DEPARTMENT OF MINERALS AND ENERGY

Secretary: Sir Lenox Hewitt O.B.E.

DIVISION OF NATIONAL MAPPING

Director: B.P. Lambert O.B.E.

TECHNICAL REPORT 19

HEIGHTING BY VERTICAL ANGLES TORRES STRAIT ISLANDS 1973

by

D.P. Cook and J.B. Steed

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1974

HEIGHTING BY VERTICAL ANGLES - TORRES STRAIT 1973

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1. INTRODUCTION

1.1 Purpose of the Survey

For tidal studies in the shipping lanes of Torres Strait the Department of Transport required accurate heights at eight sites for proposed tide gauges, referred to the mainland datum.

As shown in Division of National Mapping Technical Report 12 on The Adjustment of the National Levelling Survey 1970-71, mean sea level rises about 1.7 metres over 1900 km from Bundaberg to Cape York then falls again to its original value at Centre Island, 800 km to the south west. Mean sea level in the Torres Strait area is not usable as an equipotential reference surface.

At Bamaga, mean sea level at the Red Island Point tide gauge, determined over the 1968-70 period, had been incorporated in the Australian Height Datum, and it was decided to start the survey there.

An accuracy of 5 cm in height at the eight gauge sites was the original goal. Bets Reef, 90 km north east of Thursday Island, was so far from the mainland that it was considered to be beyond the range of any projected method and it was dropped, leaving seven sites, at Albany, East Strait, Twin, Wednesday, Hammond, Goode and Booby Islands.

A diagram of the scheme, also showing angular standard errors and lengths of lines is on the following page.

1.2 Reason for the Report

While the opportunity was taken to recover and strengthen a number of old geodetic surveys in the area, dating back over fifty years, this was considered a routine operation within the Division.

This report deals only with the techniques developed to carry heights over long lines across water. They will most likely be used again within the Division and copies of forms, program listings and step by step instructions for observing and calibration of bubbles are included for that purpose. The methods may also be of use to others.

2. SELECTION OF METHOD

Only three methods were considered:

Differential (or spirit) levelling was used of course for levelling over land through Galloways to Albany Passage but no development of it was thought practicable for lines of 25 km and more over water.

Hydrostatic levelling was considered to be impracticable on the grounds of expense and the necessity for special equipment.

Vertical angles as observed on normal first order traverse consist of two sets observed with at least fifteen minutes separation in time, and repeated on a second day. One set consists of three or four pairs and a pair consists of a double pointing on face left followed by a double pointing face right.

All readings are made with the alidade bubble centred and the rejection criterion is a range of more than eight seconds in a set.

It was decided to try and develop this method.

3. EQUIPMENT

All observations were made with Wild T3 theodolites with graduated alidade bubbles. They were all "double reading", with one minute micrometer run and two minute minimum reading on the circles. Generally observations were made to 10 volt Lucas lights, fully variable through rheostats.

Observing tents were used at all-times to shelter the instruments from wind and sun and the theodolite tripods stood on steel pegs.

For communications each station used an NEC ATR 400PI mini-transceiver, on a frequency of 470 MHz which gives interference free reception. Their line-of-sight range is more than 90 km.

An automatic level was used to read the heights of instrument and target, and also for level connections to other marks, whilst the slope distances between stations were measured with Model 8 laser Geodimeters.

The alidade bubbles on Wild theodolites are normally not graduated. It is however a simple matter to have a transparent scale fixed on top of the bubble. The graduation interval is not critical as only the separation of the bubble ends is read. All National Mapping T3s have graduations at 2 mm per division. Home-made attachments are used to illuminate the alidade bubbles.

No other special equipment was used.

4. TRIAL OBSERVATIONS

4.1 Site

In order to test the methods available a line was selected in Canberra, 4.2 km long, passing low over Lake Burley Griffin for most of its length. It was hoped that this line would be fully saturated with water vapour, as would be the lines in Torres Strait.

The terminal points of the line were connected by conventional third order levelling to the Canberra city network to give a direct comparison with results obtained from the trigonometric heighting.

4.2 Results

Measurements of reciprocal vertical angles across the lake were carried out on four different days over a two month period in various weather conditions, at different times of the day and by different methods. One set was observed over sunset, generally thought to be the worst time of day for vertical angle observations.

The results were:

DAY	TIME OF DAY	DIFF. HT.	STD ERROR	DIFF. FROM LEVELLED VALUE
	Hours	Metres	Metres	Metres
1	1100	2.860	.003	+.016
2	1145	2.907	.006	-.031
	1330	2.923	.005	-.047
3	1115	2.867	.004	+.009
	1215	2.871	.004	+.005
	1410	2.881	.003	-.005
4	1710	2.912	.005	-.036
	1800	2.893	.006	-.017

Levelled difference in height = 2.876 m

On Day 1 observations were made continuously for forty minutes, giving the result shown in the above table. As well as this however means and standard errors were computed using the program at Annex A, on the Hewlett Packard 9100B calculator, for every minute from one to forty. By inspection of the results it was seen that no improvement was evident in the standard error of the mean after about twenty minutes. For this reason it was decided that twenty minutes was sufficient for one set of observations.

