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AIR SURVEY COMMITTEE—No. 5.

CALIBRATION
OF
SURVEYING CAMERAS

BY

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THE WAR OFFICE.

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CHAPTER I.

THE CALIBRATION CONSTANTS.

1. *PRINCIPAL POINT AND DISTANCE.*

For surveying purposes, the ideal photograph is a true central perspective. A single point may be found, in relation to such a photograph, at which any two images on the photograph will subtend an angle equal to the corresponding angle from the camera station during exposure. This point, from which the photograph appears *angle-true* to the original landscape, is known as the *internal perspective centre*. The importance of this point to the surveyor needs no comment.

The position of the perspective centre may be defined in a variety of ways. It will be fixed in position, for instance, by specifying its distances or subtended angles from three* points on the photograph. The most *convenient* method is to suppose a perpendicular dropped from the perspective centre to the plane of the photograph. The perspective centre will then be located by specifying the position of the foot of this perpendicular, known as the *principal point*, and the length of the perpendicular, known as the *principal distance*, intercepted between the perspective centre and the plane of the photograph. The perpendicular itself is known as the *plate perpendicular*.

If the position of the principal point is referred to certain *collimating marks* rigidly connected with the camera lens through the camera body and forming images on the photographic plate or film, and if the plane of the photograph is made to occupy a constant position relative to the camera lens, then it is clear that the principal point and distance will remain constant for a whole series of photographs taken with a particular camera. Measurements defining the principal point and distance of a photograph are consequently functions of a particular camera and the process of determining them may be described as *calibration of the camera*. The first essential in the design of a surveying camera is to ensure constancy of the principal point and distance of a series of photographs exposed in it. We shall see later that this is not an easy matter under the very different conditions of calibration on the ground and exposure in the air.

It should be noted that the perspective centre, and the calibration constants dependant upon it, are defined *geometrically* in terms of the angle-true properties of the perspective centre, and not in terms of the optical properties of the camera lens. To avoid misconception, let us investigate this point a little further.

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* Actually, four are necessary to avoid all ambiguity.

Lying on the axis of any centred optical system, whose performance is supposed for the moment to be perfect, there are two *nodal points* which possess angle-true properties ; angles from the *front node* to various objects in the landscape are equal to the corresponding angles from the *rear node* to images on the photograph. Consequently, the rear node is the internal perspective centre and the front node is the "camera station" or external perspective centre. Camera objectives of the type usually fitted to air cameras are very nearly symmetrical, and in this case the two nodes will be close to each other and to the centre of the lens.

The plane of the photograph is normally set as nearly perpendicular to the optical axis of the lens as possible. Consequently, the latter is an approximation to the plate perpendicular. The point in which the optical axis cuts the plane of the photograph is known as the *optical centre* and, within the same limits, is clearly akin to the principal point.

The *focal length* of the lens for light of a particular wave length is defined as the intercept on the optical axis between the rear node and the point at which a sharp image of a distant object is formed. Since the camera must be focussed for distant objects, the focal length of the lens will approximate to the principal distance of the camera.

The terms perspective centre, principal point and principal distance are thus closely allied to rear node, optical centre and focal length respectively, but the two sets of terms must not be confused. Focal length is at best an indefinite expression ; in practice, the nodal points do not altogether possess angle-true properties, and unless the centreing of the various components of the lens is perfect, even the term "optical axis" may lose much of its significance. The surveyor is interested directly in the measurement of angles as from the camera station to the original landscape, and he should define and determine his calibration constants accordingly, without any intermediate reference to indefinite terms not directly connected with the problem.

2. *THE EFFECT OF LENS DISTORTION.*

So far we have considered the case of an ideal photograph possessing an unique perspective centre. Practically, however, all photographic images are subject to systematic displacements from their true perspective positions, which are primarily due to optical distortion introduced by the lens system. What effect have these displacements on the position of the perspective centre, and on the derived calibration constants ?

Systematic lens distortion is radial from the optical centre and is a function of the distance of the image considered from the optical centre. The optical centre will always be close to the principal point and the position of the latter will consequently be unaffected. For all practical purposes, the effect is confined to a variation in principal distance.

Suppose that o (Fig. 1) is an image point, subject to a distortion oo^1, so that the image actually appears at o^1. The true perspective centre is at C, and the principal point is at P. In practice, the displacement oo^1 is always so small that it may be considered radial from P

instead of from the near-by optical centre. Draw o^1C^1 parallel to oC to intersect the plate perpendicular in C^1. Then it is clear that the immediate neighbourhood of o^1 will appear angle-true from C^1, which is accordingly the perspective centre for this region. The principal distance for a small area of the photograph surrounding o^1 is

$$PC^1 = \frac{Po^1}{Po} \cdot PC$$

whence the ratio of the local principal distance to the true principal distance is

$$\frac{f^1}{f} = \frac{Po^1}{Po}$$

From this we obtain the relation

$$\frac{f^1 - f}{f} = \frac{oo^1}{Po}$$

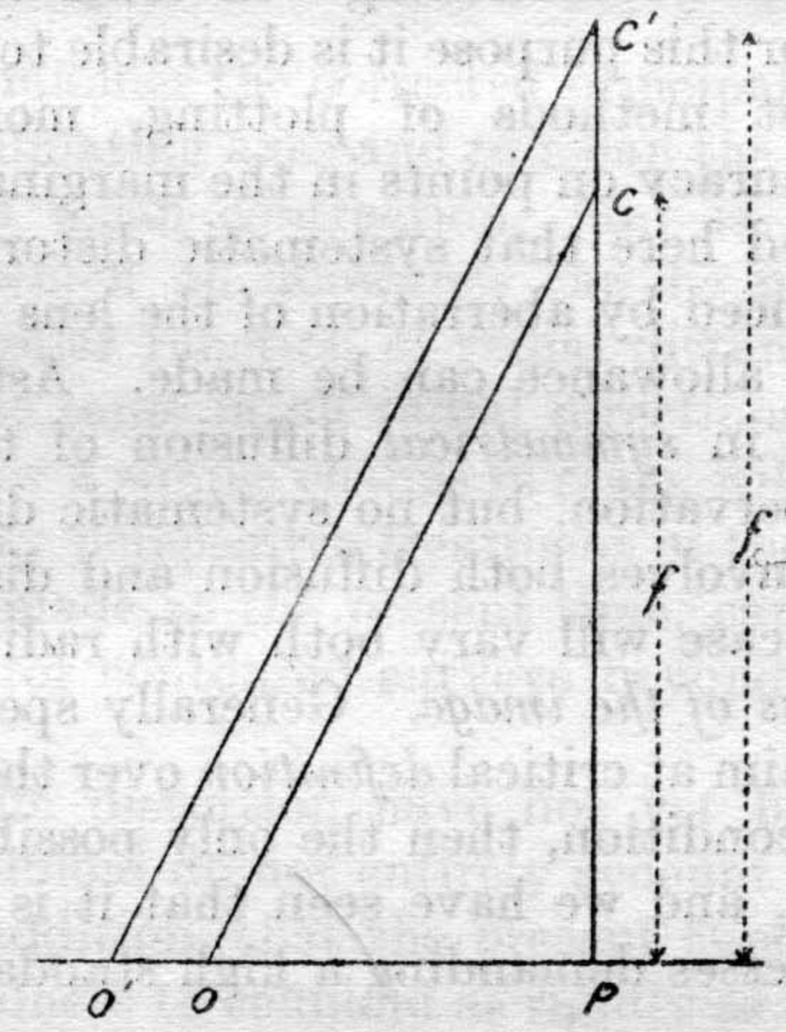

FIG. 1.

The principal distance is thus increased (or decreased) in direct proportion to the ratio of the distortion in a particular region to the radial distance of this region from the principal point. If this ratio is constant over the whole area of the photograph, being due to a simple change in scale, then the principal distance would also remain constant over the whole area.

In the case of systematic lens distortion, the ratio of distortion to radial distance is not usually constant, but, since the distortion may be considered as some function of the radial distance, it will be sensibly constant for a particular circular *zone* of points at an approximately

equal distance from the principal point.* The principal distance is thus completely defined if we specify the particular zone to which it refers. A zone is usually specified by stating its *obliquity*, or the angle o^1C^1P. (Fig. 1.)

If we assume that the principal distance for a particular zone applies to the whole photograph, then other zones will not generally appear angle-true. A particular zone might be chosen to give the best average result, in exactly the same way as the surveyor will often use a "scale-factor" to decrease the maximum scale error of a map projection. This case seldom arises in practice, however, since a *precise* knowledge of principal distance will only be required where some form of plotting machine is to be used. In almost all such machines the photograph is reprojected through a similar and similarly placed lens to the camera lens, which will accordingly re-introduce the same distortion in the reverse sense. In this case it is merely necessary to ensure that the principal distances of the camera and of the projector are the same *for the same zone*, in order that the bundle of rays emerging from the projector lens shall be angle-true. In any case, the ruling factor will be the necessity for good "conditioning" in order to arrive at the best determination, and for this purpose it is desirable to choose a zone in the marginal field. Most methods of plotting, moreover, require the greatest degree of accuracy on points in the marginal field.

It should be noted here that systematic distortion is not the only error which is introduced by aberration of the lens system, but it is the only error for which allowance can be made. Astigmatism and field-curvature will result in *symmetrical* diffusion of the image, involving possible errors of observation, but no systematic displacement. Coma, on the other hand, involves both diffusion and displacement, and the displacement in this case will vary both with radial distance *and with the degree of brightness of the image*. Generally speaking, the design of a survey lens should aim at critical *definition* over the whole field. If the lens fulfills this one condition, then the only possible source of error is systematic distortion, and we have seen that it is a simple matter to allow for this in processes demanding a high standard of precision.

3. *OTHER FACTORS AFFECTING THE CALIBRATION CONSTANTS.*

Distortion of Photographic Materials.

The photographic image is formed on a sensitised emulsion carried by gelatine and based on a glass plate or a celluloid film. Between exposure and utilisation of the developed photograph certain changes may occur in the emulsion, or in the base, or in the relation between the two, which will give rise to changes of relative position and hence to perspective errors.

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* In the case of badly centred lenses this statement is not strictly true; distortion will vary with the particular radial direction. But in view of other sources of error to which air photographs are subject, it is usually a sufficiently close approximation.

Apart from the effects of temperature, which will be considered later, errors of this nature may be considered entirely negligible in the case of glass plates. Film, however, is subject to quite considerable distortion during the process of development and drying, and this distortion will have an appreciable effect on the calibration constants.

Firstly, the film is subject to a mean contraction of the order of $\frac{1}{4}$ per cent., the exact amount of which depends on such factors as the past history of the celluloid base and relative humidity between exposure and plotting. It cannot be estimated with any certainty, and will vary subsequently with age and weather conditions. We have seen in the last article that a mean contraction of this nature can easily be allowed for by a proportional reduction in principal distance. Suppose, for example, that the true distance between the tips of opposite collimating marks in the camera, or as measured on a glass plate exposed in the camera, is D and that the principal distance obtained by measurement of a glass calibration plate is f. Then the corrected principal distance for a film, on which the corresponding measured distance is D^1, will be $\frac{D^1}{D} f$. For the reasons given in the next paragraph it will be advisable to determine in this manner the corrected principal distance by measurement along both collimating axes and to mean the results.

In addition to a mean contraction, film is subject to irregular displacements, for which no allowance can be made. The distortion is not, in fact, a regular function of radial distance from the principal point, and is not the same in all radial directions. Residual displacements, above or below a simple change in scale, will consequently appear as direct errors in any perspective treatment of the photograph, and for this reason film, as made at the present time, cannot be used for such precise purposes as for contoured surveys beyond the limits of ground control.

The causes of film distortion have not yet been fully established. The properties of celluloid do not entirely account for the trouble and it seems probable, in addition, that the present type of gelatine emulsion cannot be made to adhere to celluloid as rigidly as to glass.

The distortion of photographic printing paper during development and drying is of the same order as that of celluloid film, but rather more irregular. A pronounced stretch in one direction is introduced by the use of a squeegee, so that prints for survey purposes should be unglazed. They may be rough dried or washed in methylated spirit which may even be burnt off without appreciable increase in distortion.

