THE GEOID IN AUSTRALIA - 1971

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C O N T E N T S

Figures

1. INTRODUCTION

1.1 The geoid is the equipotential surface of the earth's gravitational and rotational field which coincides with mean sea level. This surface is everywhere at right angles to the direction of gravity. Due to the irregular distribution of material in the earth's crust, and its variation in density, the force of gravity varies from place to place, not only in amount, but in direction; but unlike the surface of the ground which departs from a surface of revolution by several kilometres at slopes of up to ninety degrees, the geoid seldom departs from a well chosen spheroid by as much as 100 metres, at slopes rarely exceeding one minute of arc.

1.2 The computation of surveys would be difficult on the irregular geoid surface, and computations have usually to start before the shape of the geoid is known. Computations are therefore made on a spheroid, a geometrical surface of revolution, so chosen as to fit the geoid, or some part of it, as well as current knowledge permits. The size and shape of a spheroid is defined by two numbers: the length of the major semi-axis, a, and the flattening, f. To define a datum on which computation can proceed, it is also necessary to define the location of the spheroid's centre, and the direction of its minor axis.

1.3 The separation of the geoid above or below a spheroid is denoted by N, positive or negative, measured in metres. Values of N can be determined by astro-geodetic levelling, by gravity, or by observations to artificial satellites.

1.4 This report describes the determination of the geoid in Australia relative to the Australian Geodetic Datum, by the method of astro-geodetic levelling. Gravity observations have been used to fill in detail, but the gravity results have everywhere been adjusted to the astro-geodetic framework.

1.5 A preliminary geoid was calculated for Papua and New Guinea and some adjacent islands using those astronomical determinations which had geodetic coordinates on the Australian Geodetic Datum, - see Appendix 1.

2. GEODETIC DATUM

2.1 The geodetic network that extends over the Australian mainland and some adjacent islands has been computed on the Australian Geodetic Datum (AGD). After a series of investigations by the Division of National Mapping (see Mather and Fryer, 1970, for a summary), a reference spheroid called the Australian National Spheroid (ANS) was adopted in 1965. The values of the major semi-axis and the flattening are:

$$a = 6378160 \text{ metres}; \qquad 1/f = 298.25$$

The ANS is almost identical to Reference Ellipsoid 1967 (Geodetic Reference System 1967, p.12) the difference being less than 0.003 in the value of 1/f. In 1970 the minor axis of the ANS was re-defined to be parallel to the Conventional International Origin.

2.2 Johnston Geodetic Station (Lambert, 1968, p. 83), sometimes called the Johnston Origin, was assigned a value of N = 0 metres (Bomford, 1967, p. 58). The values of the geodetic coordinates at Johnston were determined such that the means of the deflections of the vertical at 274 astro-geodetic stations spaced evenly across the continent were close to zero. Scale errors in the geodetic network due to non-coincidence of the geoid and spheroid were thereby kept small. The AGD 1970 is defined by the ANS and the geodetic coordinates of Johnston Geodetic Station:

Latitude: 25°56' 54".5515 S

Longitude: 133°12' 30".0771 E

Spheroidal Height: 571.2 metres

2.3 By 31 December 1970 approximately 1150 stations on the AGD had astronomic determinations of latitude and longitude. Re-observations at 110 stations were also made during 1966-70. The astronomical determinations in this period were made almost exclusively with Kern DKM3A theodolites using the impersonal eyepiece micrometer for almucantar longitude observations and circum-meridian altitude observations for latitude (Bomford, et. al., 1970).

2.4 A readjustment of the major horizontal control network of Australia is scheduled for 1972. Several traverses in the original adjustment have been remeasured with either MRA4 Tellurometers or Laser Geodimeters and strengthened with the observation of simultaneous reciprocal azimuths. The values of N along lines of traverse must be known if distances are to be reduced to the spheroid. A preliminary geoid map for Australia was produced in late 1966 (Fischer and Slutsky, 1967, p. 331). About 550 astro-geodetic stations on the AGD were available for this determination.

2.5 In 1971 a new geoid map for Australia was determined. The first stage of this project was the computation of sections of primary geoidal profiles along traverses where the astronomical station spacing was generally less than 35 kms. These sections formed large loops which were broken up by geoidal profiles along traverses where the astronomical stations spacing was often in excess of 50 kms. A weighted least squares adjustment provided values of N for 1133 astro-geodetic stations.

2.6 Values of N and the deflections of the vertical were gravimetrically computed at 51 geodetic stations and at 1679 points on a half degree grid inside the loops formed by the

geoidal profiles. The gravimetrically computed values were adjusted, loop by loop, into the system defined by the adjusted values of N at the astro-geodetic stations on the loop perimeter. A total of about 3000 values were available for the automatic contouring of maps of the deflections of the vertical and N.

2.7 All geodetic values used in the 1971 geoid determination were from the 1966 Geodetic Adjustment and should be free of errors greater than 0.1 seconds of arc (Bomford, 1967, p. 69). The astronomic observations have been performed in the period 1950 to 1970. Since 1963, 917 stations have been observed with DKM3A theodolites. A single batch of computer programs, combined into one program known as ASTRO in 1967, have been used to reduce all observations. A homogeneity exists throughout the astronomic and geodetic information on the AGD and this has added greatly to the certainty of the results.

3. FORMULA

3.1 Only one formula is required for the computation of the change in the value of N between two stations. The accuracy with which the geoid is mapped is solely dependent on the rate of change of the value of the deflection of the vertical. It is impossible to set rigid standards for the maximum distance between astro-geodetic stations but, in geoidally disturbed areas, distances over 20 km should be avoided. If the rate of change of the deflection of the vertical between two stations is non-linear, the increment in N between these stations computed from equation (3.4) will be in error.

3.2 Following the convention of the Bureau International de L'Heure and taking longitude (λ) positive west and latitudes (ϕ) positive north, the deflections of the vertical in the meridian ξ (Xi) and in the prime vertical η (Eta) directions are represented by:

$$\xi = \phi_A - \phi_G \qquad \text{....(3.1)}$$

$$\eta = (\lambda_A - \lambda_G) \cos \phi \qquad \text{....(3.2)}$$

where the subscripts A and G respectively refer to astronomic and geodetic values.