It was also observed from the results that the time of day seemed to have less effect on the results than the quality of the target, so that pointing accuracy determined the quality of the final result. This was as it theoretically should be, since the observations were truly simultaneous over symmetrical topography.

The test justified going ahead with the proposed survey, though further refinements would have to be developed for field conditions, where much longer and less controlled lines would be encountered.

4.3 Weather Conditions

During the tests over Lake Burley Griffin, it was very cold and calm, with some fog on one occasion. These conditions are conducive to layering and therefore far from ideal for trigonometric heighting.

On the other hand, during the job in the Torres Strait, it was consistently warm with better than twenty knot winds blowing day and night. This ensured a thoroughly mixed atmosphere, conducive to accurate heighting by this method.

4.4 Methods

In all observations a double pointing was made on face left, then two double pointings on face right, followed by another double pointing on face left. This procedure was repeated until the set was completed. The time was noted at each pointing. Rather than zeroing the bubble at each observation, it was read against the graduations and a correction to the reading was calculated from the bubble calibration. After each observation the bubble was moved across zero with the final motion against the spring of the alidade bubble screw and had time to settle before the next pointing.

For the observations made on Days 1 and 2 on the test line in Canberra, watches were synchronised and observations commenced at the same time. Each observer then continued at his own pace for a given period of time.

The angles thus observed at each end were then graphed against the same time scale and since the time interval between each observation was small, of the order of one minute, it was assumed that any change in the coefficient of refraction was linear. With this in mind, pseudo-simultaneous reciprocal angles were read off the graph for every minute over the period of observation. The angles thus obtained were then used to calculate a difference in height for the line.

At first the angle reduced from each face left and face right single pointing at each end was graphed and a face left and face right angle for each end interpolated for each minute.

Later, in an effort to reduce work, angles reduced from the means of each face left and face right double pointing were graphed and then simultaneous angles interpolated from the graph. This cut down the amount of reductions by about fifty percent.

From the results it became clear that the second, streamlined, method was no less accurate than the first.

Because the proposed survey was to be run on a tight time schedule, involving a helicopter and boats, it was decided to try and reduce the computations involved even further.

For this reason observations on Days 3 and 4 were made as before, but exactly simultaneously, by having the booker at the slower observer's end call the instant of pointing for both ends, over the two way radio. This meant that the angles observed could be used directly to calculate the difference in height, with a minimum of reductions.

With a little practice, this method proved to be quite successful and no real difference could be detected in the quality of the results of the various methods.

It was decided to adopt the method used on Days 3 and 4, but to still book the time of the observation at each end. This was a safety measure in case radio contact was lost or one of the observers got out of step, (reading on face left when the other observer was on face right). It also enabled the results at each end to be compared if necessary, at a later date, for the purpose of rejecting any poor observations. As it turned out none of these measures was necessary.

With the adopted method, the booker must be thoroughly conversant with the procedures, as he is particularly busy, especially if he is also calling the pace over the radio.

5. FIELD METHOD

After a short time of actually using the adopted method in the field, several refinements were included to improve the quality of the work and to make observing easier.

5.1 Bubble

Firstly, it was decided to use a footscrew, rather than the alidade bubble screw, to move the alidade bubble. This meant that the observer did not have to worry about moving the bubble against the spring and since the footscrew is less sensitive, accurate movement of the bubble was quicker and easier.

To avoid difficulty and parallax in reading the alidade bubble, it was found best to keep the movement of the bubble within the range +2 to -2 divisions.

It was also decided to keep a running total of the bubble readings on FL and FR and to try and keep these totals equal, since if the total FL - FR is zero, the mean of all readings is independent of the bubble value, thus eliminating the effect of error in the bubble calibration.

5.2 Instrument and Target Heights and Location

As originally intended, all heights of instrument and targets were measured using a level and tape, thus enabling, by using two setups of the level,

two independent measurements of the heights to millimetre accuracy.

To ensure complete accuracy any eccentric target light was always set at right angles to the line observed, so that angles at both ends were observed over the same distance. This would not be so if the lights were placed on line.

Where possible the light was plumbed beneath the theodolite, ensuring that the observed point would be correct.

It was even found over very short lines in good conditions, that it was possible to observe directly to the later type of vertical circle light, which is colinear with the trunnion axis of the theodolite. With the older model of the light it was possible for the light not to be level with the trunnion axis on steep sights.

In some cases, such as existed on Tuesday Island, it was impossible to set the target light either at right angles or underneath the theodolite. In such a case, care had to be taken in the computations that the distance used was the slope chord distance from the theodolite to the target. This meant, a different distance had to be used for the calculation from each end.