Curvature of Commercial Plates.

We have so far spoken of the " plane " of the photograph, and defined the calibration constants in reference to this " plane." Commercial photographic plates are, however, seldom flat ; a straight edge placed across a plate selected at random will usually show a pronounced concavity on the emulsion side. Glass for the manufacture of ordinary plates is blown in the form of a " sausage " which is cut and opened out while

still hot. The curvature of the "sausage" does not entirely disappear when the sections are laid out to cool. The inside surface of the "sausage," being the smoother and the less liable to faults, is made to carry the emulsion, and this cause alone would account for the concavity mentioned above. In addition to the method of manufacture of the glass, the emulsion contracts slightly on drying and exerts a definite pull on the glass surface, resulting in further concavity. As a result, the concavity alters slightly after developing and drying.

The geometrical effect on the calibration constants of using a slightly curved plate is illustrated by Fig. 2. If, for the purpose of specifying principal point and distance, the equivalent plane of the photograph is assumed to be 1—1, then points such as 0 in the marginal field will be subject to perspective errors, since the plane of the photograph for such points should be clearly taken as 2—2. If, on the other hand, 2—2 is assumed, then the central field will be in error. It will be seen, however, that a change in principal distance, tantamount to such a change in the

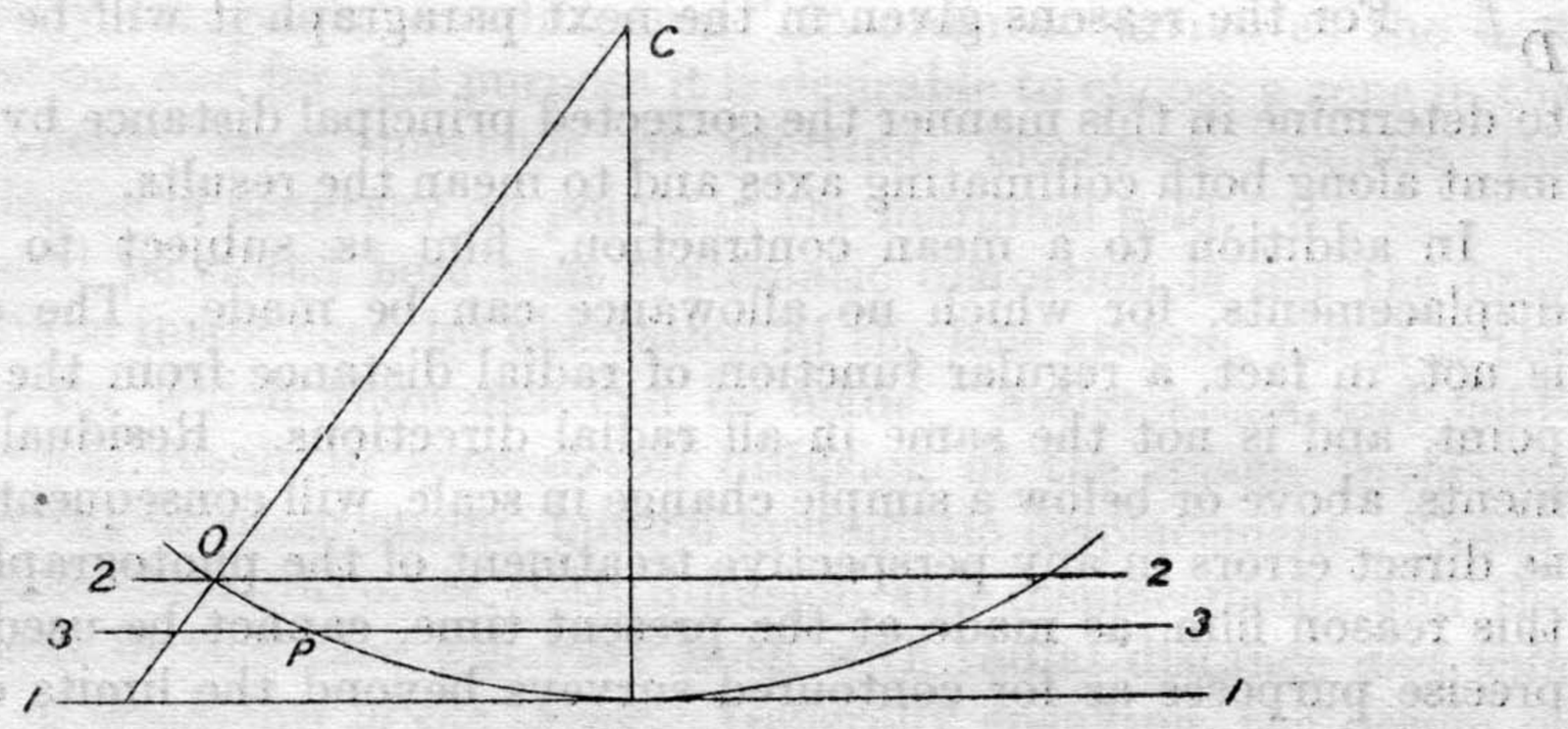

FIG. 2.

choice of an equivalent plane for the curved photograph, affects the angular subtense of points in the central field less than that of points in the marginal field. As in the case of lens distortion, an average approximation may be obtained by choosing a mean plane, such as 3—3, and this will be done automatically during calibration of the camera by observing on such images as P. Where the plate is replaced for plotting in a projection camera similar to the air camera, and may be held to have retained the same curvature, there will be no residual error provided that, as in the case of lens distortion, the principal distance is calibrated and set for the same obliquity. This statement assumes that the curvature due to emulsion tension does not alter appreciably during developing and drying and that the effect of curvature is equal on all points at the same obliquity. Neither of these assumptions is entirely correct, but they are reasonable approximations to make in practice. Some such reprojection process will, in all probability, be used where a high order of precision is required.

The effects of curvature may be entirely eliminated, should it prove necessary to do so, by using flat glass (such as patent plate) in the first instance and by adding an unsensitised "emulsion" on the back of the plate in order to equilibrate emulsion tension.

Pressure Plate Refraction.

In the Service F.8 camera, or "Eagle" camera, the celluloid film is pressed flat against a plate-glass "pressure plate" by a spring pressure pad. Under these circumstances, the film may be considered as lying flat at the moment of exposure and the source of error discussed in the last paragraph is consequently not operative. A further source of error is, however, introduced by refraction at the surface of the glass pressure plate.

If, for instance, C in Fig. 3 is the true perspective centre, giving rise to a true principal distance CP, then an image-forming ray CN should

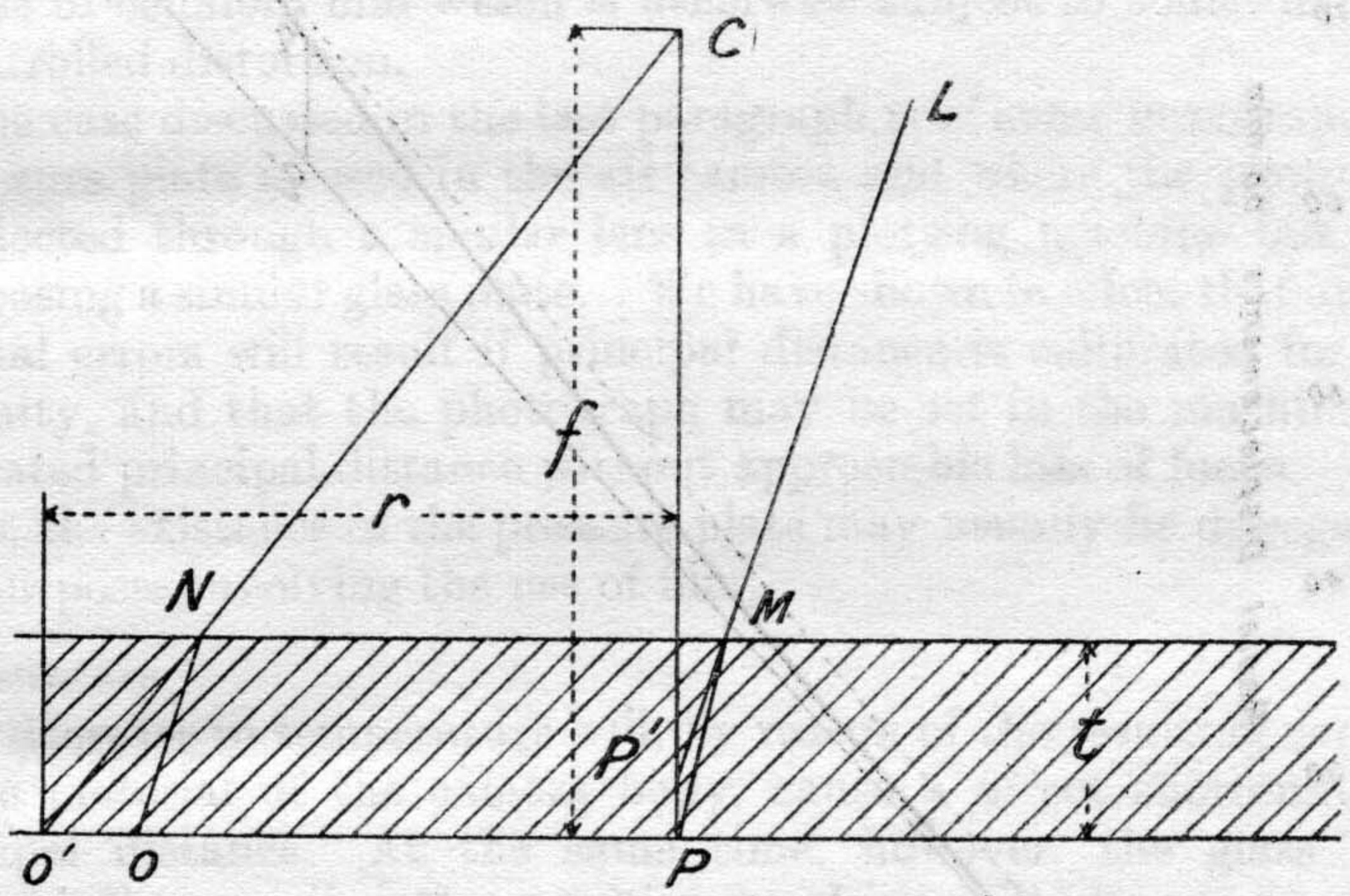

FIG. 3.

produce an image at o^1. The ray CN is, however, bent at the surface of the pressure plate nearer to the normal to this surface, and actually forms an image at o.

If μ is the refractive index of the glass, it can be shown that the amount of the displacement oo^1 is given by the expression—

$$t \cdot r\left(\frac{1}{f} - \frac{1}{\sqrt{\mu^2 (r^2 + f^2) - r^2}}\right)$$

where the remaining symbols refer to quantities marked on Fig. 3. For representative values of the constants μ, t and f, the amount of the displacement for different values of r is shown by the graph in Fig. 4. This graph is very nearly a straight line, showing that the ratio $oo^1 : r$ is very nearly constant, and we have already seen that a displacement of this nature is equivalent to a change in principal distance. Calibration by observation on points at an obliquity represented by O (Fig. 4) will automatically determine the adjusted principal distance corresponding

to the straight line *PO*. The use of this adjusted principal distance will involve residual errors represented by the difference in ordinates between the straight line and the curve, and these residual displacements, amounting in an average case to no more than 0·04 mm., may often be neglected in view of the larger sources of error inherent in the use of celluloid film.