3.3 The deflection of the vertical ζ (Zeta) in a particular azimuth α may be computed from

$$\zeta = - \xi \cos \alpha + \eta \sin \alpha \qquad \text{....(3.3)}$$

3.4 If ζ_A and ζ_B are the deflections of the vertical at the astro-geodetic stations A and B in the azimuth AB, the difference in N values between these stations may be expressed as

$$N_B - N_A = \tfrac{1}{2}(\zeta_A + \zeta_B)\, L_{AB} \qquad \ldots.(3.4)$$

where L_{AB} is the distance between A and B.

4. DATA

Astronomic Observations

4.1 All astronomic observations have been made in accordance with the specifications for astronomical determinations set out in (Bomford, et. al., 1970).

4.2 The FK4 star catalogue has been exclusively used since 1962 for all star predictions and reductions. The corrections applied to latitude and longitude observations are given in Bomford (1963).

5. GEOIDAL PROFILES

5.1 The positions of the available astro-geodetic stations were plotted on a map of the geodetic traverses and 125 sections of geoidal profiles joining 76 junction stations were selected. The total length of the profiles was 49,407 km of which 24,650 km had an average astro-geodetic station spacing of less than 35 km. The numbering system of the 125 sections is shown in Figure 1.

5.2 A total of 49 loops were formed by the geoidal profiles. The loop numbering system - see figure 1 - follows closely the numbering system used in the 1966 Geodetic Adjustment of the Australian Geodetic Datum (Bomford 1967, figure 1). The average loop length was 1656 km.

5.3 The values of N and the deflections of the vertical at 51 geodetic stations and 1679 points on a half degree grid inside the loops were gravimetrically computed by Prof. R. Mather. The method of computation has been described in several publications, notably (Mather 1970, p. 55 et. seq).

6. COMPUTER PROGRAMS

6.1 The computation of the increments in N - see equation 3.4 along the geoidal profiles was performed by program GEOID specially written for this project. PUNGEOID is a modified version of GEOID which provides punched output in a format which is directly suitable for the program GEOIDADJ - see section 6.4. All programs prepared by the Division of National Mapping were written in Fortran IV for the CDC 3600 computer at the CSIRO Division of Computing Research in Canberra.

6.2 The main adjustment program is called DETERGEN. It is a modified version of the levelling adjustment program LEVELONE which can simultaneously adjust 216 height differences amongst 139 junction points. The height of at least one

junction point must be held fixed. This program provides information regarding the variance - covariance matrix of the adjusted junction points as well as a plotted histogram of the adjustments. A normal distribution curve is superimposed on the histogram. All sections of the geoidal profiles are automatically weighted inversely to their length and an additional weighting factor for each section may be included. The factor used is described in Chapter 6 below.

6.3 Program DETERGEN calculates the minimum value of the least squares adjustment and also the variance of the observed quantities. The expected variance must be included in the data. The quotient of these two variances is compared with F test values in statistical tables according to the degrees of freedom in a particular solution. This enables a check to be made on the value of the estimates variance. The occurrence of gross errors may also be detected from this test.

6.4 Linear re-adjustment inside each section of geoidal profile is required once values of N have been obtained for the junction stations from DETERGEN. Program LINADJ was written for this purpose for the National Height Adjustment and was modified to provide a revised printout and punched output for the adjusted stations in the 1971 geoid determination. This modified version was named GEOIDADJ. The data for GEOIDADJ consisted of the punched output for a section of geoidal profile from PUNGEOID together with the adjusted values of N at the terminal junction stations from DETERGEN. The punched output from GEOIDADJ was in a format suited to Prof. R. Mather's programs, which meshed the gravimetrically computed values of N to the adjusted astro-geodetic values.

6.5 The program ROBBINS was used to compute bearings and distances between adjacent astro-geodetic stations whenever these values were not directly available from output of the 1966 Geodetic Adjustment. This situation arose on the secondary profiles where astronomical observations were not available at every station. National Mapping programmed the formula of ROBBINS in 1964 and have used it extensively for geodetic calculations. (Robbins, 1962, p. 301 et. seq.).

7. ASSISTANCE OF GRAVIMETRY AND GEOLOGY

7.1 Computations at a geodetic station using gravity anomaly data, which is available over the majority of the Australian mainland on a tenth degree grid, provide good estimates of the average values of the deflections of the vertical for the region surrounding that station. Isolated, local anomalies will not be reflected in the gravimetrically computed values of the deflections of the vertical unless the local gravity field has been closely sampled around a specific station. The astro-geodetic deflections of the vertical at that station indicate the local anomaly. If the adjacent astro-geodetic stations in the profile are not close enough

to detect any irregular change in deflection values then the geoid as mapped either side of the anomalous station will be erroneous.

7.2 The values of the deflections of the vertical may change rapidly in regions of relatively flat topography. In Australia changes of 25 seconds of arc in either Xi or Eta within 60 kms. were discovered. In one region of South Australia another change of 25 seconds of arc occurred within the next 90 kms of the astro-geodetic levelling section. If a geoidal profile runs through a geoidally disturbed region the station spacing must be reduced to ensure reasonable loop closures. Topographic, tectonic and gravimetric information can be used to decide which regions are likely to be geoidally disturbed. A local irregularity which has an 8 second of arc influence on a deflection of the vertical will cause a 1 metre error in astro-geodetic levelling along a line 50 km long.

7.3 The substitution of gravimetrically calculated values of the deflections of the vertical for their astro-geodetic counterparts can be justified when reasonable evidence such as that listed below is available:

7.3.1 The station is in an area whose geological structure is markedly different from that of the surrounding region;

7.3.2 The gravity anomaly field is well represented with observations, ensuring that the gravimetrically calculated values of the deflections of the vertical are reliable;

7.3.3 The adjacent astro-geodetic station spacing is too large when the topographic and geological conditions are considered;

7.3.4 The astro-geodetic and gravimetric deflections of the vertical disagree much more than usual;

7.3.5 The misclosure of loops indicate a weakness in the region.

7.4 Six of the 1133 astro-geodetic stations were investigated. Four stations had their astro-geodetic values of the deflections of the vertical replaced by gravimetric ones whilst two stations were removed completely from all computations.
A 432 (Queensland): Latitude S 20°21'; Longitude E 139°12'.