5.3 Number of Observations

It was quickly realised that three of the booking sheets, as at paragraph 5.6, would take twenty to twenty five minutes to complete, so it was decided to observe three complete pages rather than to a time limit. This meant that all lines had the same number of pointings and also gave a check, in that both observers should finish three complete pages on the same pointing, otherwise someone was out of step.

5.4 Observing Procedure

To obtain the best results all normal first order observing procedures were followed. That is, all final movements of the tangent screw were made in a clockwise direction against the spring and were backed off again after each pointing. Similarly the micrometer drum was used consistently in the one direction, and all pointings were made close to the centre of the cross hairs.

After the target lights were aligned and the intensity adjusted using the rheostats, and the watches synchronised, the observations proceeded as below, each pointing taking place the instant the booker called, "4 - 3 - 2 - 1 'point'."

Each pointing consisted of:

1. Intersecting the target with the horizontal cross hair, the booker noting the time of the pointing.

2. Reading the vertical circle

3. Reading the alidade bubble, estimating to 1/10 of a division and giving the appropriate sign as indicated on the booking sheets.

4. Moving the alidade bubble off a small amount with a footscrew.

5. Moving the cross hair off the target ready for the next pointing.

Two such pointings were made on face left, then four on face right and four on face left and so on. At the end of the allotted time or number of pointings, the observations finished on two pointings on face left.

Any number of pointings may be used, provided there is an equal number of face left and face right pointings to cancel any errors due to vertical circle index error.

5.5 Booking

During the observations the booker noted the time and vertical circle readings in the Observed Time and Observed Vertical Circle columns of the booking sheets, as shown on the following pages. At the same time he noted the bubble readings in the Observed Bubble column and summed the pairs of bubble readings for each face, writing the answer in the Bubble Sum columns, Left or Right as appropriate. For each subsequent double pointing a running aggregate of FL and FR bubble readings was entered in the appropriate column.

On the Wild double reading T3 the sum of the seconds of a double pointing was entered during observations when possible to give a complete reading, as shown in the Vertical Circle Reduced column.

5.6 Field Reductions

The Vertical Circle Reduced reading was still only half the vertical angle and so had to be doubled and the result entered in the Vertical Angle + or - column. This valuewas then corrected for the alidade bubble reading.

Because the angle read on one face was only half the actual vertical angle, the bubble correction to be applied to the vertical circle on one face was the algebraic bubble sum multiplied by half the bubble value. This correction with the appropriate sign from the algebraic bubble sum, was added to the absolute value of the angle in the Vertical Angle + or - column, to give the corrected vertical angle.

The procedure to reduce the observations from vertical circle reading to vertical angle on one face is as follows:

1. Add the seconds of a double pointing, from the Vertical Circle Observed column, noting any change in the minutes, and enter the result in the Vertical Circle Reduced column.

2. Double the angle formed by the degrees in the Observed column and the minutes and seconds in the Reduced column and enter the result in the Vertical Angle + or - column.

3. Sum the observed bubbles for a double pointing, enter it in the Bubble Sum column, multiply this by half the bubble value and enter the result in the Correction column, with the same sign as the Bubble Sum.

4. Apply this correction to the value in the Vertical Angle + or - column and enter the answer in the Vertical Angle Corrected column.

5. In the Vertical Angle Absolute column enter 180° plus or minus Corrected Vertical Angle. This answer is the true vertical angle measured on one face only.

