Airborne Heighting During Australian National Antarctic Research Expedition (A.N.A.R.E.) 1960

By

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In 1959, the Antarctic Division of the Department of External Affairs bought and modified a D.C. 3 aircraft for aerial surveys in Antarctica; thus it became possible to use equipment which was too heavy or bulky for use with the previously available aircraft.

The aircraft was fitted with a Marconi AD 2300A Doppler radar, a Wild RC9 survey camera for vertical photography and K17 survey cameras for oblique photography. As with previous Antarctic Division aircraft, the D.C. 3 was fitted with a Polar Path gyro compass, radio compass and radar altimeter in addition to the more normal navigational aids.

The radar altimeter fitted was an S.C.R. 718/D. Height presentation was in the form of two lobes on a circular trace on a cathode ray tube, one being the adjustable zero mark and the other the return signal. The scale round the tube was graduated 0′-5,000′ in 50′ intervals. Range was up to 40,000′.

On his appointment in September, 1959, to the Division of National Mapping of the Department of National Development, as surveyor with the 1960-61 A.N.A.R.E., the author obtained a copy of " Bureau of Meteorology Working Paper 57/1349—October, 1959 " on Barometric determination of elevation by G. U. Wilson (Empire Survey Review No. 118, Vol. XV, October, 1960), and the following is a description of the field and office techniques in the practical application of this method and the accuracy obtained. Further reference to Bureau of Meteorology Working Paper 57/1345 in this report has been abbreviated to B.M.W.P.

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Flying and Recording

Wherever possible flights were so placed that they could cross existing or future runs or would start from over the sea and return to over the sea at some distant point. In this way a good check was kept on the reliability of the heights obtained. Start points, which were always on the coast, were approached from some 10 miles out to sea on the heading of the proposed track and at the operating pressure altitude and power settings for the run and the drift determined. This drift was laid off the heading so that the approximate desired track could be flown.

As soon as the run in was commenced, a stop watch mounted on the instrument panel was started and readings of the indicated air speed, outside air temperature, pressure altitude, radar height, drift, distance and heading commenced. These were made at one minute intervals and it was found most convenient to record pressure and radar altimeter readings on the minute and the other information on the half minute. As there were about four radar altimeter readings over the sea before crossing the coast, these were meaned to give a true altitude of the pressure altitude flown. As the coast was crossed, an observer on the Williamson drift sight would call the instant of passage which was recorded at the time and Doppler distance and used as datum for subsequent distances.

It was found necessary to read heading and pressure altimeter at each time interval, due to the inability of the pilots to fly the heading or pressure altitude sufficiently accurately. As an aid to fixing the position of the coast crossing on some runs, an R.C.9 photo was taken, and a similar procedure was adopted at the finish of each run if over an identifiable feature.

The Polar Path compass was checked about every half hour of flight and immediately before and after each run to determine gyro error and gyro drift rate. Heading checks were made on the sun with a Kollsman periscopic sextant, sun azimuth being pre-computed for an instant and the sextant observer calling several intersections, over a period of about one minute, to the navigator, who read compass heading at each and deduced a mean compass error.

Operating height for most runs was in the region of 12,500 ft. pressure altitude to ensure adequate ground clearance inland as the pressure altitude should not be changed during the run and it is necessary to have a minimum 3,000 ft. ground clearance. Oxygen was not available for the crew, so to avoid errors due to carelessness caused by anoxia on long runs it was thought desirable to have two recorders each booking half the required information, allowing ample time to record and check without haste. This was particularly necessary with the radar altimeter, when the height is presented as a " lobe " on a circle alongside the scale on a cathode ray tube. All other information is presented as a dial reading except for Doppler distance which is summed on counters. Improvements in the method of recording and presentation will be discussed later.