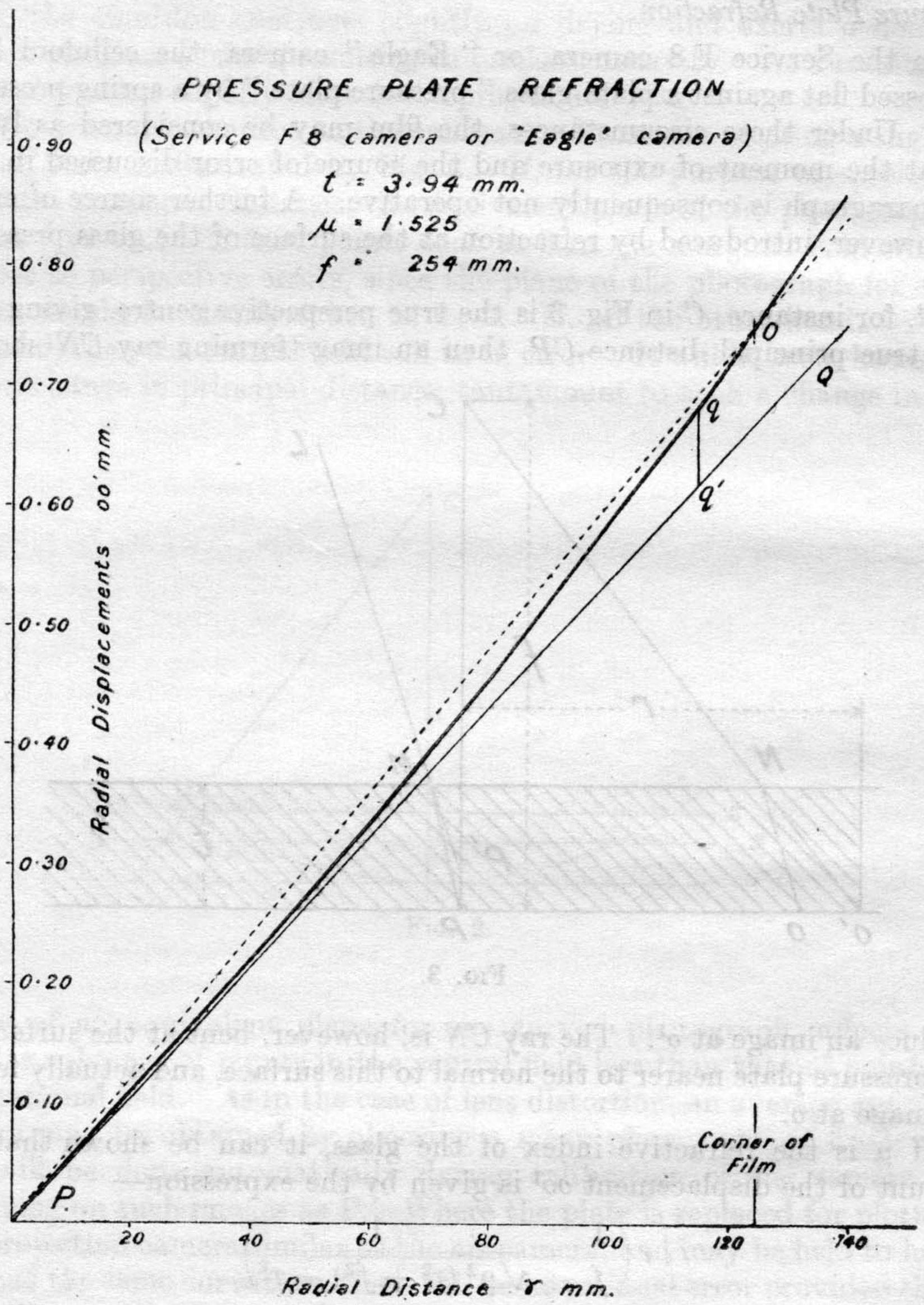

Fig. 4.

It is of interest to consider what the principal distance would be if the glass pressure plate were removed. Rays emitted from a distant object corresponding to an image at *P* (Fig. 3) are refracted through the outer zones of the lens into the direction *LM*. These rays are also refracted at the surface of the glass plate and form an image in sharp focus at *P*. If the glass plate were removed the image would be formed

in sharp focus at P^1 and it would thus be necessary, in order to preserve focus, to shorten the true principal distance by PP^1. It can be shown that—

$$PP^1 = t\,\frac{(\mu - 1)}{\mu}$$

A shortening of the principal distance by this amount is illustrated by the straight line PQ in Fig. 4, and it is apparent that the introduction of a pressure plate merely introduces residual displacements of qq^1, when due allowance is made for the change in principal distance required to preserve focus. Mathematically, the residual displacement qq^1 is roughly proportional to the cube of the tangent of the angle of obliquity, and as this is the form of displacement introduced by primary lens distortion, it could be corrected in the design of the lens. Actually, the maximum value of qq^1 in a representative case is 0·08 mm., and this may often be neglected in view of the fact that the whole question only arises from the use of celluloid film which is otherwise subject to somewhat greater uncontrolled distortion.

The case discussed in the last paragraph is of some importance where a pressure plate is used in the air camera and where the photograph is reprojected through a similar lens in a plotting machine but without interposing a similar glass plate. We have shown in effect that only small residual errors will result if principal distance is calibrated for a mean obliquity, and that the photograph may be set in the machine at this calibrated principal distance without appreciable loss of focus. In other words, the existence of the pressure plate may usually be disregarded for any purposes involving the use of film.

Temperature.

A decrease in temperature will, by reason of the resulting contraction in the material of the camera body, cause a direct shortening of the principal distance. At the same time, however, the glass plate or celluloid film carrying the sensitive emulsion will also contract, and if the degree of contraction were the same for both plate and camera body it is apparent that angles subtended at the perspective centre under different conditions of temperature would remain unaltered. In this case, we could calibrate the camera at a relatively high temperature on the ground, expose at a low temperature in the air and yet use the calibrated principal distance for plotting at the ground temperature.

Actually, the co-efficients of expansion of the metallic camera body and the glass photographic plate or celluloid film are not the same, and we are not justified in assuming that the calibrated principal distance holds good at the low temperature of photography and remains good at the high temperature of plotting.

Suppose that—

t = Calibration Temperature.
t_1 = Exposure Temperature.
t_3 = Plotting Temperature.
and α_1 = Co-efficient of Expansion of Camera Body.
α_2 = Co-efficient of Expansion of Glass.

The proportional decrease in principal distance due to contraction of the camera body will be $\alpha_1 (t_1 - t_2)$. Between exposure and plotting the plate *expands* in the proportion $\alpha_2 (t_3 - t_2)$ and this will occasion a proportional *increase* in principal distance of the same amount, in order that the angle-true properties of the perspective centre may be preserved. Consequently the nett proportional increase in principal distance between calibration and plotting is—

$$\alpha_2 (t_3 - t_2) - \alpha_1 (t_1 - t_2)$$

To get a clearer idea of the effect, let us take a numerical example. The camera is constructed of an aluminium alloy, B.E.S.A. Specification No. 3L. 11, whose co-efficient of expansion is ·000022 per °C. The co-efficient of expansion of glass is ·000009. To choose quite possible values suppose that the temperatures of calibration, exposure and plotting are respectively 15° C., 0° C. and 20° C. Then the proportional correction to be applied to the calibrated principal distance is an increase of—

$$\cdot 000009 \times (20 - 0) - \cdot 000022 (15 - 0)$$
$$= - \cdot 00015.$$

If the calibrated principal distance were 165·00 mm., then the corrected principal distance would be—

$$165 \cdot 00 - 165 \times \cdot 00015$$
$$= 164 \cdot 98 \text{ mm.}$$

Under ordinary conditions the correction is consequently very small, but it may prove desirable to apply it in precise work outside the limits of ground control. Even then, however, only very rough indications of temperature would be required.

The variation of principal distance arising from expansion is probably the greatest source of temperature error, but it is not necessarily the only one. It is likely, for instance, that a further displacement may be imparted to the perspective centre by the effect of temperature changes on the lens, particularly as regards an alteration in the separation of its components.

Shutter Distortion.

The slit of a focal plane shutter takes an appreciable time—about one-tenth second—to travel across the plate or film, with the result that bands of the photograph are exposed at different times and consequently from different air stations. If the slit moves in the direction of flight, the actual effect is an elongation or compression of the photographic image in the direction of travel of the slit, depending on whether the slit is travelling in the same direction as the aeroplane or in the opposite direction. If, on the other hand, the slit travels at right angles to the direction of flight, a square on the ground would appear roughly as a rhombus on the photograph. The result in the first case would be a simple change of scale in one direction and in the second, an angular distortion unaccompanied by change of scale, if the speed of the slit

were uniform. This, however, is not the case; a certain amount of acceleration is unavoidable.

Further complications are introduced by varying tilts and yaws during the whole time of exposure, and in practice, it becomes impossible to estimate with any certainty the effect of this source of error. That it has an effect on the applicability of a principal distance determined by calibration on the ground is sufficiently obvious.

For this and other reasons, focal plane shutters are rapidly dying for purposes of precise survey.

Faults in Camera Construction.

The permanence of the calibration constants between calibration and successive exposures in the air should always receive attention in the design of surveying cameras. This implies at the outset a rigid construction for the camera body, in order that the lens may remain in a constant relation to the focal register under severe conditions of vibration and rough handling. The fewer removable or adjustable parts there are between the lens and focal register the better, and, for this reason, the focal register should form part of the camera body and not part of the magazine.

The plate or film must lie firmly in contact with the focal register during exposure. It will not meet the case to rely solely on a gravity feed for this purpose; the plate or film must be definitely pushed into contact with the focal register and held there mechanically while the exposure is made.

Collimating marks, constituting a reference system for the indication of the principal point, should form an integral part of, or be rigidly attached to, the camera body. They may not be carried in the magazine and so be subject to varying displacements, depending on the fit of the magazine on the camera body.

4. *CHOICE OF POINTS FOR OBSERVATION.*

Various sources of systematic error in the photograph have been discussed in the preceding Article. Broadly speaking, we have seen that these errors are negligible in the neighbourhood of the principal point and that they are roughly constant for a particular obliquity. Consequently, if we confine observation to points situated in the centre of the photograph and to points at an approximately equal distance from the centre, we shall determine the calibration constants for rays at an obliquity corresponding to this *zone* of the photograph. The use of calibration constants determined in this manner ensures that angles subtended at the perspective centre between points in this zone or in the centre of the photograph will be correct. Angles subtended by points in other zones will not be correct, unless the photograph is treated by some process of perspective reprojection, but they will be determined with a reasonable approximation. If, on the other hand, calibration is effected by observation on points in very different zones, then the

determination will be effected by relatively large systematic errors, and as a result a perspective centre may be obtained which gives rise to disproportionally large errors in angles subtended by other points.

Various attempts have been made to allow for the effect of systematic distortion during calibration by expressing the distortion in terms of the radius of the zone. Mr. Fourcade,* for instance, has tested a photographic lens with extreme care and finds that its distortion may be expressed to a very high degree of accuracy in terms of the second and fourth powers of the radius. There is, however, no justification for assuming that this rule holds in all cases. On the contrary, theoretical considerations would require an expression involving the cube and ascending *odd* powers of the radius, and the fact that an equivalent empirical formula can be designed to fit a particular case does not warrant the universal adoption of that formula. The objection to using the theoretical series itself is that the coefficients of the higher powers are not necessarily small, and that several terms of the series would consequently need to be included. Short of independent measurements on the particular lens, in fact, there appears to be no fully practical method of making a numerical allowance for distortion during the process of calibration.

In all the methods described in this paper it will be assumed that systematic distortion is constant in a particular zone, and that it varies from zone to zone without discontinuity. No assumptions will be made regarding the absolute *amount* of distortion present, and for this reason the determination will be confined to the centre and to one zone of the photographic field.

5. *CALIBRATION OF MULTI-LENS CAMERAS.*

For instrumental air surveys it will be usual, in place of a camera exposing a single photograph, to use a multi-lens camera, consisting of two or more complete camera units, whose axes are at a fixed inclination to one another. The purpose of such multiple cameras is solely to extend the field photographed from a single air station. Before photographs taken in them can be used for plotting, it is necessary, in addition to the principal point and distance of each camera unit, to determine the angular position occupied by each plate in relation to its neighbours.

Suppose, for the sake of simplicity, that we are dealing with a bi-lens camera exposing two plates at the same time. Each perspective unit may be reconstructed, and each plate replaced accordingly in a suitable projection camera for plotting, from a knowledge of principal point and distance. In addition, we require to replace the two perspective units (each consisting of a plate and its appropriate perspective centre) in the same relation as they occupied at exposure, except that one complete perspective unit may be moved for any small distance parallel to itself.