HEIGHTING BY VERTICAL ANGLES

OBSERVER W. JEFFREY
RECORDER R. SMALL
INSTRUMENT T3 82990
ALIDADE BUBBLE VALUE 6·25"/DIVISION
BUBBLE VALUE ÷ 2 3·125

LEVEL READINGS

STN. MK. + 1·812
INSTRUMENT ± + 0·026
HT. INST.
ABOVE STN MK + 1·838

STN. MK. + 1·812
TARGET ± − 0·769
HT TARGET
ABOVE STN MK + 1·043

DOUBLE READING T3

F. L. 90° 01′ IS +VE
F R 90° 01′ IS −VE

FACE	TIME		VERTICAL CIRCLE		VERTICAL ANGLE + OR −	BUBBLE				VERTICAL ANGLE + OR −	
	OBSERVED	MEAN	OBSERVED	RED'D		OBSD	SUM L (AGGR)	SUM R (AGGR)	CORR'N	CORRECTED	ABSOLUTE
L	21 16 55 / 17 20	21 17 08	89 58 49·3 / 58 49·1	59 38·4	179 59 16·8	−0·1 / +0·0	−0·1		−0·31	179 59 16·49	−00 00 43·51
R	19 20 / 19 50	21 19 35	90 00 11·4 / 00 11·2	00 22·6	180 00 45·2	+0·3 / +0·2		+0·5	+1·56	180 00 46·76	00 46·76
R	20 20 / 20 50	21 20 35	00 12·7 / 00 11·8	00 24·5	180 00 49·0	−0·3 / −0·1		−0·4	−1·25	180 00 47·75	00 47·75
L	21 50 / 22 22	21 22 06	89 58 49·9 / 58 49·3	59 39·2	179 59 18·4	−0·4 / +0·0	−0·4		−1·25	179 59 17·15	00 42·85
L	22 55 / 23 22	21 23 08	58 49·6 / 58 49·7	59 39·3	179 59 18·6	−0·1 / −0·1	−0·2		−0·62	179 59 17·98	00 42·02
R	24 22 / 24 52	21 24 37	90 00 11·6 / 00 11·4	00 23·0	180 00 46·0	+0·2 / +0·3		+0·5	+1·56	180 00 47·56	00 47·56
R	25 22 / 25 57	21 25 40	00 11·7 / 00 11·9	00 23·6	180 00 47·2	+0·1 / +0·0		+0·1	+0·31	180 00 47·51	00 47·51
L	26 52 / 27 22	21 27 07	89 58 48·9 / 58 50·6	59 39·5	179 59 19·0	+0·6 / −0·6	+0·0 (−0·7)	(+0·7)	+0·0	179 59 19·00	00 41·00

FACE LEFT
BUBBLE −
BUBBLE +

FACE RIGHT
BUBBLE −
BUBBLE +

"TOP TO THE RIGHT BUBBLE IS POSITIVE"

REDUCED R.S. CHECKED on HP 14 NOV 73

DAY & DATE THU 1 NOV 73
STATION OBSERVED ENTRANCE
AT STATION DOUBLE

HEIGHTING BY VERTICAL ANGLES

Page of

DOUBLE READING T3

OBSERVER
RECORDER
INSTRUMENT
ALIDADE BUBBLE VALUE "/DIVISION
BUBBLE VALUE ÷ 2

LEVEL READINGS

STN. MK. + INSTRUMENT ± _____ HT. INST. ABOVE STN MK

STN. MK. + TARGET ± _____ HT TARGET ABOVE STN MK

F. L. 90° 01′ IS +VE

F R 90° 01′ IS −VE

FACE	TIME		VERTICAL CIRCLE		VERTICAL ANGLE + OR −	BUBBLE				VERTICAL ANGLE + OR −	
	OBSERVED	MEAN	OBSERVED	RED'D		OBSD	SUM L (AGGR)	SUM R (AGGR)	CORR'N	CORRECTED	ABSOLUTE

FACE LEFT

BUBBLE −

BUBBLE +

FACE RIGHT

BUBBLE −

BUBBLE +

"TOP TO THE RIGHT BUBBLE IS POSITIVE"

REDUCED CHECKED

DAY & DATE

STATION OBSERVED

AT STATION

6. OFFICE COMPUTATIONS

On return to the office all field computations were checked using programs on the Hewlett Packard 9100B computer. The program at Annex A was used to determine the standard error of the mean slope angle for each line. The angles both ways on each line were summarised on the form on the following page.

Because the value for the alidade bubble calibration was sometimes slightly different to that used in the field reductions, a correction was computed and applied where necessary to the slope angles of the lines. A calculation form and example are on the following pages.

Using the corrected angles the differences in height were calculated again on the 9100 B.

The results of these calculations are shown in the diagram on the following page.

Orthometric corrections were found to be negligible.

7. ALIDADE BUBBLE CALIBRATION

Where the bubble aggregates on each face were not close to equal the bubble value became important. The one second micrometer divisions on a double reading T3 are in fact four seconds of arc; things are not as precise as they first appear.

Calibrations done in the field differed by up to 0.4 seconds per division from calibrations done some months previously and this was ascribed to the difference in temperature. Changes in bubble length, viscosity and radius had probably occurred. The field calibrations were adopted.

Values derived from hand plotting of the calibrations were adopted for field calculations. All calibrations were later computed by a linear regression formula giving the best straight line fit to the observed values. The slope of the line between readings +2 and -2 was generally adopted. Outside these points the graph usually started to curve, at least partly due to parallax when reading the bubble.

The formula used is in the Hewlett Packard program library for the 9100B calculator, Part No. 09100-70803.

The calibration procedure is set out at Annex C, as it is not a job which surveyors do every day.

8. ADJUSTMENT

The height differences were adjusted by program TRIGHT, which is an adaptation of LEVEL 1 described in Technical Report 12. It is a least squares adjustment by observation equations.

8.1 Weights

The height difference obtained by differential levelling over the 35 km from Galloways to Somerset was held fixed by allotting a very high weight.