METHOD OF REDUCTION OF AIRCRAFT HEIGHTING

In B.M.W.P., the formula for the height of the ground at point " D " below an air station " C ", is given as—

$$hd = ha + (hb - ha) + (hc - hb)p - (hc - hd)$$

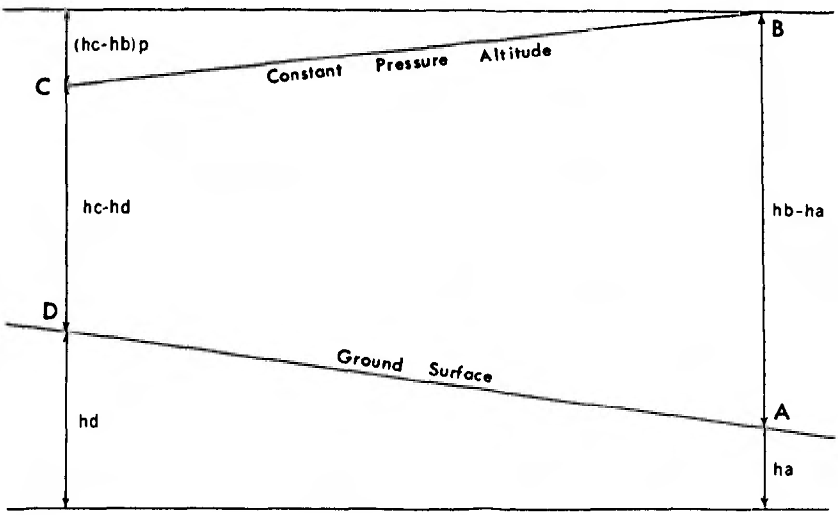

Fig. I. B.M.W.P. Height Formula Diagram.

Where—

ha = elevation of terrain at start point " A ";
hd = elevation of ground point required;
(hb — ha) = the height of the pressure altitude flown above the start point " A ";
(hc — hb)p = difference of height between the air station " B ", vertically above " A ", and " C " vertically above " D ";
(hc — hd) = the difference in height between the ground station " D " and the air station " C " above it. In effect this is the radar altimeter reading above the point the elevation of which is required.

Again from B.M.W.P.—

$$(hc - hb)p \approx (\emptyset c - \emptyset b)p = \bar{f}\, d\, \bar{A}\, \overline{\sin \alpha}$$

where—

$\bar{f} = 2\omega\, \overline{\sin.\ lat.}$;
d = distance;
$\bar{A}$ = true air speed;
α = drift angle;
ω = angular velocity of earth's rotation.

This term is negative for port drift and positive for starboard drift.

ha, will be known.

The term (hb — ha) is obtainable as outlined in B.M.W.P., pages 6 and 7, but is more practically the radar altimeter reading over the start point.

And (hc — hd) is as stated above the radar altimeter reading at " C ".

Term (hc — hb)p $\approx \bar{f}d\bar{A}\,\overline{\sin\alpha}$ is obtainable by direct computation but as this is cumbersome on long flights with frequent heights required, the author has prepared a " Nomogram for the reduction of Barometric levels " (Fig. 2), which greatly facilitates the reduction.