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* See "The Principal Point and Principal Distance in Photogrammetry," by H. G. Fourcade, "Transactions of the Royal Society of South Africa," Vol. XVI. Part I, 1928, and previous papers by the same author in the same publication.

It is clear that where we are concerned solely with the photography of distant objects, such a small parallel displacement has no practical effect on the conditions of reconstruction.

In order that we may replace the two perspective units in the above manner, certain angular measurements must be made on the camera, which give rise to a set of *angular calibration constants*. For simplicity in the subsequent setting of a plotting machine these constants will take the form indicated in the next paragraph.

Designate the two perspective units A and B, and imagine a parallel to the plate perpendicular of B drawn through the perspective centre of A. Even if the two actual plate perpendiculars are skew, the plate perpendicular of A will now define a *plane* with such a parallel to that of B. The angle made by the trace of this plane on plate A with the line joining the principal point of A to a specified collimating point will be known as the *swing constant* of A. The swing constant of B is defined similarly. The angle enclosed by the two plate perpendiculars, or by intersecting parallels to them, is known as the *Tilt constant*.

There are thus three angular constants for a bi-lens camera, and a little consideration will show that these three constants are necessary and sufficient to re-establish the *angular* relation of the two perspective units. For a tri-lens camera we have three similar constants for each pair of perspective units, although the resulting nine constants will not all be different.

This branch of the subject will be better understood after reading Chapter V on methods of obtaining the angular constants.

CHAPTER III.

METHODS INVOLVING THE MEASUREMENT OF ANGLES ONLY.

13. *GENERAL PRINCIPLES.*

EVERY satisfactory method of calibration yet devised, at any rate as regards principal distance, depends on relating linear distances in the focal plane of the camera to the corresponding angles subtended at the perspective centre. We may, as in the last Chapter, measure both these distances and angles, or else we may start with either distances or angles already known and measure the other quantity. The methods discussed in the present Chapter start with known distances by placing an accurately graduated glass scale in the focal plane. Measurements are then made of the perspective angles between the graduations of this scale.

14. *DIRECT MEASUREMENT OF PERSPECTIVE ANGLES.*

To obtain a direct measure of perspective angles we might, for instance, clamp the camera rigidly on the graduated head of a dividing engine or other large divided circle, C (Fig. 12). The graduations of the scale in the focal plane (G for example), could then be observed through the camera objective O by means of a *fixed* telescope, T. Other graduations would then be brought on to the fixed line of collimation of the telescope by rotating the camera on its dividing head, and the resulting difference in perspective angles would be read off directly on the divided circle.

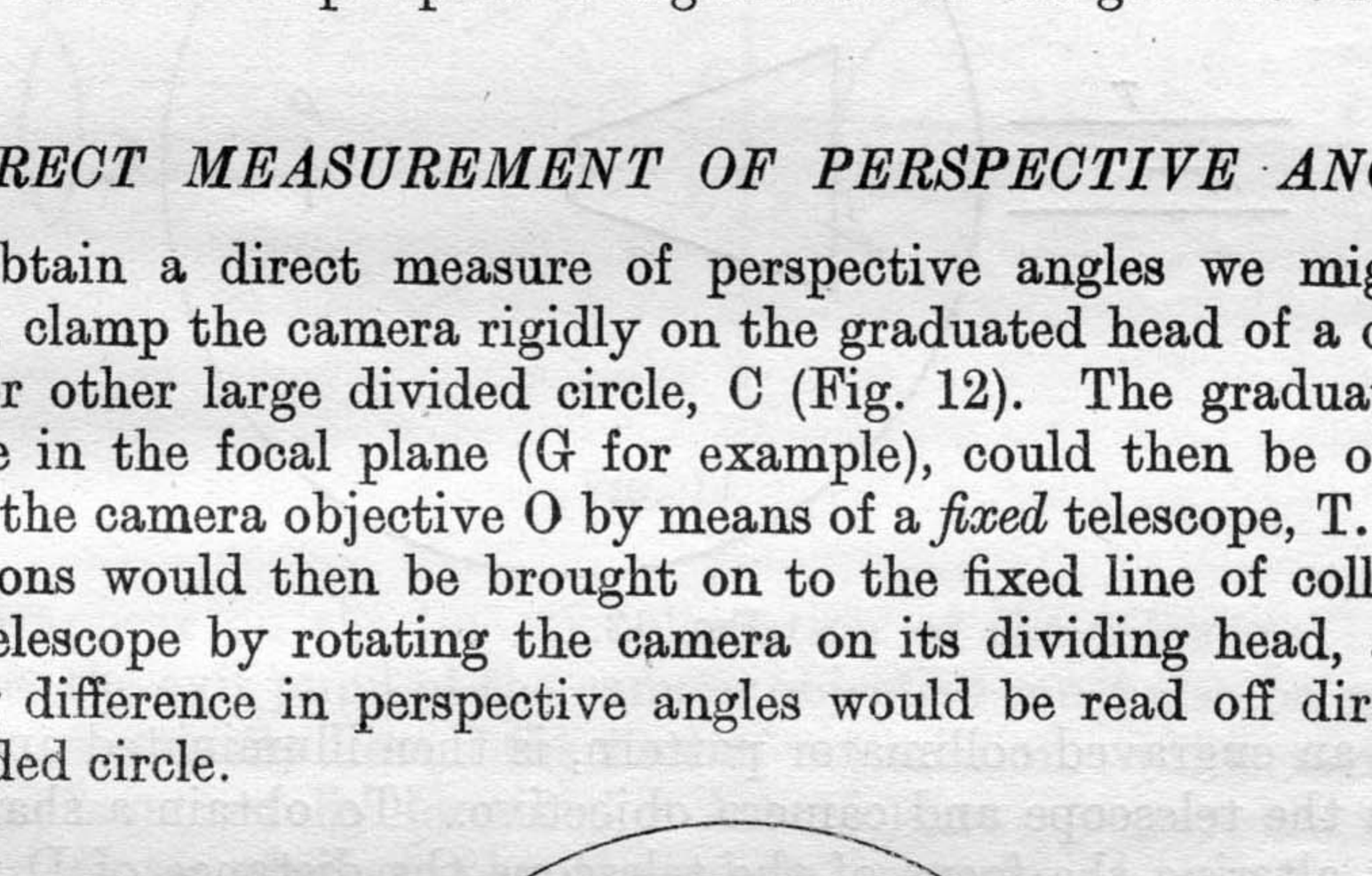

FIG. 12.

In order that successive graduations may be sighted, it would be necessary to place the camera objective O on the axis of rotation of the divided circle. A further condition is, however, desirable to ensure accuracy, in that the emergent beam B from the camera objective should be symmetrical in all cases about the line of collimation of the telescope. Otherwise errors may be introduced by the aberrations of the telescope system. This condition is fulfilled, firstly by placing the *front node* of the camera objective on the axis of rotation of the divided circle, and, secondly, by arranging the position of the telescope so that the exit pupil of the camera objective appears symmetrical about the line of collimation. The latter condition may be gauged with sufficient exactitude by eye, the actual exit pupil being seen in the telescope by focussing down.

The correct position of the front node is very easily determined by the usual method. Focus the telescope for infinity either by sighting a distant natural object or by focussing through the camera objective on to a mark in the focal plane of the camera. An object, D. (Fig. 13),

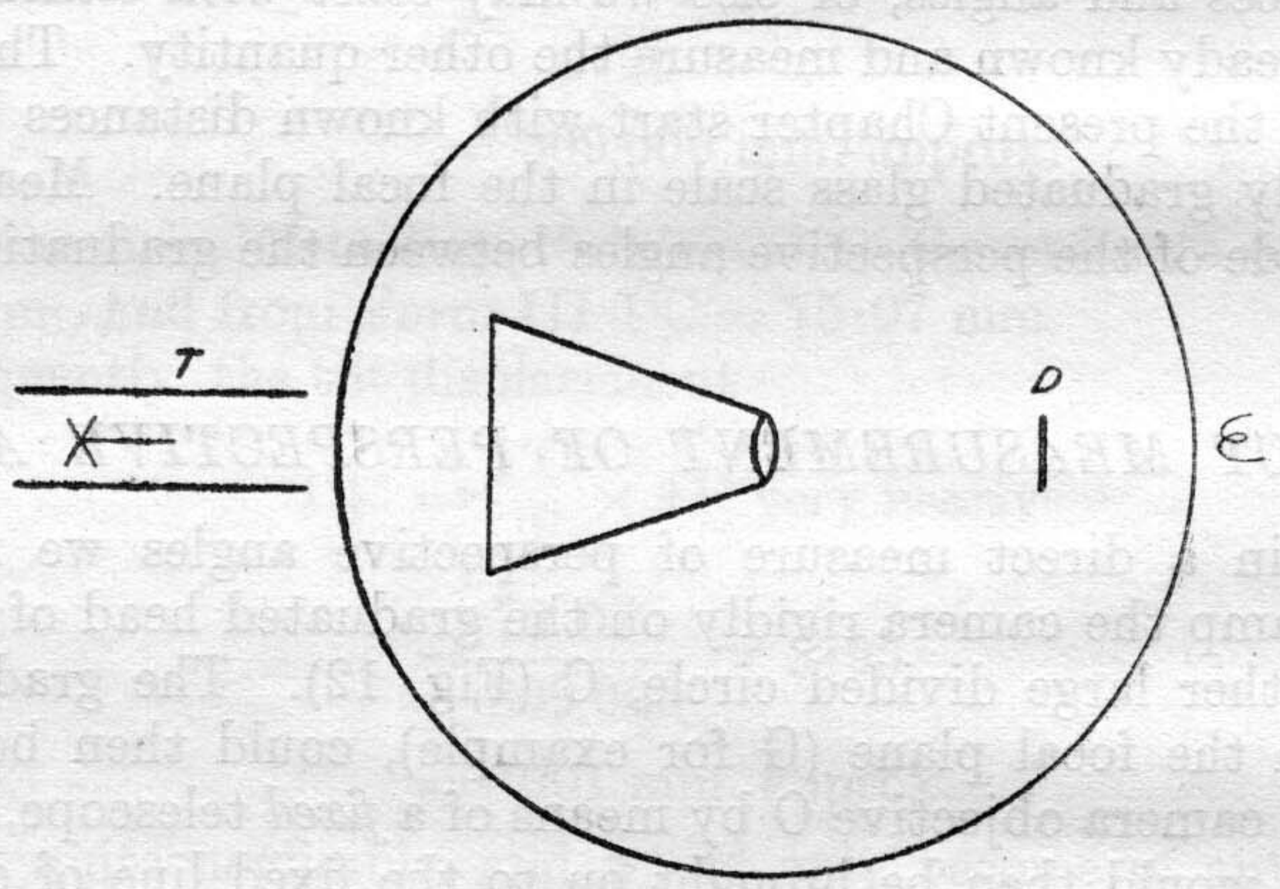

FIG. 13.

such as an engraved collimator pattern, is then illuminated and viewed through the telescope and camera objective. To obtain a sharp image without altering the focus of the telescope the distance of D from the camera objective may be varied, but thereafter D should be held fixed. If the front node is correctly centred on the axis of the divided circle a small rotation of the latter would not involve a shift of the image of D relative to the cross hairs of the telescope. Consequently the correct position of the front node will be secured by moving the camera by trial until this condition is fulfilled. It has been found that an eccentricity of one millimetre in the position of the front node of a 6-inch lens does not measurably affect the subsequent readings, so that no very great accuracy is required. If the position of the front node has been marked on the lens barrel by the manufacturers, or can be so marked from a drawing supplied by the manufacturers, then a plumb-line hung over the centre of the divided circle will, in fact, furnish a sufficiently exact means of setting up the camera.

There is yet another point which must receive attention in positioning the camera. The camera is preferably mounted on a stand furnished with three levelling screws. The known distances in the focal plane and the measured perspective angles must not only correspond, they must also refer to a particular collimating axis. This implies, firstly, that the scale must be so placed as to read along a collimating axis, and, secondly, that the collimating axis must be arranged to lie parallel to the surface of the dividing circle.