VERTICAL ANGLES

DAY & DATE

TIME	FACE LEFT	FACE RIGHT	MEAN
MEANS			

THEOD : ½ BUB VAL : BUBBLE ΣL: ΣR:

INST. HEIGHT: TARGET HEIGHT: AT STATION

MEANS			

THEOD: ½ BUB VAL: BUBBLE ΣL: ΣR:

INST. HEIGHT: TARGET HEIGHT: AT STATION:

REDUCED CHECKED OBSERVED STATION:

VERTICAL ANGLES

DAY & DATE *Thu 1 NOV 73*

TIME	FACE LEFT	FACE RIGHT	MEAN
21 17 15	- 0° 05′ 47.″8	- 0° 05′ 47.″4	- 0° 05′ 47.″6
	48·6	47·7	48·2
	46·4	48·0	47·2
	46·9	45·1	46·0
	47·8	48·6	48·2
	46·6	47·3	47·0
	43·7	47·0	45·4
	48·3	48·1	48·2
	47·9	49·7	48·8
	46·2	48·5	47·4
	48·4	49·8	49·1
21 47 15	47·2	48·2	47·7
MEANS	- 0° 05′ 47.″2	- 0° 05′ 48.″0	- 0° 05′ 47.″6

THEOD: *T3 83450* ½ BUB VAL: *3.″24* BUBBLE ΣL: *-1·1* ΣR: *+1·0*

INST. HEIGHT: *+ 1·687 m* TARGET HEIGHT: *+ 0·848 m* AT STATION *ENTRANCE*

21 17 08	- 0° 00′ 43.″5	- 0° 00′ 46.″8	- 0° 00′ 45.″2
	42·8	47·8	45·3
	42·0	47·6	44·8
	41·0	47·5	44·2
	43·3	47·6	45·4
	42·5	48·0	45·2
	42·4	49·1	45·8
	40·9	48·7	44·8
	41·3	49·5	45·4
	42·5	49·7	46·1
	44·0	47·0	45·5
21 47 07	42·1	47·9	45·0
MEANS	- 0° 00′ 42.″4	- 0° 00′ 48.″1	- 0° 00′ 45.″2

THEOD: *T3 82990* ½ BUB VAL: *3.″12* BUBBLE ΣL: *-1·3* ΣR: *+1·3*

INST. HEIGHT: *1·838 m* TARGET HEIGHT: *+ 1·043 m* AT STATION: *DOUBLE*

REDUCED *J.C.* CHECKED *G.D.* OBSERVED STATION: *ENTRANCE*

CORRECTION TO SLOPE FOR CHANGE IN ALIDADE BUBBLE VALUE

Σ F. L. BUBBLE	− 1.1	Σ F. L. BUBBLE	−1.3
Σ F. R. BUBBLE	+ 1.0	Σ F. R. BUBBLE	+ 1.3
NEW $\frac{\text{BUB VAL / DIVISION}}{2}$	3.13	NEW $\frac{\text{BUB VAL / DIVISION}}{2}$	3.21
OLD $\frac{\text{BUB VAL / DIVISION}}{2}$	3.24	OLD $\frac{\text{BUB VAL / DIVISION}}{2}$	3.12
NEW − OLD $(\frac{\Delta}{2})$	− 0.11	NEW − OLD $(\frac{\Delta}{2})$	+ 0.09

$$\text{CORRECTION TO SLOPE} = \frac{\Delta/2}{\text{Number of double pointings on each face (4 per page)}} \times \frac{\Sigma\,\text{F. L. BUBBLE} - \Sigma\,\text{F. R. BUBBLE}}{2}$$

PREVIOUS SLOPE A to B	− 0° 05′ 47.6″	PREVIOUS SLOPE B to A	− 0° 00′ 45.2″
CORRECTION	+ 0.01	CORRECTION	−0.01
CORRECTED SLOPE A to B	−0° 05′ 47.6″	CORRECTED SLOPE B to A	−0° 00′ 45.2″
STATION A	ENTRANCE	STATION B	DOUBLE
DAY, DATE & TIME	Thu 1 NOV 73		2117 − 2147

DIFF HT A to B = SINE SLOPE A to B × DIST A to B + HT INST A − HT TARGET B

DIFF HT B to A = SINE SLOPE B to A × DIST B to A + HT INST B − HT TARGET A

DIFF HT A to B	− 21.280	DIFF HT B to A	− 1.861

MEAN A to B − 9.710

CALCULATED JBS CHECKED JC 22 NOV 73

on HP 30 DEC 73

CORRECTION TO SLOPE FOR CHANGE IN ALIDADE BUBBLE VALUE

Σ F. L. BUBBLE	Σ F. L. BUBBLE
Σ F. R. BUBBLE	Σ F. R. BUBBLE
NEW $\frac{\text{BUB VAL / DIVISION}}{2}$	NEW $\frac{\text{BUB VAL / DIVISION}}{2}$
OLD $\frac{\text{BUB VAL / DIVISION}}{2}$	OLD $\frac{\text{BUB VAL / DIVISION}}{2}$
NEW - OLD $(\frac{\Delta}{2})$	NEW - OLD $(\frac{\Delta}{2})$

$$\text{CORRECTION TO SLOPE} = \frac{\Delta/2}{\text{Number of double pointings on each face (4 per page)}} \times \frac{\Sigma\,\text{F. L. BUBBLE} - \Sigma\,\text{F. R. BUBBLE}}{2}$$