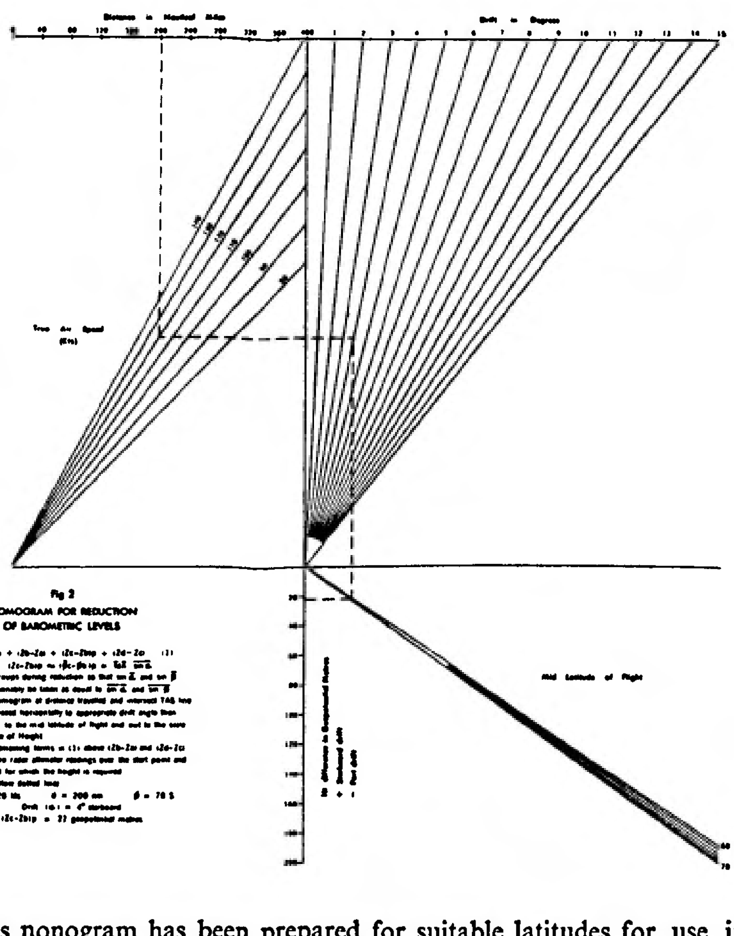

Fig 2
NOMOGRAM FOR REDUCTION OF BAROMETRIC LEVELS

This nonogram has been prepared for suitable latitudes for use in Antarctica, but can be modified to suit any latitude.

The entry arguments are true air speed in knots, distance in nautical miles, mean drift angle and mean latitude of flight. For convenience, the result (Øc — Øb)p has been divided by " g " (using g = 980 cm./sec./sec.) for the reason explained in B.M.W.P., pages 3 and 6 and tabulated directly as change in altitude.

Method

As stated the drifts and indicated air speed and outside air temperatures are recorded for each minute of the flight. These are grouped over sections where they are approximately equal and the mean drift and mean indicated air speed are noted for each group.

Drift as found is correct within the accuracy of the instruments and requires no further reduction.

Indicated air speed must be converted to rectified air speed by application of the instrument and position error corrections. This correction will vary between different aircraft and for the Dakota A65-81, was of the order + 3kts over the operating range, but varied with indicated air speed.

The rectified air speed is further corrected to the true air speed and this correction is dependent on the rectified air speed itself, pressure altitude and outside air temperature which in turn must be corrected for instrument error and frictional heating. The value of true air speed is most conveniently found by use of the Dalton Navigational Computer, with the above entry arguments. An error of 5° in air temperature produces approximately 1 knot error in true air speed as does an error of 500 ft. in pressure altitude.

Distances are next taken out of the observer's log for each group of similar drifts and indicated air speeds. The Doppler distance, as recorded, is in error by an amount dependent on the ground speed and must be corrected before using this term to compute any change of geopotential. Ground speed as determined by the Doppler is found by dividing change in distance by change in time and is then corrected to true ground speed from the calibration table for the instrument. From this true ground speed a true distance is deduced.

Having corrected all observed values, the nomogram now gives directly the difference in altitude between the air station above the start point and that above any other ground point, where the elevation is required.

This is applied to the previous terms ha and (hb — ha) to give the ground elevation at point " D ".

As ground speed may vary along a flight line, it will be found convenient to plot a curve of time against distance, after this has been corrected for Doppler ground speed error, for each leg of the flight in which ground speed is sufficiently constant to be meaned. Each spot height as determined above may now be plotted against time and so save tedious correction of every ground distance, and used to form a profile of true elevation of the ground along the track.

It remains now to plot the track made good and place on it, either spot heights, or contour intervals.

This is done in the following manner if a gyro compass is used:—

(1) Gyro error and drift is determined from heading checks, and the amount of gyro drift applied proportional to time for gyro error at any instant. This is applied to the gyro heading to give true heading.

(2) True heading is corrected for drift to give track.

(3) Ground speed, as determined by Doppler, is corrected as described, and—

(4) Track made good is plotted.

This may conveniently be plotted on the same sheet as the ground profile and the time to distance graph, and heights transferred to it to show their distance along and across a desired track.

Example for heighting run 3, from Tilley Nunatak to Hansen Mountains, is attached. (Fig. 3.)