7.4.1 This station is situated approximately 50 km north-west of Mount Isa. It is the junction station for four sections the distances to the nearest stations in these sections being 29, 47,75 and 237 km. The astro-geodetic value of eta is 4.40 seconds of arc different from the gravimetric one. The gravimetric coverage in this region is amongst the best for the Australian mainland. Several outcrops of differing geological structure occur in the vicinity of A 432 and the region is topographically rugged. The station is situated near a large granite outcrop in an area of strongly folded geosynclinal

deposits which have been intruded by basic and acid igneous rocks. The gravimetric deflections of the vertical were adopted.

Rarys Lease (Queensland) Latitude S 20°46'; Longitude E 140°02'

7.4.2 This station is situated on a small granite outcrop in a region of strongly folded geosynclinal sediments. Rarys Lease is near Mary Kathleen between Cloncurry and Mount Isa in a mountainous region of geological structure similar to that near A 432. The distances to the adjacent stations are 50 and 53 km, nearly double the average station spacing for that section of geoidal profile. Xi and Eta are respectively 1.66 and 4.11 seconds of arc different from the gravimetric values. The gravimetric values were adopted.

Bullaway (New South Wales) Latitude S 31°12'; Longitude E 148°56'

7.4.3 The astronomical determinations for this station have been observed on the side of a mountain as the astronomical contractor was unable to get the observational equipment to the summit. The observation station was a distance of 2.5 km from the summit and about 500 metres below it. The values of the deflections were no doubt influenced by the summit. Bullaway is a junction point with adjacent station spacings of 45, 46 and 137 km. The adjacent stations are in a region of gently folded continental deposits which are geologically older than the formation around Bullaway, which consists of continental and marine sediments. Misclosures of adjacent loops all indicated a weakness at Bullaway and the gravimetric substitution of values Xi and Eta improved these misclosures by about 1 metre.

Druid (New South Wales) Latitude S 29°59'; Longitude E 146°34'

7.4.4 The spacing to the adjacent stations are 31 and 40 km, significantly more than the 23 km average station spacing for the section. Druid is situated in a region of aluvium and residual soil, whilst the other stations in this section are in a region of gently folded continental deposits. The astro-geodetic and gravimetric values of the deflections of the vertical differ by approximately 3 seconds in both Xi and Eta compared with differences of less than 0.5 seconds for comparisons at most other stations in the region. The adjacent loop misclosures decreased by approximately 0.5 metres when the gravimetric deflections were adopted.

Peveril (New South Wales) Latitude S 31°20'; Longitude E 142°48'

7.4.5 Peveril lies in a section of geoidal profile running east-west and its astronomic value of Eta differs from the gravimetric by 11 seconds of arc. Astronomical stations adjacent to Peveril indicate that a large local Eta deflection exists in this region. The gravimetric coverage in the area is less reliable than for most sections of the continent. The

rate of change of Eta deflection in this region must be rapid and irregular. Further astronomic and gravimetric studies are planned for this region and the results of these investigations should clarify the situation. Studies of gravity anomaly maps in New South Wales indicate the possible presence of deep rooted anomalous structures which are not related to obvious topographic features. Peveril is situated on the edge of a granite outcrop near the junction of strongly folded Devonian geosynclinal deposits and recent continental sediments. To assume a linear rate of deflection change between adjacent stations was considered more inaccurate than to simply leave Peveril out of all computations.

A 445 (Northern Territory) Latitude S 19°58'; Longitude E 136°

7.4.6 In the vicinity of A445 several small deposits of undeformed limestone and sandstone occur in a region of older folded conglomerate and dolomite. Loop closures, geological information and comparisons with gravimetric data suggested something was wrong at this station. To preserve the astro-geodetic nature of the geoid determination project, in lieu of gravimetric substitution of deflection values, A 445 was removed from all computations.

7.5 There was a great temptation to substitute gravimetric values of the deflections of the vertical whenever substantial misclosures appeared in loops of geoidal profiles. The use of gravimetric and geological information was restricted to the stations A 432, Rarys Lease, Bullaway, Druid, Peveril and A 445 so that the astro-geodetic nature of the geoid determination could be preserved.

7.6 The consultation of medium to large scale geological maps during the planning stages of geoidal profiles does seem to be beneficial and this practice is recommended. If there is no alternative to running a profile through a geoidally disturbed area, the spacing of astronomic stations must be decreased to ensure that reasonable results will be obtained.

8. WEIGHTING SYSTEMS

8.1 In paragraph 6.2 the weighting system in the program DETERGEN was briefly mentioned. Each section of geoidal profile was automatically weighted proportionally to the inverse of its length, and then this inbuilt weight coefficient was multiplied by a factor appropriate to that section.

8.2 From purely practical reasoning, a special weight factor based on the average station spacing in each section was proposed. Experience gained with loop misclosures indicated that where the average station spacings were comparatively small, the loop closures were accordingly low. Large station spacings in a section inevitably produced large loop misclosures.

8.3 The weighting scheme proposed was:

$$\frac{100}{\text{average station spacing}} \qquad \ldots.(8.1)$$

The factor 100 was merely to scale the weights between 0.5 and 6.0. Some typical examples of this weighting scheme are listed below.

Section Classification	Average Station Spacing	Weight Factor
Very poor	150 km.	0.66
poor	100 km.	1.00
fair	60 km.	1.33
good	30 km.	3.33
Very good	20 km.	5

8.4 Adopting equation (8.1) for the additional weight factor, the total weight coefficient in the program DETERGEN was:

$$\frac{1}{\text{total length}} \times \frac{100}{\text{average station spacing}}$$

$$= \frac{100 \times (\text{no. of stations in section} - 1)}{(\text{total length of section})^2} \qquad \ldots.(8.2)$$

$$= \frac{(\text{constant}) \times (n - 1)}{L^2} \qquad \ldots.(8.3)$$

where n = number of stations in section
and L = section length

Theoretical Basis for Additional Weight Factor

8.5 Assume the weighting system is to be based on the expected accuracy of the astronomical determinations. Re-observations at 110 astronomic stations indicate a standard deviation for latitude observations of $\pm$ 0.38 seconds of arc and $\pm$ 0.76 seconds of arc for longitude determinations. This implies that, in general, the deflection of the vertical possesses a standard deviation of about $\pm$ 0.8 seconds of arc.