PREVIOUS SLOPE A to B	PREVIOUS SLOPE B to A
CORRECTION	CORRECTION
CORRECTED SLOPE A to B	CORRECTED SLOPE B to A
STATION A	STATION B

DAY, DATE & TIME

DIFF HT A to B = SINE SLOPE A to B × DIST A to B + HT INST A - HT TARGET B

DIFF HT B to A = SINE SLOPE B to A × DIST B to A + HT INST B - HT TARGET A

DIFF HT A to B	DIFF HT B to A

MEAN A to B

CALCULATED CHECKED

Program TRIGHT gives observations of height difference a weight of $1/d^2$, where d is the distance between stations, on the basis that the relevant error propagates in direct proportion to the distance. In addition a specific weight, shown as 'weight indicator' in the output, may be entered for each line in accordance with the estimated accuracy of the height difference.

In this survey all observations were reciprocal and simultaneous but internal standard errors (SE) ranged from 0.1 to 0.4 seconds.

For a first adjustment the value of $1/SE^2$ was entered for the specific weight on each line, giving a range in the specific weights from 1 to 16, and this was then multiplied internally by $1/d^2$ to give the total weight. Where a line was observed twice the mean difference of height was used but the standard error of the whole observation was calculated separately. For two sets of twelve mean slopes the combined standard error was $0.5(SE_1^2 + SE_2^2)^{\frac{1}{2}}$

A second adjustment was made with uniform specific weight on all lines. A comparison between the two adjustments showed that, apart from Booby and Despair, whose heights changed 0.053 and 0.047 m respectively, largely because of the long 27.7 km non-symmetrical line between them, heights on all stations changed by no more than 0.022 m, with an average of 0.007m.

The mathematical model for the second adjustment was free of distortions caused by weights, other than the simple function $1/d^2$, and from its variance-covariance matrix the relative standard errors could be calculated, as set out in paragraph 8.3. Because of the insignificant changes in height between the two adjustments and the desirability of knowing the relative standard errors, the second adjustment, with angular observations on all lines given an equal weight indicator was adopted.

8.2 Variance Ratio Test

From the mean angular standard error of 0.25 seconds for all lines and the mean length of line of 11.3 km, a standard error in height at the end of each line, called S, was estimated at 0.15 m.

A variance factor $\bar{S}^2 = \frac{\Sigma W.e.e}{R}$ was calculated internally by the adjustment program, where W is the weight of an observation, e the adjustment to that observation and R the number of redundant observations.

If the mathematical model is free of undue distortions due to mistakes or unsuspected systematic errors the quantities S^2 and $\bar{S}^2$ should be approximately equal (see paragraph 6.3.3 of Project Surveying (1.)).

The variance ratio $\bar{S}^2/S^2$ is not normally distributed but is distributed according to the F distribution $F\,0.95_{r1\ r2}$, where r1 is the number of redundant observations in $\bar{S}^2$ and r2 is the degrees of freedom in S^2, when a 0.05 level of significance is adopted. In this adjustment r1 was 17 and r2 was taken as infinity, assuming the estimator 0.015 to be accurate. From the nomogram at page 444 of "Project Surveying" (1) F was found to be 1.6.

1. P. Richardus et al. "Project Surveying" 1966. Published by North-Holland Publishing Company. Amsterdam.

The variance ratio calculated for the adjustment was 1.3, smaller than the F value, showing that the mathematical model and selected observations were free of undue distortions.

The adjusted heights, test values and the variance covariance matrix are on the following pages.

