics would be highly desirable. Geometric satellite connections to Eurasia will help. Also the completion of the terrestrial free-air gravity anomaly chart of Australia will be quite useful, particularly if gravity coverage in the surrounding waters and islands should become available.

REFERENCES

(1) Australian Commonwealth Gazette, No. 84, 6th October, 1966.

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(4) I. Fischer: A Revision of the Geoid Map of North America, J. Geoph. Res. Vol. 71, No. 20, Oct., 1966.

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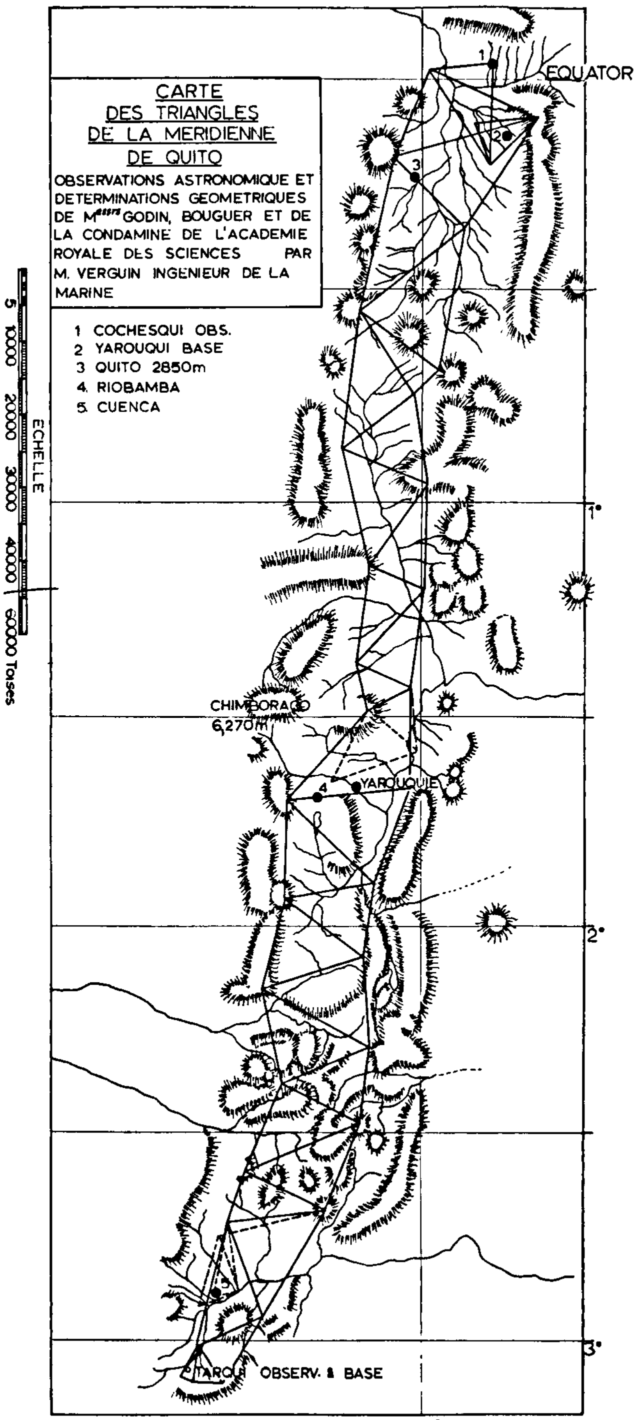

French arc in Peru
1735–1742

FIG 3.

*FROM BOUGUER — FIGURE DE LA TERRE PARIS, 1749.