8.6 If S_1 is the distance between the first two stations in a section then the expected standard deviation (s.d.) can be expressed as:

$$\text{s.d.} = (0.9 \sqrt{2}\, S_1)/(2 \times 10^5) \qquad \ldots.(8.4)$$

Between 2nd and 3rd stations in section,

expected s.d. = $(0.9\sqrt{2}\,S_2)/(2 \times 10^5)$ ....(8.5)

Between n - 1 and nth stations in section,

expected s.d. = $(0.9\sqrt{2}\,Sn-1)/(2 \times 10^5)$ ....(8.6)

Total expected s.d. in section, E, is given by

$$E = (0.9\sqrt{2}\sqrt{S_1^2 + S_2^2 + \ldots + S^2_{n-1}})/(2 \times 10^5) \quad \ldots.(8.7)$$

If S_{av} represents the average station spacing and it can be assumed that

$$(n-1)\,S_{av}^2 = S_1^2 + S_2^2 + S_3^2 \ldots + S_{n-1}^2 \quad \ldots.(8.8)$$

then $$E = (0.9\sqrt{2}\,S_{av}\sqrt{(n-1)})/(2 \times 10^5) \quad \ldots.(8.9)$$

The assumption in equation (8.8) is generally valid for geoid profiles in Australia.

8.7 S_{av} may be expressed as:

$$S_{av} = \frac{L}{n-1} \quad \ldots.(8.10)$$

Inverting and squaring equation (8.9) gives:

$$\frac{1}{E^2} = \left(\frac{2 \times 10^5}{0.9\sqrt{2}}\right)^2 \frac{(n-1)}{L^2} \quad \ldots.(8.11)$$

$$= (\text{constant}) \times \frac{(n-1)}{L^2} \quad \ldots.(8.12)$$

8.8 In a least squares solution the weighting factor should be inversely proportional to the variance. Equation (8.12) satisfies this requirement if the expected standard deviation, E, is considered representative for a section of astro-geodetic levelling.

8.9 The agreement of equations (8.3) and (8.12) led to the adoption of 100/(average station spacing) as the special weight factor for each section of geoidal profile in the program DETERGEN.

TABLE 1

LOOP LENGTHS AND MISCLOSURES

Loop Number	Length	Misclosure	Misclosure/ (Loop Length)½	Loop Number	Length	Misclosure	Misclosure/ (Loop Length)½
	km	metres	metres/km^{½}		km	metres	metres/km^{½}
1	1,002	-0.73	-0.023	49	2,275	-1.72	-0.036
3	1,887	-4.68	-0.108	51	1,985	+0.80	+0.018
5	3,424	+1.14	+0.019	52	1,792	-1.54	-0.036
7	2,185	-0.02	0.000	53	2,574	+3.51	+0.069
10	1,705	+2.49	+0.060	55	1,614	+0.41	+0.010
12	1,663	+8.64	+0.212	56	1,846	-2.53	-0.059
14	1,365	+3.68	+0.100	58	1,838	-0.17	-0.004
15	1,028	+0.47	+0.015	61	2,056	-3.31	-0.073
16	1,563	+2.56	+0.065	62	1,494	+2.66	+0.069
18	721	+3.11	+0.116	64	1,076	+1.25	+0.038
19	777	-5.35	-0.192	65	992	+0.20	+0.006
22	1,211	-1.58	-0.045	66	2,025	-1.04	-0.023
24	2,484	+1.57	+0.032	68	1,920	+1.12	+0.026
26	1,623	-2.18	-0.054	69	1,564	-2.67	-0.068
27	1,911	+0.36	+0.008	70	2,056	+0.46	+0.010
30	897	-1.32	-0.044	71	1,990	+0.68	+0.015
34	2,125	-3.58	-0.078	75	1,479	-0.26	-0.007
35	1,911	+2.23	+0.051	76	1,431	+2.70	+0.071
36	1,373	-1.46	-0.039	82	813	-1.30	-0.046
37	869	-0.17	-0.006	84	1,260	-3.28	-0.092
41	1,456	-1.87	-0.049	86	1,882	+2.12	+0.049
42	2,139	+0.94	+0.020	87	1,917	-1.66	-0.038
43	1,165	+2.71	+0.079	88	3,118	-0.02	0.000
44	1,339	+4.41	+0.121	93	1,080	+2.13	+0.065
45	1,260	-3.14	-0.088				

Average Misclosure (with regard to sign) = +0.14 metres
Average Misclosure (without regard to sign) = +2.00 metres
Average Loop Length = 1,656 km
Average Misclosure/(Loop Length)^{½} (without regard to sign) = 0.052 metres/kms^{½}

9. RESULTS

9.1 An analysis of the loop misclosures appears in table 1. The misclosures for all loops are illustrated in figure 2. The misclosures in the large loops formed by the primary geoidal profiles are shown in figure 3. All misclosures were computed in a clockwise sense.

9.2 An inspection of figure 2 and table 1 indicates three regions in Australia where an explanation of the loop misclosures is warranted.

9.2.1 The large misclosures of +8.64 metres in loop 12 around Arnhem Land is undoubtedly caused by the very large station spacing in section 85. Only five stations span a distance of 1000 km and these stations are either on the coast line or on islands. The rate of change of the deflection of the vertical in such regions is likely to be unstable. The decision to include section 85 in the final adjustment was based on the results of preliminary adjustments when this section was alternatively included and omitted. The resultant changes in nearby junction stations values did not exceed 20 cms. Section 85 had a low weight - see equation (8.12) - and thereby received most of adjustment required to close loop 12.

9.2.2 The relatively large misclosures of -4.68, +3.11 and -5.35 metres in loops 3, 18 and 19 in the region near Adelaide are caused by the lack of astro-geodetic stations in this area. Seven station spacings in these loops are over 100 km, two in excess of 200 km. A densification of astro-geodetic stations in this region will be necessary before any future astro-geodetic geoid determinations.

9.2.3 Loop 44 has a misclosure of +4.41 metres. Sections 121 and 122 which cross Bass Strait have three station spacings in excess of 100 km, one of which exceeds 200 km. Section 121 is scheduled for strengthening during the 1971 field season.

9.3 The mean misclosure, without regard to sign of the 49 loops was 2.00 metres. A histogram of the 49 misclosures with a normal distribution curve superimposed is shown in figure 4. The standard deviation of the superimposed curve is $\pm$ 2.35 metres.