The most convenient form of scale consists of a glass plate engraved with two graduated medial lines crossing at right-angles in the centre of the plate. These medial lines should pass through three of the collimating marks, and for this purpose the plate-holder must be provided with slow-motion screws in order to furnish the necessary translation and rotation movements. (A fuller description is given in Art. 15.)

The collimating axis, R S (Fig. 12), is then brought into parallelism with the surface of the divided circle by rotating the latter in order to sight R and S in turn, and by turning the levelling screws until both

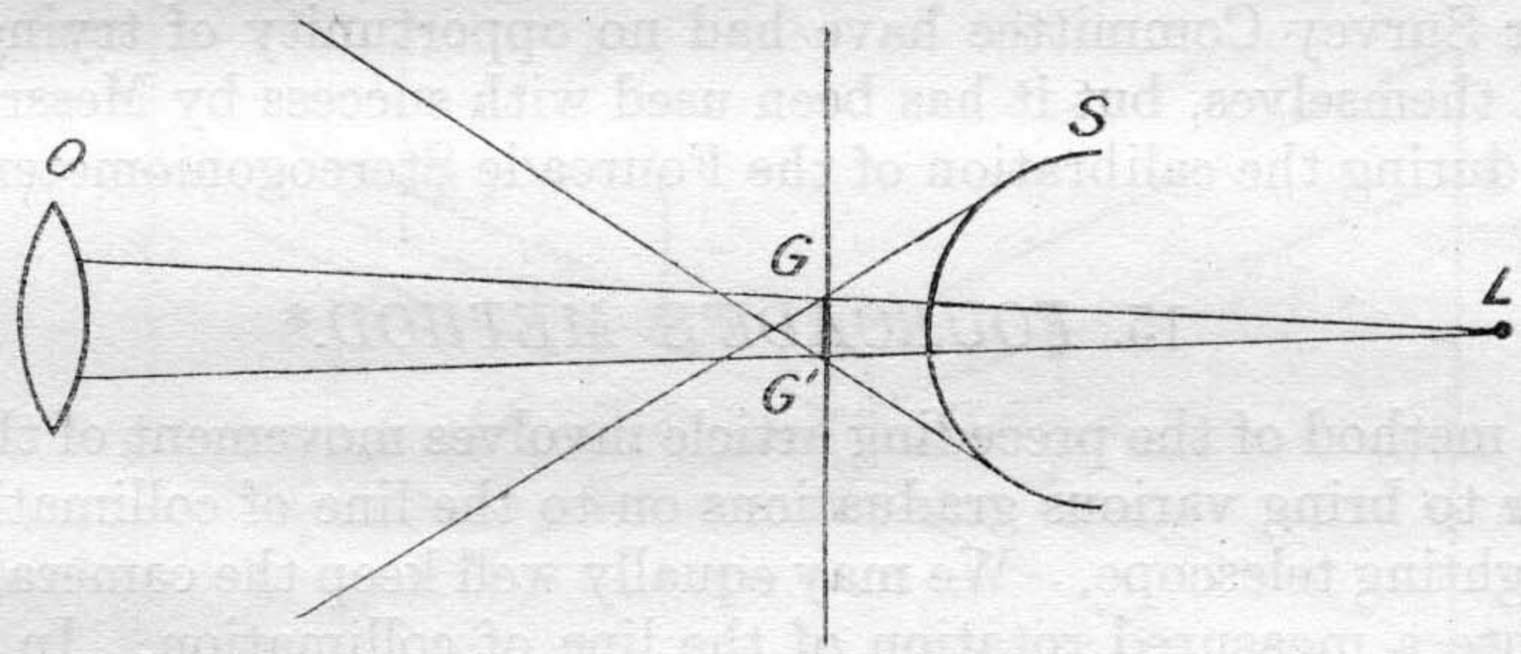

FIG. 14.

R and S appear on the line of collimation of the telescope. At the same time, the exit pupil of the camera objective must also be arranged to fall symmetrically about the line of collimation. (This method of adjustment is more fully described in Art. 15.)

In normal air photography the whole aperture of the objective is utilised, so that, theoretically speaking, the whole aperture should also be utilised during calibration. This implies that the diameter of the telescope object glass should at least equal the aperture of the camera objective. Unsymmetrical aberrations of the outer zones, which cause an apparent shift of the image during photography, will not otherwise receive consideration during calibration. The point need not, however, be laboured in dealing with the high-class lenses usually employed in precise camera construction. The same reasoning applies also to the choice of a suitable source of light for illuminating the scale graduations. If, for instance, a " point " source of light L (Fig. 14), is placed at a long distance from the graduation G G′, the incident beam L G G′ will not fill the aperture of the camera lens O. In these circumstances a fictitiously sharp image of the graduation G G′ will usually be formed, whose position

does not necessarily coincide with the mean centre of the more diffuse image obtained by utilising the whole aperture. The correct method of illumination is to use a large source of diffused light S, such as an opalescent globe, and to place this source of light as close as possible to the graduations.

Once the apparatus has been set up, angular readings are taken to each of the collimating marks R S, and to each of the intermediate scale graduations. The whole process is then repeated for graduations along the other collimating axis after turning the camera about its axis through 90° and re-adjusting the apparatus. Computation of the calibration constants may follow the methods of the last chapter, or, if the central graduation has been arranged as recommended to fall close to the centre of the field, the method of Art. 16 may be used with advantage.

The method described in this article is direct, and is likely to appeal to those manufacturers whose equipment includes a sufficiently large divided circle. It is unlikely that it will be used to any extent in Service workshops, and for this reason it has not been described in great detail. The Air Survey Committee have had no opportunity of trying out the method themselves, but it has been used with success by Messrs. Barr & Stroud during the calibration of the Fourcade Stereogoniometer.

15. *FOURCADE'S METHOD.**

The method of the preceding article involves movement of the camera in order to bring various graduations on to the line of collimation of the fixed sighting telescope. We may equally well keep the camera fixed and substitute a measured rotation of the line of collimation. In this case, an ordinary theodolite is used instead of a fixed telescope.

We have seen in the last article that the line of collimation must always pass close to the front node of the camera objective. A possible way of securing this condition would be, for the sake of illustration, to set up the theodolite afresh for observation on each graduation and to measure angles from some *distant* reference object.

Suppose, for instance, that the theodolite has been set up at T (Fig. 15), and sighted on the graduation G. By trial, the position and height of the theodolite have been chosen so that the exit pupil of the camera objective (seen as an illuminated "cat's eye" by focussing down the telescope), appears roughly symmetrical about the cross hairs of the telescope when G is sighted. Geometrically, the emergent beam from the camera objective consists of parallel light, and, subject to rough symmetricality of the exit pupil, the exact position of the theodolite

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* For further information see "The Principal Point and Principal Distance in Photogrammetry," by H. G. Fourcade, "Transactions of the Royal Society of South Africa," Vol. XVI, Part I (1928). This paper also contains an alternative method, using two pentagonal prisms, which has not been tried out by the Committee, and which is consequently not described in the present chapter. The paper also contains a valuable discussion on the general aspects of the whole question.

does not affect the result. The angle α between G and the distant reference object R is then measured. The theodolite is next set up in a similar manner at T′, and the angle β between R and the graduation G′ is measured. The difference $(\beta - \alpha)$ represents the perspective angle corresponding to the length of G G′. Before commencing readings the position of the camera must be so arranged that the collimating axis containing G G′ is roughly level, but R need not be at the same level. Apart from a graduated scale, this method needs no special apparatus and is quite capable of giving accurate results. It is, however, somewhat tedious.

To avoid moving the theodolite between successive observations, Fourcade introduces in the line of collimation a pentagonal prism P (Fig. 16), giving a constant deviation of approximately 90°. Movement of this prism enables different graduations of the glass plate to be sighted.

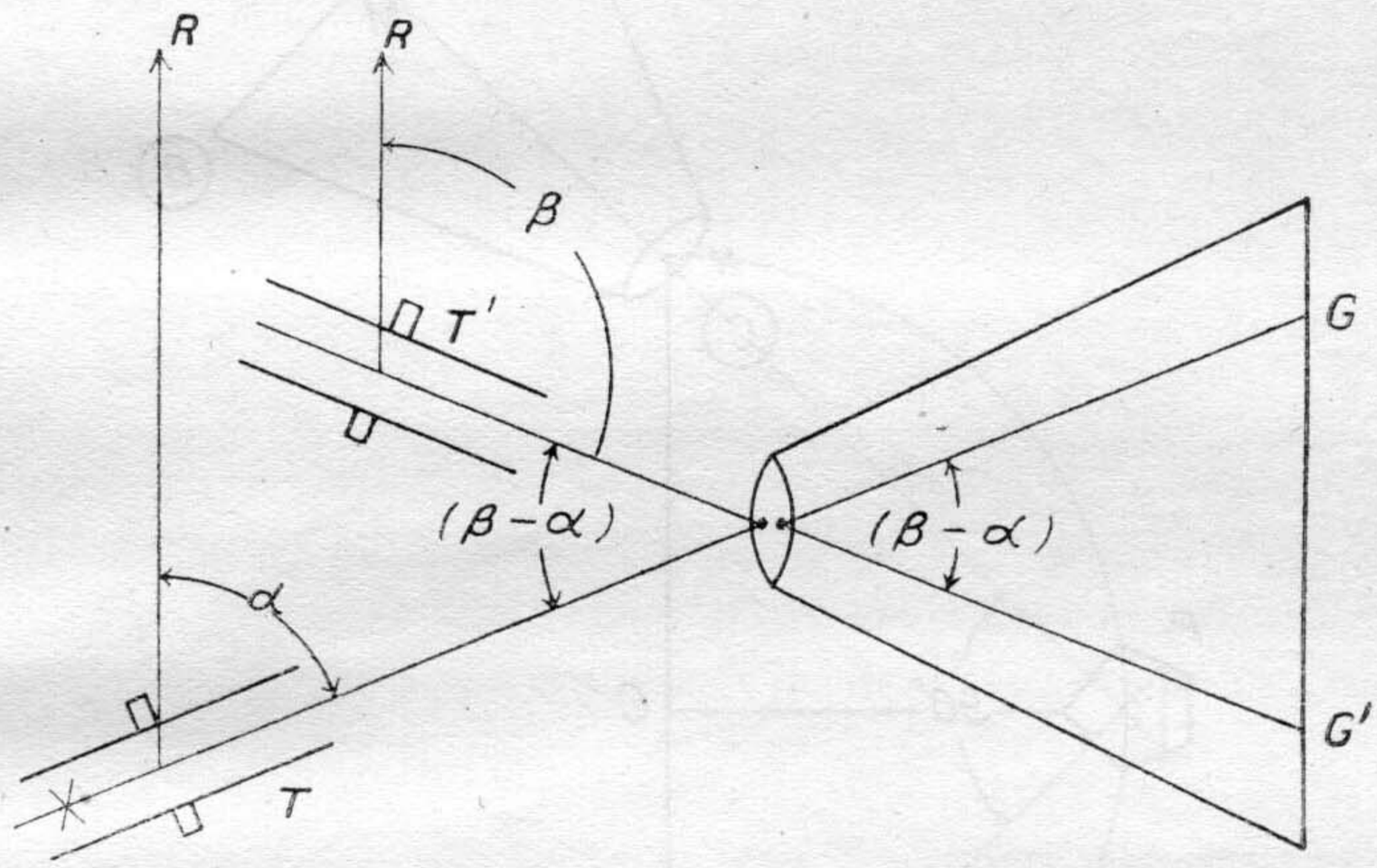


Fig. 15.

The line of collimation must in all cases pass through the front node N (Fig. 16), and the vertical axis X of the theodolite. Since the deviation of the prism is 90° the prism must accordingly move on a semi-circle passing through N and X and centred on C, the midpoint of NX. More exactly, the perpendiculars from N and X to the faces of the prism must intersect on this semi-circle, so that the prism must also be capable of rotation about an axis lying on the semi-circle. An exact set-up on these lines is unnecessary, and would, moreover, be difficult to arrange in practice, but the general principles of setting on the semi-circle are, however, followed. The prism is provided with adjustments which may finally be effected at each observation.