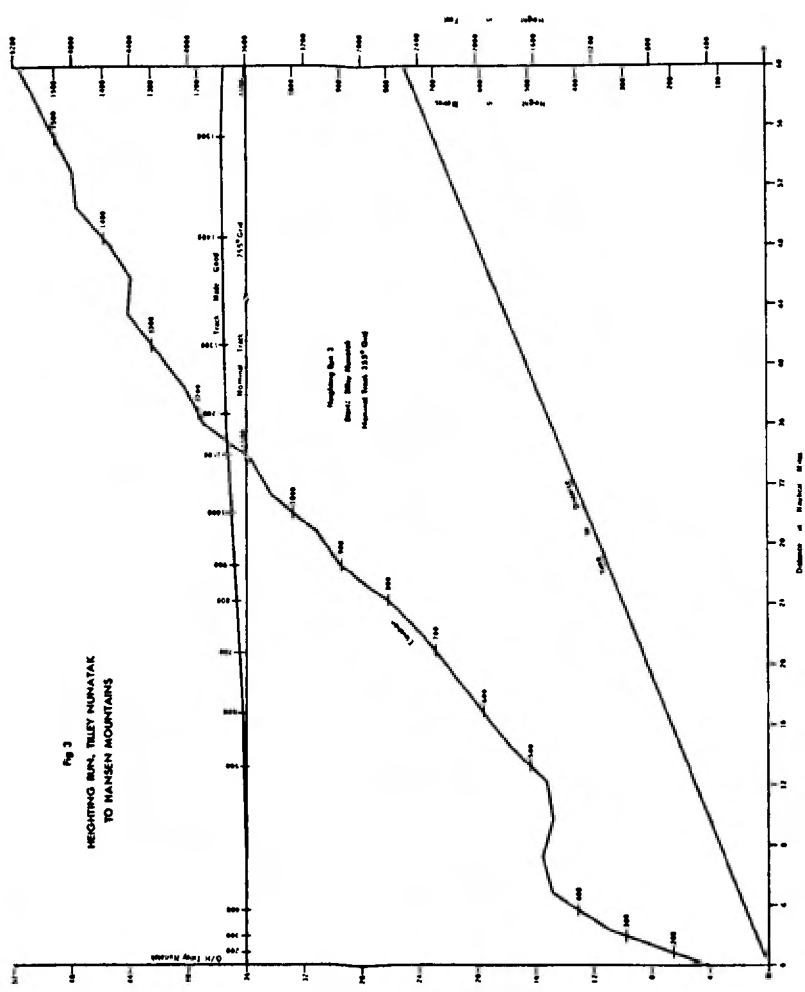

Fig 3
HEIGHTING RUN, TILLEY NUNATAK TO HANSEN MOUNTAINS

ACCURACY ATTAINED IN HEIGHTING RUNS

Each run was plotted from its start point as accurately as possible on 1: 500,000 scale and the heights of crossover points found from the profile for each run.

Where separate runs intersect, the mean difference of heights was found to be 27 feet per 100 nautical miles of flight, on 6 crossovers, with a mean actual difference of 15 feet. On flights where the run was closed from sea level to sea level, the mean misclosure was 10 feet per 100 nautical miles. Unfortunately, due to the destruction of the aircraft during a blizzard, it was not possible to complete the heighting programme, which would have given crossovers or sea level closes on all runs.

Largest crossing differences appear when a run near normal to the contours crosses one which is parallel to them, as would be expected if part of the difference in height is due to inaccuracies in the plot of the track or in the determination of aircraft heading.

Factors Affecting Accuracy

In the formula ;

$$hd = ha + (hb - hc)p + (hb - ha) - (hc - hd);$$

ha will be known.

(hb — ha) and (hc — hd), are radar altimeter readings at B and C respectively, and are the greatest source of error with the equipment as installed in Dakota A65-81. Errors in (hb — ha) are reduced to a minimum by the method of meaning several readings over the sea on the run in for commencement of the run.

Accuracy of radar altimeter heights, determined by altimeter S.C.R. 718/D, is quoted as within $\pm$ (50 ft. $\pm$ $\frac{1}{4}$ per cent of indicated height). In practice, the spread of several readings over a surface of constant elevation is relatively small (in order of 50 ft. for six readings) and it may be assumed that the initial height of the pressure plane flown as found from the mean of several readings is accurate within $\pm$ 20 ft. No method of minimising error in term (hc — hd) is available short of fitting different instruments and this must be taken as $\pm$ (50 ft. $\pm$ $\frac{1}{4}$ per cent R.A.), where R.A. is the radar altimeter reading over the point whose elevation is required. For Dakota operations in Antarctica, where altitude was limited to 12,500 ft., this error will reach a maximum of $\pm$ (50 + 30) = $\pm$ 80 ft.