9.4 The sum of the 49 loop misclosures was + 8.25 metres. This value is, in fact, the misclosure of the perimeter, which includes several sections of large station spacing. The perimeter formed by the loops of primary geoidal profiles misclosed by +1.35 metres - see figure 3. These primary profiles were the main framework of the new geoid.

9.5 A histogram of the adjustments to the sections of geoidal profile, in metres is displayed in figure 5. Two of the adjustments are outside the range of this diagram. These occur in sections 85 and 113 in the regions indicated in paragraphs 7.2.1 and 7.2.2.

9.6 A better indication of the accuracy of the geoidal profiles may be obtained from figure 6 where a histogram of the adjustments per square root of section length (metres/$km^{\frac{1}{2}}$) is shown. A normal distribution curve with a standard deviation of $\pm$0.035 metres/$km^{\frac{1}{2}}$ is superimposed.

9.7 In paragraph 6.3 a description of the F test calculation in DETERGEN was presented. The value indicated for the variance of the 124 sections of geoidal profile in the final geoid adjustment was 0.0066 $metres^2$ per km of section length. This value was influenced by the large adjustments to the sections of geoidal profile where the average station spacing exceeded 35 km. For geoidal profiles with an average station spacing less than 35 km, a variance of 0.0030 $metres^2$ per km of section length was obtained.

9.8 The printed output of the DETERGEN geoid adjustment is included as Annex A. Figure 7 shows the numbering system for the junction stations and should be consulted when examining Annex A. Figure 1 may also help.

9.9 The largest standard deviation in N at any junction station with respect to Johnston Geodetic Station was $\pm$1.5 metres. The largest standard deviations were at the junction stations on geoidal profiles around the coast line of Australia. Some caution must be attached to the quoting of the standard deviations of N at junction stations as experience has shown that values of N along a section of widely spaced astro-geodetic stations may change dramatically with the inclusion of more astronomical observations.

10. GRAVIMETRIC INTERPOLATION INSIDE LOOPS OF GEOIDAL PROFILES

10.1 Professor R. Mather at the University of NSW computed values of N and the deflections of the vertical at all the astro-geodetic stations used in the geoid determination. Gravimetric computations were also made at 51 geodetic stations along the profiles of widely spaced astro-geodetic stations and at 1679 points on a half degree grid. Comparisons of the gravimetric and the adjusted astro-geodetic values of N were made for each loop of the geoidal profiles. The results of these comparisons, loop by loop, enabled the adjustment of all the gravimetric values into the system defined by the adjusted astro-geodetic network. The adjustment to a gravimetric value of N at a point inside a loop was a function of the differences between the adjusted astro-geodetic and gravimetric values around the loop perimeter and the squared inverses of the distances from the perimeter stations to that point. In this manner the astro-geodetic framework of 1133 stations was supplemented with 1730 gravimetric values.

11. CHOICE OF N AT JOHNSTON GEODETIC STATION

11.1 In 1966 a value of N = 0 was assigned to Johnston Geodetic Station - see paragraph 2.2. This value was retained for all calculations in the 1971 geoid determination. A

preliminary geoid map was drawn using values of N at 1133 astro-geodetic stations after the section re-adjustments by program GEOIDADJ - see paragraph 6.4.

11.2 This map indicated a minimum value of -4 metres in South Australia and maximum values of +15 to +18 metres on the coastal fringes in Western Australia, Northern Territory New South Wales and Tasmania. In selecting N at Johnston, it is desirable that the average value of N over Australia should be close to zero so that the reduction of measured distances from the geoid to the spheroid should cause no overall change in scale.

11.3 An earth-centred orientation is essential for a world datum, but for local use in Australia a datum which closely fits the geoid is most appropriate. If observed distances are inadvertently reduced only to the geoid, the error made in assuming these distances to be at spheroid level will be less than 2 parts per million (ppm).

11.4 While a choice of N = -6 at Johnston makes the magnitude of the three positive anomalies around the coast (+11, +9 and +12) balance the single negative anomaly of -10 metres in the Nullarbor, it does not balance areas of positive and negative values of N. Only about 30% of Australia has negative values, and to achieve approximate equality in area, a value of N = -8 at Johnston would be better.

11.5 We need, however, not merely to keep values of N close to zero; we also need to make the changes of spheroidal heights, (N + H), and hence of scale, between the 1966 and 1972 geodetic adjustment close to zero, so that the 1971 geoid may refer equally to the results of the 1972 geodetic adjustment. The height of Johnston above the geoid was determined by the 1972 national levelling adjustment to be 566.3 metres, or 4.9 metres lower than the height used in 1966, which was carried in by vertical triangulation from Port Augusta. The change in H around the coast is no doubt close to zero: but we can expect an average change in H along any traverse joining Johnston to the coast of about - 2 metres.

11.6 On balance, therefore, it seemed best to adopt a value of N = -6 metres at Johnston.

12. GEOID MAPS

12.1 A total of approximately 3000 values were available for the automatic contouring of the geoid and the deflections of the vertical maps - see figures 8, 9 and 10. Figure 11 depicts the deviation of the plumb-line at approximately 1150 astro-geodetic stations on the Australian Geodetic Datum. Plotting of the deviations on this map was performed by Engineering Computer Services, Sydney. The program which obtains the deviations of the plumb-line was specially prepared by the Division of National Mapping.

13. INTERPRETATION OF DEFLECTION MAPS

13.1 The contour map of the meridian component of the deflection of the vertical shows a pronounced trend in the east-west direction. The prime vertical deflection contour map likewise shows a trend in the north-south direction. In the following paragraphs it will be shown that these trends in deflection diagrams are to be expected whenever the geoid surface is curved.

13.2 Xi is related to the total deflection of the vertical by the relationship

$$\xi = \zeta \cos A \qquad \text{....(13.1)}$$

and eta by

$$\eta = \zeta \sin A \qquad \text{....(13.2)}$$

where A is the azimuth of the total deflection vector.

13.3 In any region where the geoid surface is curved, the total deflection vector will vary in azimuth, and probably in magnitude, from point to point, causing values of Xi and eta to change. Between the azimuths 0° and 45° the value of the sine function moves through more than 70% of its range, so that if ζ remains constant in magnitude - see figure 12 - nearly $\frac{3}{4}$ of the eta contour lines will have a north-south tendency. In the range 45° to 90° only $\frac{1}{4}$ of the eta contours occur. Likewise the value of the cosine function varies through more than 70% of its range between the azimuths of 45° and 90° and the contour lines of Xi appear horizontally polarised.