Arrangement of Apparatus.

For the trial of this method at the Royal Aircraft Establishment, Farnborough, the apparatus was disposed on a roughly levelled surface table. A pivot hole was drilled in the table at C (Fig. 16).

The camera was carried on a three-screw levelling table to which it could be rigidly attached. Two of the levelling screws were arranged to lie as nearly as possible in the focal plane, with the third screw under the camera axis. It is an advantage in the subsequent setting if the third screw lies roughly under the front node, but this usually requires a special table for a particular type of camera. The front node of the lens was marked by measurement on the outside of the camera body from drawings supplied.

The scale consisted of a piece of patent plate-glass cut to the size of photograph exposed in the camera (half-plate in the present instance),

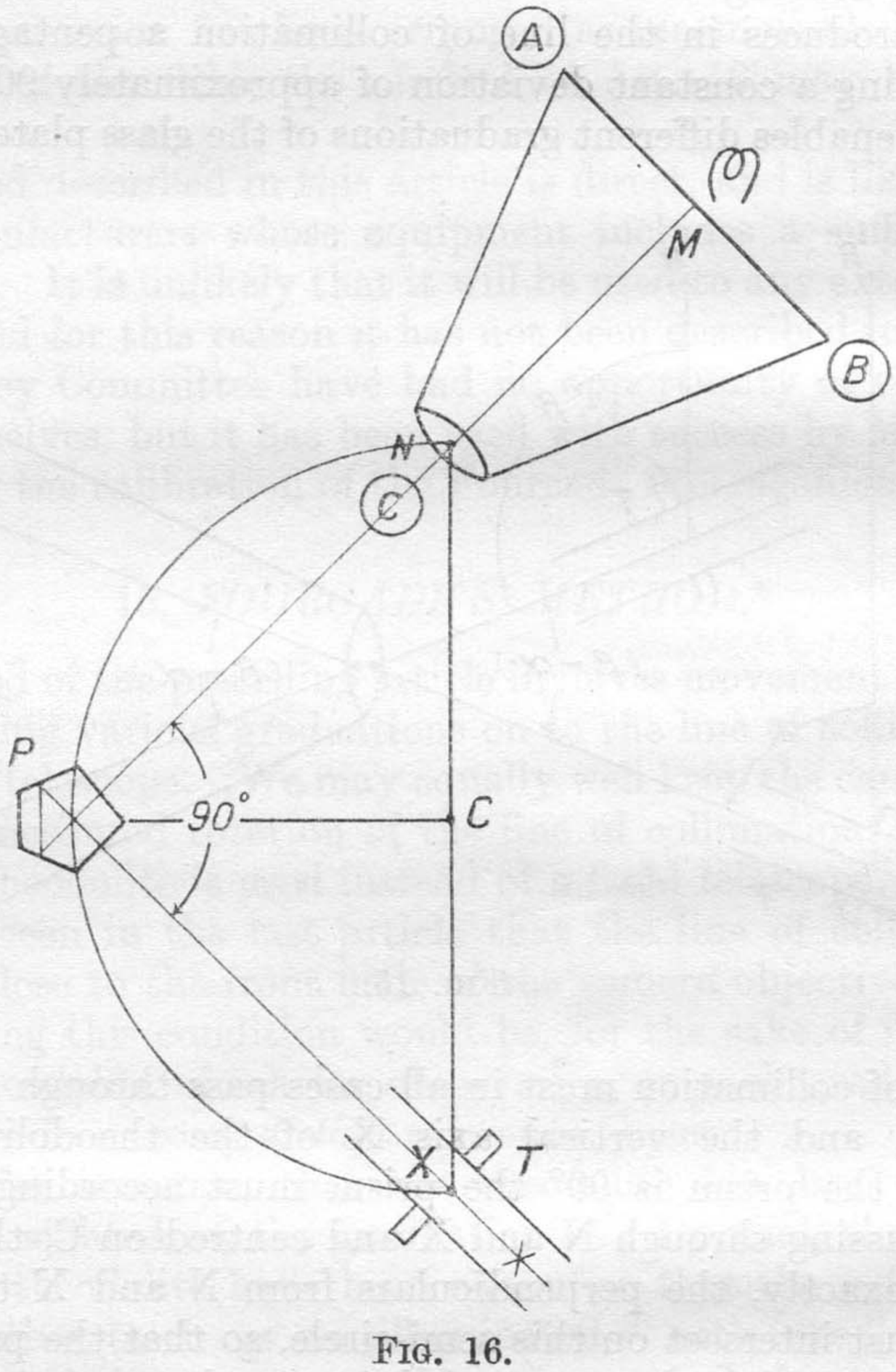


FIG. 16.

and provided with two engraved medial lines at right angles. Each medial line carried an accurately engraved centimetre scale centred on the intersection of the two medial lines. The finished plate was supplied by Messrs. Barr & Stroud.

The plate was carried in a specially constructed frame (Fig. 17), which could be attached to the camera body and which served to press the plate firmly against the focal register. Within the frame, the plate was held in position by springs working against three screws, A, A and B. (Fig. 17). Movement of the screws A and A in opposite senses rotates

the plate ; movement of the screws A and A in the same sense translates it, while the third screw B translates the plate in the right-angled direction. By means of these screws the medial lines of the plate were made to pass through three of the collimating marks, with the result that either scale lies along a collimating axis and the intersection of the medial lines (or the zero of the scales) lies close to the centre of the field. An ordinary magnifying glass was used to observe the collimating marks from the back of the camera during this adjustment.

The graduations on the plate were illuminated by a large opalescent globe, placed, for the reasons given in the preceding article, as close as possible to the focal plane.

The theodolite was a Watts " Tavistock " model fitted with electric illumination of the circles. Strictly speaking, the diameter of the object glass should equal, or be greater than, the aperture of the camera lens, for the reasons given in the preceding article.

The faces of the prism, supplied by Messrs. Barr & Stroud, should be larger than the aperture of the lens and should be plane to within about three seconds, as different parts of the faces will inevitably be utilised for different observations. The prism was placed on a table furnished with levels and corresponding levelling screws, the table being carried on a spindle which could be adjusted in height. The spindle in turn was attached to a carrier which could be moved along an arm pivotted at C (Fig. 16).

Positions for N and X (Fig. 16), were first marked on the surface table, collinear with C and equidistant from C. The length chosen for N C and C X depends mainly on the shortest distance for which the theodolite may be focussed, as it is necessary, during the setting up, to sight on the lens mount. In the present instance, N X was made equal to 6-ft. Plumb-lines were then hung over N and X and the front node and vertical axis of the theodolite respectively were placed under the bobs. These plumbed positions were checked after any subsequent movement of the camera and theodolite during setting up. By means of packing pieces placed under the foot-screws of the theodolite, the height of the transit axis above the table was arranged to be roughly equal to the height of the camera objective at about the middle of the runs of the camera stand levelling screws. The horizontal collimating axis of the camera was also arranged to lie roughly at the same height.

Adjustment of the Apparatus.

The theodolite was first levelled, and the vertical circle clamped with the line of collimation horizontal, due allowance being made for vertical collimation error.

The prism was next swung, on its radial arm P C (Fig. 16), close up to the object glass of the theodolite, and its height and radial distance were adjusted to correspond with the position of the object glass. For this purpose the theodolite and prism were turned in azimuth to make the line of collimation approximately perpendicular to the prism face, a sufficiently close setting being obtained by placing the rim of the object

glass in actual contact with the prism face. In this position the height and radial distance of the prism were adjusted to make the object glass lie in the centre of the prism face. The prism was then levelled.

It remained to level the camera in such a manner that both the centre of the objective, and the collimating axis along which measurements are to be made, lie at the same height as the line of collimation. Suppose that A B C in Fig. 16 represent the three levelling screws of the camera stand. The prism was first swung, and the theodolite turned in azimuth, until the intersection M (Fig. 18) of the medial lines on the glass plate appeared on the vertical cross hair of the theodolite. Since this observation is made through the camera objective, the telescope must be focussed for distant objects. M was then brought on to the horizontal cross hair by turning A and B equally in the same direction. The theodolite was next focussed down on to the lens mount and the levelling screw C was turned until the lens mount appeared symmetrically within the field of the theodolite telescope. The theodolite was then refocussed on M and the coincidence of M with the horizontal wire restored by turning A and B. Next the prism was swung and the theodolite rotated in azimuth until a collimating mark (or part of the medial line of the glass plate lying in the outer field) appeared on the vertical cross hair. By turning A and B equally in opposite directions (*i.e.*, both inwards or both outwards), this collimating mark was then brought on to the horizontal cross wire. The whole process is finally repeated as necessary.

Before commencing observations, the theodolite was carefully refocussed on M, and visual parallax on the cross hairs eliminated.

A photograph of the apparatus actually in use is reproduced in Fig. 18.

Observing.

For successive observations on each of the scale graduations the following procedure was adopted :—

(*a*) Swing the prism, and follow the motion with the azimuth rotation of the theodolite, until the required graduation appears on the cross hairs. Clamp the horizontal circle of the theodolite.

(*b*) Look over the sights of the theodolite and rotate the prism until its face is perpendicular to the line of sight. This condition may easily be gauged by equalising the internal reflections of the prism.

(*c*) Shift the source of light until the exit pupil of the camera objective, as seen in the prism, is fully illuminated.

(*d*) Swing the prism about the pivot C (Fig. 16) until the theodolite sights appear central in the prism image of the exit pupil, repeating the adjustment (*b*) as necessary. No great accuracy in these adjustments is required.

(*e*) By means of the theodolite slow-motion screw, bring the graduation finally on to the cross hairs. Read and book.

Readings are required on the central graduation, on the collimating marks, and on three or four pairs of the outer graduations. Readings in the inner field do not add greatly to the precision of the result as computed by the method in the next article, and need not be taken.

The whole procedure is finally repeated for the other collimating axis of the camera after turning the camera round through 90° about its axis and re-setting for height and level in the manner described above.

16. *COMPUTATION.*

From the known distances between the graduations of the glass plate and the corresponding measured perspective angles we may compute