Errors in the term (hb — hc)p. This is the term expressing inclination of the constant pressure plane along which the aircraft is flown.

As stated in B.M.W.P.—

$$(hb - hc)p \approx (\emptyset b - \emptyset c)p = f\ d\ A\ \overline{\sin\alpha} = K$$

where—

$f = 2\omega \sin$ lat.;

d = distance;

A = mean air speed;

α = drift angle;

ω = angular velocity of the earth's rotation.

By breaking of the flight into one degree bands during reduction the errors introduced by assuming $\overline{\sin \emptyset}$ (mean sin latitude) is equal to $\sin \bar{\emptyset}$ (sin mean latitude), may be reduced to a negligible amount, so that error in $\bar{f} = 0$.

Error in " d ".

The Doppler distance measuring was calibrated in the aircraft under operating conditions and found to have a maximum error of 4 per cent.

So that $\delta d = 0{\cdot}04d$.

$\bar{A}$, the mean air speed is derived from the rectified air speed, outside air temperature and pressure altitude. The air speed indicator is well calibrated as are the altimeter and thermometer. A maximum error in air speed would be 2kts for Dakota operations and with a minimum air speed of 80 knots, this would amount to 2½ per cent, so that—

$$\delta\bar{A} = 0{\cdot}025\bar{A}.$$

$\overline{\text{Sin } \alpha}$: Doppler drift measurement is accurate to within ½°, and by selecting during reduction, groups of drifts which are near constant, the error in mean drift may be held to $\pm$ 1°.

$$\text{So that } \delta\alpha = 1^\circ = \frac{2\pi}{360} = 0{\cdot}0174.$$

In the formula $(\emptyset b - \emptyset c)p = \bar{f}\, d\, \bar{A}\, \overline{\sin \alpha} = K$, differentiating with respect of f, d, A and α, we obtain—

$$\frac{\delta K}{\delta f} = d\bar{A}\,\overline{\sin \alpha} \qquad \text{hence } \delta K(f) = d\bar{A}\,\overline{\sin \alpha}\,\delta f$$

$$\frac{\delta K}{\delta d} = \bar{f}\bar{A}\,\overline{\sin \alpha} \qquad \text{hence } \delta K(d) = \bar{f}\bar{A}\,\overline{\sin \alpha}\,\delta d$$

$$\frac{\delta K}{\delta A} = \bar{f}\bar{d}\,\overline{\sin \alpha} \qquad \text{hence } \delta K(A) = \bar{f}\bar{d}\,\overline{\sin \alpha}\,\delta A$$

$$\frac{\delta K}{\delta \alpha} = \bar{f}\bar{d}\bar{A} \qquad \text{hence } \delta K(\alpha) = \bar{f}d\bar{A}\,\overline{\cos \alpha}\,\delta\alpha$$

Adding $\Delta K = \overline{\sin}\,\alpha\,(d\bar{A}\delta\bar{f} + \bar{f}\bar{A}\delta d + \bar{f}d\delta\bar{A}) + \bar{f}\,\bar{d}\,\bar{A}\,\overline{\cos}\,\alpha\delta\alpha$ as $\delta f = 0$, rewrite

$$\Delta K = \overline{\sin \alpha}\,(\bar{f}\bar{A}\delta d + \bar{f}d\delta\bar{A}) + \bar{f}d\bar{A}\,\overline{\cos}\,\alpha\delta\alpha \qquad (2)$$

We may now substitute for values of $\bar{A}$ and d, and evaluate for various latitudes and drift angles.