13.4 Caution must be exercised in the use of the contour maps of the deflections. The average data point spacing is approximately 50 km and the standard deviation of gravimetrically computed deflections is about 2 seconds of arc. The deflections vary irregularly in hilly regions with a wavelength much less than 50 km. Figures 9, 10 and 11 can, however, help indicate regions where the deflections are likely to be small if observation sites are carefully chosen; and regions where they are likely to be large and changing rapidly.

14. CONCLUSIONS

14.1 The 1966 values of the geodetic co-ordinates at Johnston Geodetic station did not need alteration but the value of N was changed from N = 0 to N = -6 metres.

14.2 The scale differences between the 1966 and the proposed 1972 horizontal control adjustments will be less than 2 ppm for lines along the coast and less than 1 ppm for lines radiating from Johnston to the coast.

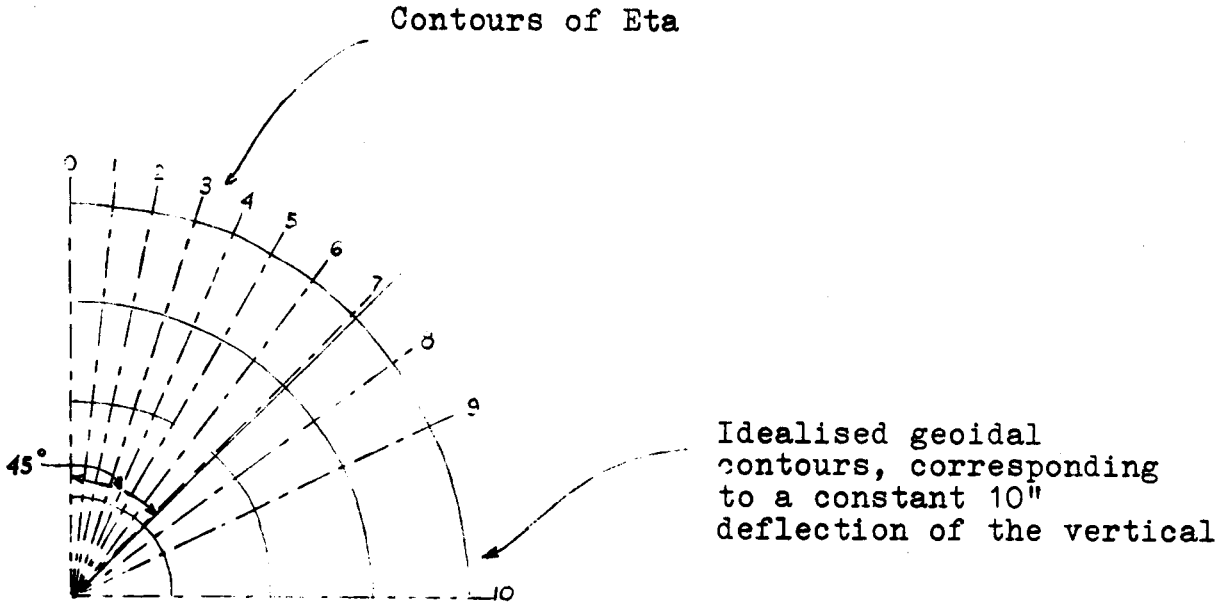

FIGURE 12

NORTH-SOUTH TENDENCY IN CONTOURS OF ETA

Notes:

1. The magnitude of the deflection of the vertical is everywhere assumed to be 10 seconds of arc in this example.

2. The contours of Eta equal to 0, 1", 2", 3", 4", 5", 6" and 7" all lie in azimuth between 0° and 45°, i.e., tending to run North-South.

3. The 8", 9" and 10" contours will, of course, lie between the azimuths 45° and 90°.

14.3 The 1971 geoid determination will be valid for the system of geodetic coordinates resulting from the 1972 adjustment. The changes in coordinates from the 1966 adjustment are not expected to exceed 3 metres.

15. ACKNOWLEDGEMENTS

15.1 Numerous officers in the Division of National Mapping have of course played a part in this project. The author of this report would however particularly like to remind the reader of the important roles played by:

15.1.1 Professor R. S. Mather of the University of NSW, in his contribution on the gravimetric side.

15.1.2 Mr A. Krisjanis, who was responsible in the years 1964-70 for the computation of all the astronomic observations.

15.1.3 All those who, often in conditions of great discomfort made the astronomical observations, those who booked for them, and those who helped them carry their equipment up and down the hills.