PROGRAM AMENDED FEB 1971 TRIG HEIGHT ADJUSTMENT OF AUSTRALIA PROGRAM TRIGHT COMPUTED ON 01/02/74

TORRES STRAIT - HEIGHTING BY VERTICAL ANGLES = 1973

ALL HEIGHTS AND HEIGHT DIFFERENCES ARE IN METRES

HEIGHTS ARE ON THE AUSTRALIAN HEIGHT DATUM

WEIGHT = WEIGHT INDICATOR/D SQUARED, D = DISTANCE IN KILOMETRES

WEIGHT INDICATORS = 1/SE SQUARED SE = 0.25

SERIAL NUMBERS FROM A	TO B	DISTANCE (KM)	WT IND	WEIGHT	OBS HT DIFF	ADJ HT DIFF	ADJ-OBS	STATION B	ADJ HT OF B
1	2	7.2	16.2	.313	-12.371	-12.351	.020	HIGH	78.019
2	4	12.4	16.2	.105	22.018	22.098	.080	ENTRANCE	100.117
4	3	9.5	16.2	.180	-14.259	-14.213	.045	DAYMAN	85.904
3	2	6.1	16.2	.435	-7.890	-7.885	.005	HIGH	78.019
3	1	6.0	16.2	.450	4.453	4.466	.013	GALLOWAYS NEW	90.370
4	5	11.6	16.2	.120	144.962	144.985	.023	SCOTT	245.102
5	6	7.7	16.2	.273	-186.042	-186.016	.026	GREEN	59.086
6	7	5.5	16.2	.536	31.314	31.328	.014	DOUBLE	90.414
7	8	6.3	16.2	.408	15.374	15.385	.011	SHERARD	105.799
8	4	9.1	16.2	.196	-5.698	-5.682	.016	ENTRANCE	100.117
5	8	10.9	16.2	.136	-139.294	-139.303	-.009	SHERARD	105.799
7	4	13.0	16.2	.096	9.710	9.703	-.007	ENTRANCE	100.117
5	7	7.6	16.2	.280	-154.677	-154.688	-.011	DOUBLE	90.414
5	9	10.7	16.2	.141	-62.254	-62.236	.018	DESPAIR	182.865
9	10	27.7	16.2	.021	-162.557	-162.439	.118	BOOBY TRIG	20.426
5	10	33.4	16.2	.015	-224.718	-224.676	.042	BOOBY TRIG	20.426
10	11	26.7	16.2	.023	83.300	83.436	.136	GOODE	103.862
11	5	11.8	16.2	.116	141.211	141.239	.028	SCOTT	245.102
6	11	6.8	16.2	.350	44.778	44.776	-.001	GOODE	103.862
7	11	12.3	16.2	.107	13.485	13.448	-.037	GOODE	103.862
11	12	8.7	16.2	.214	48.802	48.781	-.022	HAMMOND	152.643
7	12	8.8	16.2	.209	62.201	62.229	.028	HAMMOND	152.643
7	13	10.9	16.2	.136	-43.783	-43.790	-.007	TUESDAY	46.624
12	13	13.7	16.2	.086	-106.078	-106.019	.059	TUESDAY	46.624
12	14	10.1	16.2	.159	-109.387	-109.412	-.025	INCE	43.231
13	14	6.1	16.2	.435	-3.414	-3.393	.021	INCE	43.231
13	15	13.1	16.2	.094	-33.763	-33.836	-.073	EAST STRAIT	12.788
14	15	15.2	16.2	.070	-30.425	-30.443	-.018	EAST STRAIT	12.788
13	16	14.9	16.2	.073	11.793	11.820	.027	TWIN RM 3	58.445
14	16	15.6	16.2	.067	15.116	15.213	.097	TWIN RM 3	58.445
15	16	3.7	16.2	1.183	45.664	45.657	-.007	TWIN RM 3	58.445
15	17	14.8	16.2	.074	5.653	5.657	.004	ALPHA	18.445
17	18	8.0	16.2	.253	67.672	67.673	.001	YORK	86.118
18	19	7.1	16.2	.321	-76.052	-76.051	.001	FREDERICK	10.067
19	20	2.3	16.2	3.062	-8.438	-8.438	.000	SOMERSET	1.629
20	1	.1	999.0	******	88.741	88.741	.000	GALLOWAYS NEW	90.370