One hundred and fifty knots would be the approx. limit of operating range for the aircraft concerned, so that—

$$\delta A = 3{\cdot}75k = 6{\cdot}33 \text{ fps}.$$

Using, d = 100 nautical miles—

$$\delta d = 4 \text{ nm.} = 24{,}320 \text{ ft.}$$

$$\Delta K = (f \times 150k \times 4nm) \sin \alpha + (f \times 100nm \times 3{\cdot}75k) \sin \alpha + (f \times 100nm \times 150k \times \cos \alpha \times 0{\cdot}0174).$$

Evaluating—

$$\delta K_{(d)} = f\bar{A}\delta d \sin\alpha \text{ (per 100nm).}$$

Lat.	Drift				
	5	10	15	20	25
10	14	27	40	53	66
20	27	53	79	105	130
30	39	78	116	154	190
40	50	100	149	197	244
50	60	119	178	235	291
60	68	135	201	266	329
70	74	146	218	289	357
80	77	154	229	303	374
90	78	156	232	307	379

$$\delta K_{(A)} = fd \sin\alpha\delta A \text{ (per 100nm)}$$

Lat.	Drift				
	5	10	15	20	25
10	8	17	25	34	40
20	17	32	49	66	81
30	24	47	73	96	118
40	32	61	93	123	152
50	37	73	111	147	187
60	42	83	125	165	206
70	46	89	137	181	223
80	47	94	143	189	233
90	49	94	145	192	236

$$\delta K_{(\alpha)} = fdA \cos\alpha\delta\alpha \text{ (per 100nm)}$$

Lat.	Drift				
	5	10	15	20	25
10	68	67	66	64	62
20	133	132	129	126	121
30	194	192	188	183	178
40	250	247	242	236	227
50	298	294	289	281	271
60	337	333	326	318	306
70	366	361	354	345	333
80	383	379	372	362	349
90	389	384	377	367	354

Sum these to find maximum total error and divide by "g", as explained in B.M.W.P., to express the error in units of height, in this case geopotential feet.

Error in feet per 100nm.

Lat.	Drift				
	5	10	15	20	25
10	3	3	4	5	5
20	6	7	8	9	10
30	8	10	12	14	15
40	10	13	15	17	19
50	12	15	18	21	23
60	14	17	20	23	26
70	15	19	22	25	29
80	16	20	23	27	30
90	16	20	24	27	30

These individual errors are maximum and the likely errors would be about ¼ those tabulated for the operating range of a Dakota with well calibrated Doppler.

Remaining sources of error stem from errors in ground position as determined from the Doppler. The magnitude of these errors will naturally vary with the ruggedness of the terrain which is being heighted and the magnitude of position errors.

A well calibrated Doppler is quoted as being accurate to $\pm$ 1 per cent in distance and $\pm \frac{1}{2}^{\circ}$ in drift.

The most useful application of this method seems to be in conjunction with aerial photography, where spot heights are synchronised with exposures so that minor deviations from track are of no consequence.

METHODS OF IMPROVING ACCURACY

The greatest error is in the radar altimeter reading, partly due to instrumental inaccuracies, but also largely due to the poor form of presentation and consequent unreliability in reading. The radar altimeter as fitted to the aircraft used, was an SCR718/D, which presented the height as a lobe on a cathode ray tube against a scale graduated to 50 feet intervals. The fitting of an altimeter reading on either Veeder counters or a hand-swept dial, would greatly improve the accuracy.

At each radar altimeter reading, as explained earlier, the pressure altimeter is read to find deviation from a preselected altitude. Accuracy could be much improved by fitting a statoscope to avoid possible lag errors and to provide reading on a finer scale.

These last two improvements are combined in the "Canadian Applied Research Limited, Airborne Profile Recorder Mk5", for which the manufacturers quote a profile accuracy of $\pm$ 10 ft., over good terrain and $\pm$ 5 ft. for spot heights.

It would also be possible to fit a slave camera, which photographed the instruments necessary for reduction of the heights. This camera could be fitted and synchronised with a vertical camera, and photograph the panel at each $\frac{1}{2}$, $\frac{1}{4}$, etc., of the vertical camera interval, to provide a virtually continuous trace of the profile. This system would also eliminate observer errors.

Results obtained with the equipment used in Antarctica indicate that, with the previously suggested refinements, a means of heighting could be available, which would be a valuable aid to small-scale photogrammetric mapping.