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APPENDIX 1

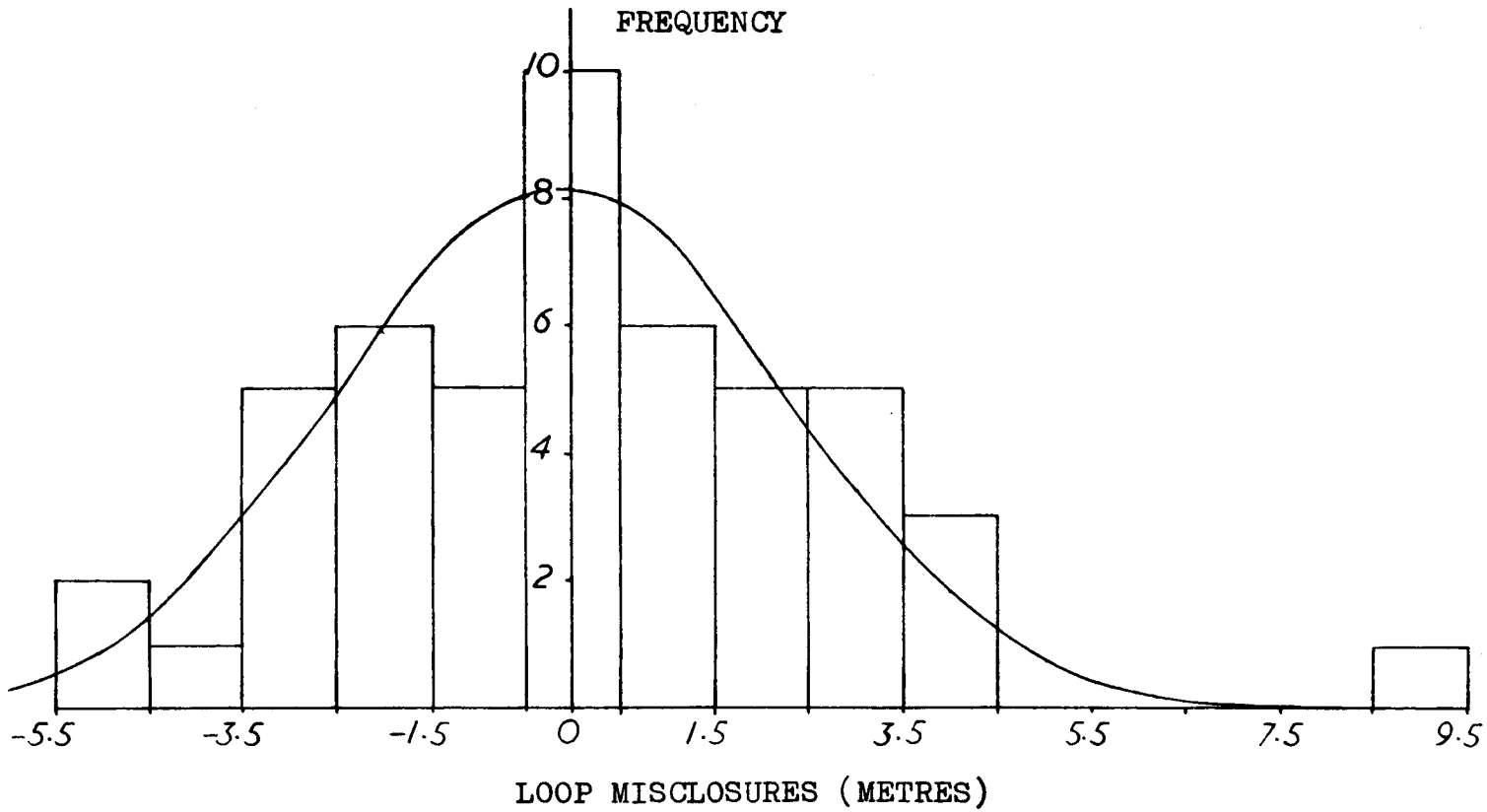
A PRELIMINARY GEOID FOR PAPUA AND NEW GUINEA

An extension of the geoid in Australia to Papua and New Guinea (PNG) was made using the 117 astro-geodetic stations which have geodetic coordinates on the Australian Geodetic Datum. The location of the astro-geodetic stations was not, in general, suitable for the method of geoidal profiles used in Australia. Only in New Britain and New Ireland were the station spacings small enough to adopt this procedure. In all other regions each station was considered as a junction station and the differences in N between adjacent stations were used directly in the adjustment.

The average misclosures without regard to sign for 57 inland loops was 2.5 metres and for 17 loops between islands 6.8 metres. The average perimeter lengths for these loops were 348 km and 772 km respectively. The accuracy of the astronomic observations is generally lower than in Australia due to the persistent cloud in the mountainous regions. The method of position lines has been used at most of the astro-geodetic stations with Wild T3 or geodetic Tavistock theodolites.

The value of N at Thursday Island was held at +1.18 metres from the Australian adjustment, corresponding to N = -6 metres at Johnston Geodetic Station. The statistical tests in program DETERGEN - see paragraph 6.3 - indicated a variance of 0.035 metres^2 per km for astro-geodetic levelling in PNG. This variance is about 5 times larger than the value in Australia. The largest standard deviation for the value of N at any astro-geodetic station relative to Thursday Island was $\pm$6 metres. This value was obtained for Buka Island, the easternmost station. On the mainland of PNG the standard deviations were about $\pm$3 to $\pm$4 metres, increasing with distance from Thursday Island.

Deflections of the vertical larger than 1 minute of arc often occur in PNG. The whole region may be described as geologically disturbed, gravity anomaly maps indicating large variations in most areas. Mountains up to 4500 metres high and deep ocean trenches contribute to the steep geoidal gradients shown in figure 13.