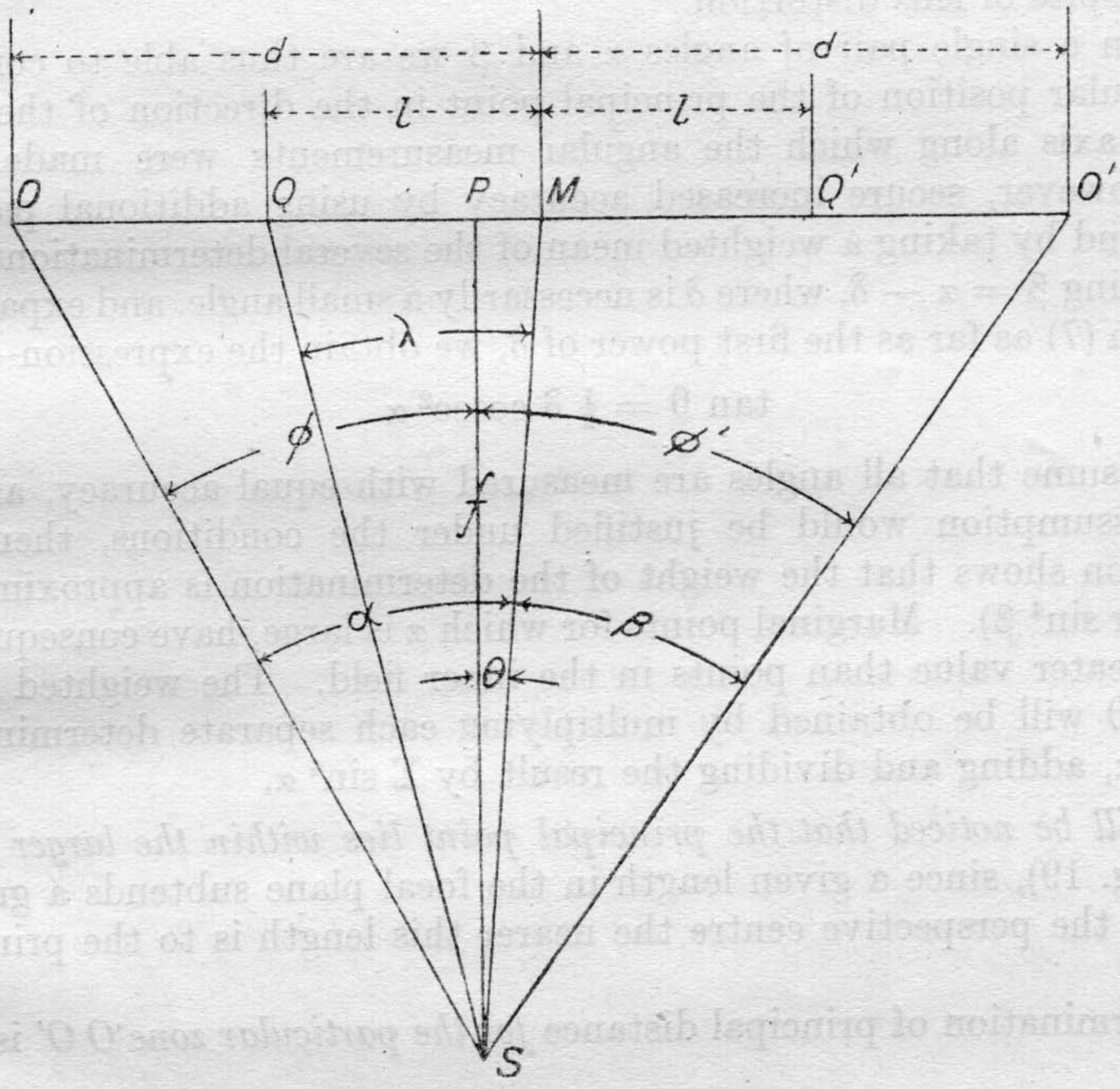


FIG. 19.

the principal point and distance by the method described in the last Chapter. The even nature of the focal plane distances, however, enables a simpler and quicker method to be employed in this case, by separating the determination of principal point from that of principal distance.

Suppose that M in Fig. 19 represents the central graduation of the scale, and that O O′ represent graduations at an equal distance d on either side of M. O M and O′ M subtend measured angles α and β at the perspective centre S. Let P be the principal point at an angular distance θ from M.

Then, by applying the rule of sines to the triangles O M S and O′ M S in turn,

$$\frac{SM}{d} = \frac{\cos(\alpha - \theta)}{\sin \alpha} = \frac{\cos(\beta + \theta)}{\sin \beta},$$

whence

$$2 \tan \theta = (\cot \beta - \cot \alpha) \quad \ldots\ldots\ldots\ldots \quad (7)$$

In deducing this expression, it will be seen that we have taken no account of the absolute value of d; it is sufficient if the two lengths O M and O′ M are equal, whatever their absolute magnitude. Now M has been arranged to lie close to the optical centre of the photograph, so that O and O′ will lie in the same zone, and will, for all practical purposes, be subject to the same amount of lens distortion. Equation (7) holds, therefore, in spite of lens distortion.

From a single pair of angles α and β we are thus able to compute the angular position of the principal point in the direction of the collimating axis along which the angular measurements were made. We may, however, secure increased accuracy by using additional pairs of angles and by taking a weighted mean of the several determinations.

Putting $\beta = \alpha - \delta$, where δ is necessarily a small angle, and expanding equation (7) as far as the first power of δ, we obtain the expression—

$$\tan \theta = \tfrac{1}{2} \delta \operatorname{cosec}^2 \alpha$$

If we assume that all angles are measured with equal accuracy, and no other assumption would be justified under the conditions, then this expression shows that the weight of the determination is approximately $\sin^4 \alpha$ (or $\sin^4 \beta$). Marginal points for which α is large, have consequently a far greater value than points in the inner field. The weighted mean for tan θ will be obtained by multiplying each separate determination by $\sin^4 \alpha$, adding and dividing the result by $\Sigma \sin^4 \alpha$.

It will be noticed that the principal point lies within the larger angle (α in Fig. 19), since a given length in the focal plane subtends a greater angle at the perspective centre the nearer this length is to the principal point.

Determination of principal distance *for the particular zone O O′* is now simple.

From Fig. 19

$$OP = OM - MP$$

or

$$f \tan(\alpha - \theta) = d - f \tan \theta \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$O'P = O'M + MP$$

or

$$f \tan(\beta + \theta) = d + f \tan \theta \quad \ldots\ldots\ldots\ldots \quad (9)$$

Adding (8) and (9)—

$$\frac{2d}{f} = \tan(\alpha - \theta) + \tan(\beta + \theta)$$

$$= \frac{\sin(\alpha + \beta)}{\cos(\alpha - \theta) \cos(\beta + \theta)} \quad \ldots\ldots\ldots\ldots \quad (10)$$

To avoid ambiguity in the sign of θ, it is better to consider the angles $(\alpha - \theta)$ and $(\beta + \theta)$ as the perspective angles ϕ and ϕ', subtended between the graduations considered and the principal point. Equation (10) then reduces to—

$$f = 2d \frac{\cos \phi \cos \phi'}{\sin (\alpha + \beta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Determinations of principal distance based on measurements in other zones will be affected by the corresponding variation in lens distortion, and without some knowledge of the extent of such distortion we must accordingly rely on a single determination. Calibration by this method of each of the three units of the tri-lens camera shows that the two values of principal distance obtained by single determinations along each collimating axis agree to within 0·01 mm. (or about 1 in 20,000), provided the same zone is considered in each case. We may thus conclude that a single determination based on careful measurement is sufficient.

By accepting the principal distance obtained from a particular zone, it is possible to calculate the distortion affecting other zones. Consider, for example, the case of a graduation Q (Fig. 19) at a distance l from the central graduation M. The measured perspective angle corresponding to Q M is λ. If the principal distance f obtained from the zone O O′ is held to apply to the zone Q Q′, which would be the case if the photograph were a true central perspective, then the angular distance of Q from the principal point P should equal—

$$\tan^{-1}\left(\frac{l - f \tan \theta}{f}\right)$$

The corresponding measured angle is $(\lambda - \theta)$ and the difference between the two furnishes a measurement of angular distortion in the zone. Within the limits of error in the measurement of angles, and in the determination of the calibration constants, an equal amount of distortion should be revealed by Q^1, the graduation corresponding to Q on the other side of the centre. It is accordingly worth while computing the distortion in order to furnish a check on the accuracy of the results.

Example.

An example of computation on the above lines is given in Form V.

The first diagram on this form should have been drawn in the observation book at the time of observation, in order that the computed principal point may be correctly related to the collimating marks.

Column (*b*) is filled in from the Abstract of Angles. If a suitable table of natural contangents is available column (*c*) may be filled in directly from it ; if not, the log. cots. may be looked out in Shortrede and the antilogarithms taken. The difference between the cotangents of α and β is also shown in this column and is halved in column (*d*) to give the corresponding determination of tan θ. The weights in column (*e*)

will be obtained with ample accuracy by looking out the natural sines and squaring twice on a slide-rule. For convenience the values of $\sin^4 \alpha$ have all been multiplied by 10,000. Column (*f*) contains a logarithmic computation for the product of columns (*d*) and (*e*). The results of columns (*e*) and (*f*) are added, the sum of column (*f*) being divided by that of (*e*) to give the accepted value of tan θ.

θ is finally applied to the angles in column (*b*) to give the corresponding perspective angles measured from the principal point, these latter angles being entered in column (*g*). θ is also applied to the measured angle to the collimating mark R in order to locate the principal point with respect to this collimating mark.

The succeeding computation of principal distance for the 5 cm. zone needs no explanation.

As a check on the whole determination, the angular distortion is next computed. The distance of the principal point from the central graduation is computed on the bottom of the form as 0·175 mm. The distance of the + 5 graduation from the principal point is accordingly (50 — 0·175), which equals 49·825 mm., and the corresponding true perspective angle will be—

$$\tan^{-1}\left(\frac{49\cdot825}{158\cdot50}\right) = 17^\circ\ 27'\ 03''$$

The measured angle (from column (*g*) on Form V), is also 17° 27′ 03″, showing that there is no residual distortion on the graduation + 5. This result might have been anticipated from the fact that the principal distance has been chosen to fit this particular zone, although an error in the principal point, which does not rest solely on measurements in the 5 cm. zone, might have introduced a residual distortion of a few seconds. The same distortion with the opposite sign would in that case have been obtained for the — 5 graduation. The following table shows the results obtained similarly for the other graduations :—

Graduation.	Distance from Principal Point.	Computed Angle from Principal Point.	Measured Angle from Principal Point.	Difference.	Mean distortion in zone.
	(mm.)	° ′ ″	° ′ ″	″	″
+5	49·825	17 27 03	17 27 03	—	—
−5	50·175	17 33 57	17 33 57	—	—
+4	39·825	14 06 16	14 06 03	13	12
−4	40·175	14 13 24	14 13 13	11	
+3	29·825	10 39 25	10 39 07	18	18
−3	30·175	10 46 44	10 46 27	17	

A similar determination for the short way of the plate provides the following data :—

f (for 17° zone) = 158·50 mm.
θ = 03′ 40″.

Angular distance of Principal Point from Collimating Mark—

P = 18° 56′ 58″.
Q = 18° 53′ 22″.

A suitable calibration sheet is illustrated in Form VI.

It will be noticed that sufficient information is given to enable the principal point to be fixed from any two of the three collimating marks should one of these marks not show up on a particular air photograph. To provide still greater latitude it is likely that four collimating marks will be provided in future models of this camera. Data has been provided in the form of perspective *angles* for locating the principal point, since this is most convenient in adjusting an angular measuring instrument such as the Fourcade stereogoniometer. The angular data could, if desired, be transformed into linear co-ordinates from the intersection of the collimating axes. Thus the co-ordinates of the principal point in the present example are—

Towards P — f tan 0° 03′ 40″.
Away from R — f tan 0° 03′ 48″.

17. *RELATIVE ACCURACY OF PRINCIPAL POINT AND PRINCIPAL DISTANCE CALIBRATION.*

An apparent defect in any system of calibration involving the measurement of perspective angles is that the principal point cannot thereby be determined to the same degree of linear accuracy as the principal distance. The extent and consequences of this comparative indetermination of principal point are now to be investigated.

In Fig. 20, P is the true principal point and O is a point in the focal plane such that O P subtends an angle α at the perspective centre S. O P S will lie on a circle of diameter O S. Suppose that the principal point and distance are subjected to small changes $d\,p$ and $d\,f$ respectively, and that each change results in an equal and opposite change in the perspective angle α. The final change in α will be zero and the perspective centre must accordingly move along the circumference of the circle, initially in the direction of the tangent S T. Hence,

$$\frac{d\,f}{d\,p} = \tan \alpha$$

For points O in the outer field, from which calibration is normally effected, α will lie between 15° and 20°, in which case $d\,p$ will approximately equal 3 . $d\,f$.