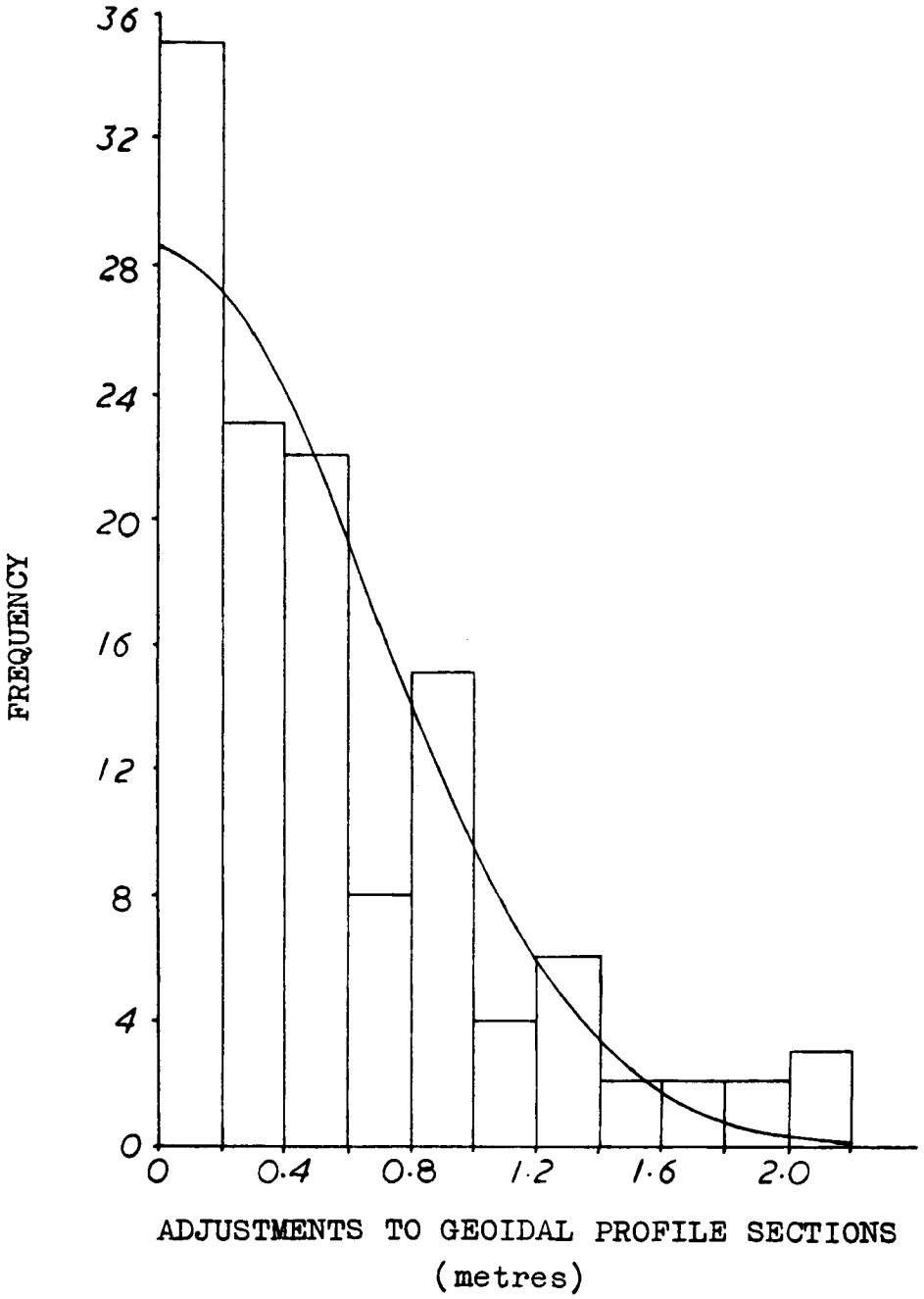


HISTOGRAM OF 49 LOOP MISCLOSURES

GEOID ADJUSTMENT 1971

Normal Distribution Curve with Standard Deviation ± 2.35 metres superimposed

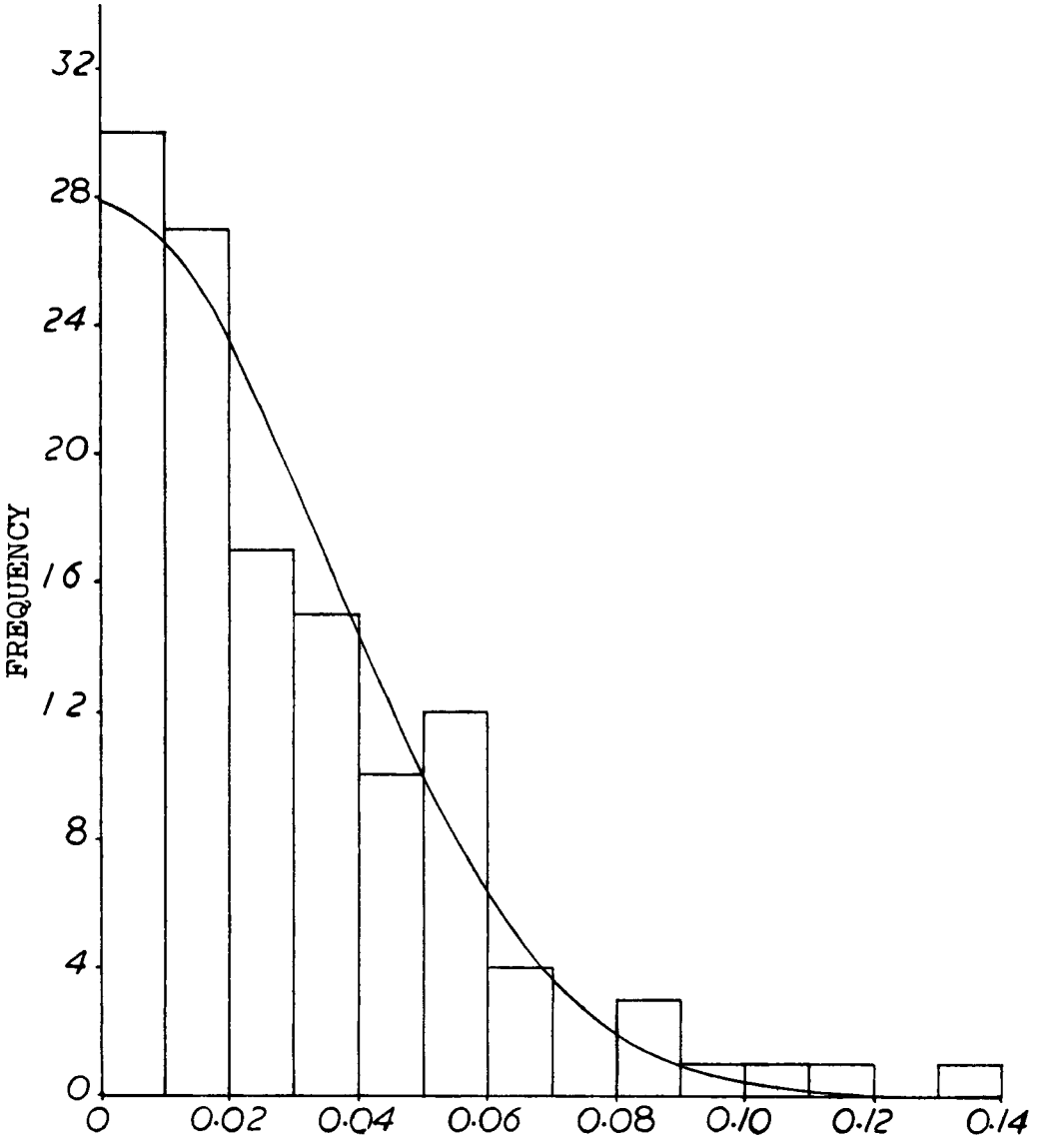
Misclosures calculated clockwise. As at January 1971



HISTOGRAM OF ADJUSTMENTS

GEOID ADJUSTMENT 1971

Normal Distribution Curve with Standard Deviation
± 0.68 metres superimposed



ADJUSTMENTS PER km.$^{\frac{1}{2}}$ OF SECTIONS OF GEOIDAL PROFILE

(metres/km.$^{\frac{1}{2}}$)

HISTOGRAM OF ADJUSTMENTS PER km.$^{\frac{1}{2}}$ OF
SECTIONS OF GEOIDAL PROFILE

GEOID ADJUSTMENT 1971

Normal Distribution Curve with Standard Deviation

±0.035 metres/km.$^{\frac{1}{2}}$ superimposed

FIGURE 6