A given error in measurement of the perspective angle may accordingly involve three times more error in the principal point than in the principal distance, and to evaluate the two quantities to the same degree of linear

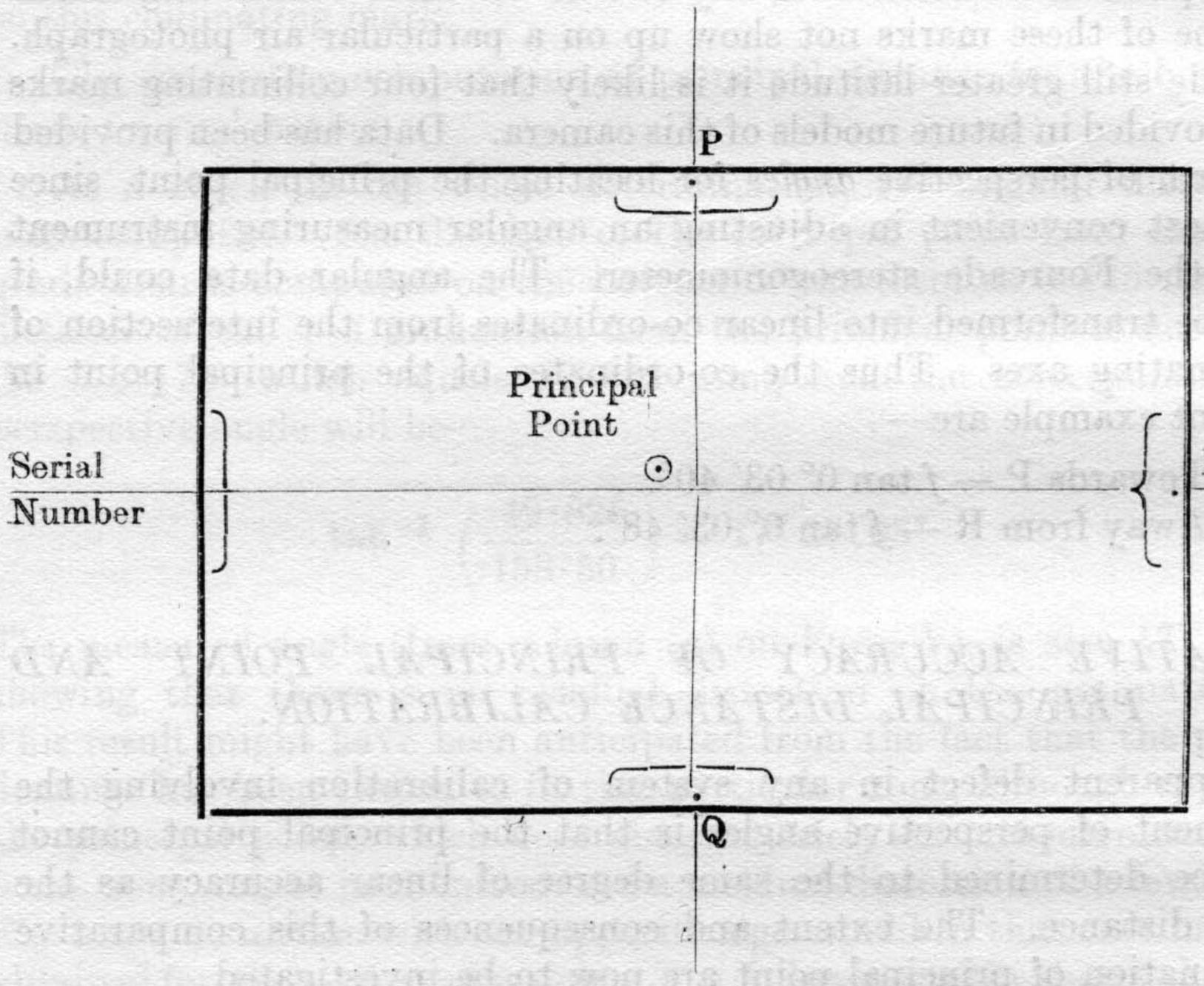
[FORM VI.

CALIBRATION SHEET.

Camera No. ... ... Tri-lens Experimental (Centre Unit).
Method ... ... Fourcade.
Temperature of Calibration ... 60° F.

$f = 158{\cdot}50$ mm. (for 17° or 50 mm. Zone).

The diagram illustrates a positive viewed from the emulsion side, or a negative viewed through the glass.



Angular Subtense of the Principal Point.

Longitudinal.	
From R	23° 13′ 59″
From PQ (on the side of PQ remote from R) ...	0° 03′ 48″
Transverse.	
From P	18° 56′ 58″
From Q	18° 53′ 22″
From the perpendicular through R to PQ (towards P)	0° 03′ 40″

accuracy we should need to make something like nine times as many determinations of principal point as of principal distance. If we consider the problem in reverse, however, it will be seen that there is no need to proceed to this extreme. Wherever the method of survey depends on the reconstruction of perspective angles—and for precise purposes this will usually be the case—the resulting linear error in the principal point matters little, since no greater error will be introduced in these reconstructed angles than the errors which occurred in the measured angles during calibration, and which gave rise in the first place to the comparatively large error in principal point.

A *direct* determination of the principal point, as the foot of the perpendicular from the perspective centre, would overcome this source of inaccuracy, but unfortunately there is at present no such direct method which is not open to serious objection. Colonel Grove-Hills, for

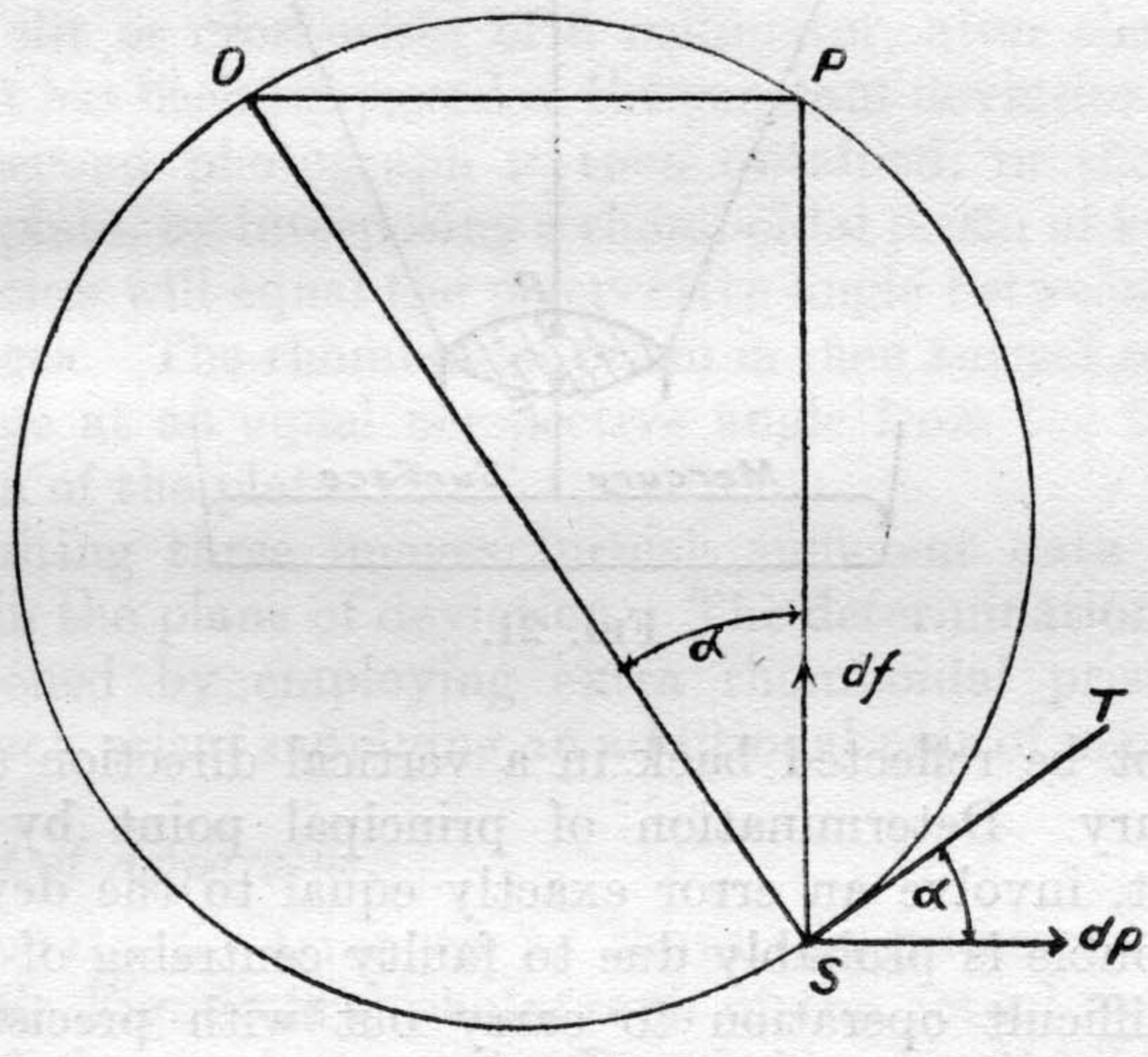


FIG. 20.

instance, has suggested inverting the camera over a trough of mercury (Fig. 21), and levelling the focal plane. The ray to the rear node R from a suitably illuminating mark situated at the principal point M, would then be vertical and would, on the usual geometrical theory, emerge from the front node F also in a vertical direction. The emergent beam of parallel light is then reflected back in a vertical direction at the horizontal surface of the mercury and finally forms an image in coincidence with M. If the mark were not initially situated at the principal point, then the image would be formed at an equal distance on the other side of the principal point. Various auto-collimation methods based on the same general principle have been described from time to time, in particular by Mr. Fourcade.*

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* See "The Principal Point and Principal Distance in Photogrammetry," by H. G. Fourcade, "Transactions of the Royal Society of South Africa," Vol. XVI, Part I, 1928, and previous papers by the same author in the same publication.

Mr. Fourcade himself, however, has since discovered that a lens introduces *deviation* which, in the case of an apparently good lens tested by him, amounted to as much as six minutes. This means that a ray entering at one node of the lens does not emerge in a parallel direction from the other node, as had previously been supposed, but is bent through a constant angle known as the deviation of the lens. Thus, although the ray M R in Fig. 21 is vertical, the emergent ray from F is not vertical

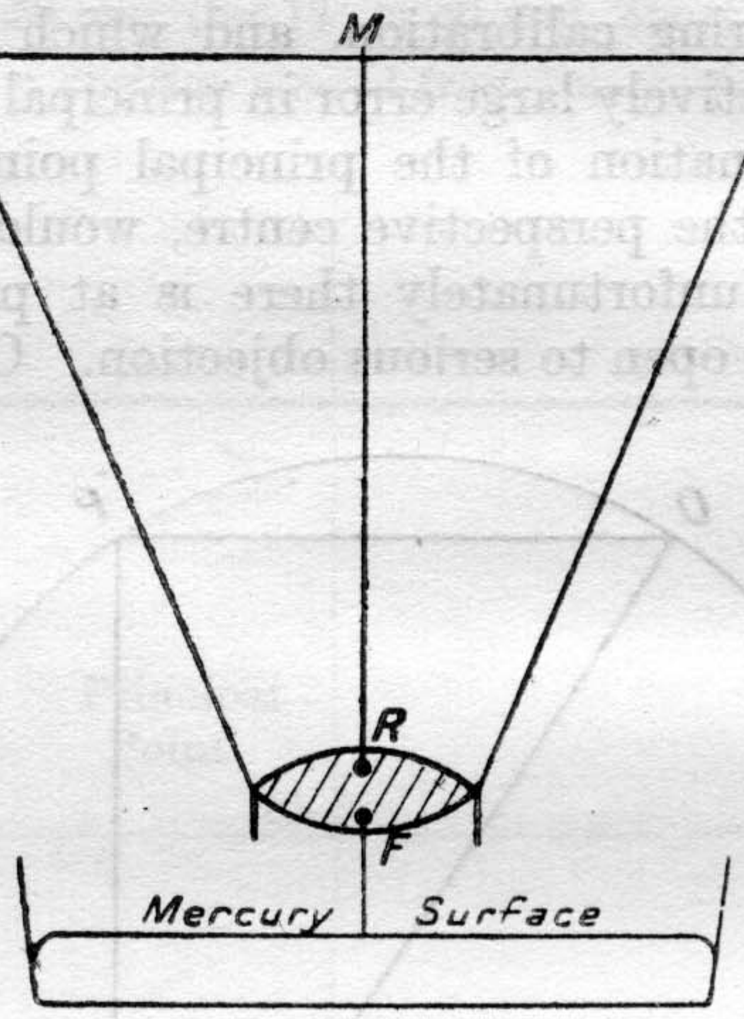

FIG. 21.

and would not be reflected back in a vertical direction at the surface of the mercury. Determination of principal point by these means would, in fact, involve an error exactly equal to the deviation of the lens. The trouble is probably due to faulty centreing of the lens components—a difficult operation to carry out with precision. A small error in centreing would, moreover, seem to involve a disproportionately large deviation.