MAX ADJ (WITHOUT SIGN) = .136 FROM 10 TO 11 MEAN (WITHOUT SIGN) = .029 MEAN (WITH SIGN) = .017

Figure 7

VARIANCE-COVARIANCE MATRIX BASED ON S = .015

2	.000399	.000205	.000249	.000228	.000225	.000224	.000232	.000228	.000226	.000222
	.000213	.000197	.000195	.000168	.000171	.000059	.000028	.000003	.000000	
3	.000205	.000339	.000260	.000239	.000235	.000235	.000242	.000238	.000236	.000232
	.000223	.000206	.000204	.000176	.000179	.000062	.000029	.000003	.000000	
4	.000249	.000260	.000939	.000863	.000849	.000848	.000875	.000861	.000853	.000839
	.000806	.000743	.000738	.000636	.000647	.000225	.000105	.000010	.000000	
5	.000228	.000239	.000863	.001473	.001334	.001271	.001200	.001465	.001411	.001321
	.001234	.001128	.001122	.000966	.000983	.000341	.000159	.000015	.000000	
6	.000225	.000235	.000849	.001334	.001559	.001340	.001209	.001339	.001373	.001428
	.001315	.001197	.001192	.001026	.001043	.000362	.000169	.000016	.000000	
7	.000224	.000235	.000848	.001271	.001340	.001386	.001223	.001274	.001293	.001323
	.001297	.001204	.001193	.001029	.001047	.000364	.000169	.000016	.000000	
8	.000232	.000242	.000875	.001200	.001209	.001223	.001431	.001200	.001198	.001195
	.001155	.001068	.001060	.000913	.000929	.000323	.000150	.000014	.000000	
9	.000228	.000238	.000861	.001465	.001339	.001274	.001200	.002910	.001939	.001340
	.001243	.001134	.001129	.000971	.000988	.000343	.000160	.000015	.000000	
10	.000226	.000236	.000853	.001411	.001373	.001293	.001198	.001939	.005478	.001464
	.001304	.001174	.001173	.001007	.001025	.000356	.000166	.000016	.000000	
11	.000222	.000232	.000839	.001321	.001428	.001323	.001195	.001340	.001464	.001672
	.001406	.001240	.001246	.001066	.001085	.000377	.000175	.000017	.000000	
12	.000213	.000223	.000806	.001234	.001315	.001297	.001155	.001243	.001304	.001406
	.001714	.001397	.001434	.001213	.001234	.000429	.000199	.000019	.000000	
13	.000197	.000206	.000743	.001128	.001197	.001204	.001068	.001134	.001174	.001240
	.001397	.001791	.001650	.001483	.001508	.000524	.000244	.000023	.000000	
14	.000195	.000204	.000738	.001122	.001192	.001193	.001060	.001129	.001173	.001246
	.001434	.001650	.001887	.001507	.001534	.000533	.000248	.000024	.000000	
15	.000168	.000176	.000636	.000966	.001026	.001029	.000913	.000971	.001007	.001066
	.001213	.001483	.001507	.001949	.001901	.000689	.000320	.000030	.000000	
16	.000171	.000179	.000647	.000983	.001043	.001047	.000929	.000988	.001025	.001085
	.001234	.001508	.001534	.001901	.002031	.000672	.000313	.000030	.000000	
17	.000059	.000062	.000225	.000341	.000362	.000364	.000323	.000343	.000356	.000377
	.000429	.000524	.000533	.000689	.000672	.001318	.000613	.000058	.000000	
18	.000028	.000029	.000105	.000159	.000169	.000169	.000150	.000160	.000166	.000175
	.000199	.000244	.000248	.000320	.000313	.000613	.000699	.000066	.000000	
19	.000003	.000003	.000010	.000015	.000016	.000016	.000014	.000015	.000016	.000017
	.000019	.000023	.000024	.000030	.000030	.000058	.000066	.000073	.000000	
20	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	

Figure 8

TEST VALUES

MINIMUM =	.004943848	
S-BAR-SQ =	.000290815	
S-BAR-SQ/S-SQ=	1.292509191	BASED ON S = .015

STATION ERROR is the standard error of the station height in relation to the fixed points, GALLOWAYS NEW (1) or SOMERSET (20).

SERIAL NUMBER	ADJ HEIGHT	STATION ERROR
1	90.370	
2	78.019	.020
3	85.904	.018
4	100.117	.031
5	245.102	.038
6	59.086	.039
7	90.414	.037
8	105.799	.038
9	182.865	.054
10	20.426	.074
11	103.862	.041
12	152.643	.041
13	46.624	.042
14	43.231	.043
15	12.788	.044
16	58.445	.045
17	18.445	.036
18	86.118	.026
19	10.067	.009
20	1.629	.000

Figure 9

8.3 Statistics

As mentioned at paragraph 8.1 the standard errors of the mean slope for each line ranged from 0.11 seconds to 0.40 seconds. These were derived from at least 48 simultaneous pointings at each end. The mean was 0.25 seconds over 36 lines. Nine lines were observed twice, because of triangle misclosures or because they were single line traverse, and their combined standard errors are included in these figures.

The angular standard errors bore no relation to distance; the shortest line, 2.3 km, was not the best and the longest, 33.4 km, was not the worst although it was nearly so due to the poor seeing conditions. The lighthouse next to the trig was not visible, nor even the island itself; only the light could be seen through the thick haze.

The average height closure for 20 triangles was 72 mm; their average perimeter was 32.2 km.

The 35 km from Galloways to Somerset, across Cape York, was levelled to third order specifications with wooden staves, but attained a precision approaching first order. The average misclose between adjacent bench marks was 4 mm, or 2.5 $(K)^{\frac{1}{2}}$ mm. The total misclose was 20 mm, or $3.4(K)^{\frac{1}{2}}$ where K is the distance in kilometres. As mentioned above the levelled values were held fixed.

The variance - covariance matrix provided from the adjustment of the network enabled a relative standard error to be calculated for any two points in the network, using the formula

$$\text{Relative standard error A to B} = (\text{variance A} + \text{variance B} - 2\ \text{covariance AB})^{\frac{1}{2}}$$

Relative Standard Errors were calculated for all combinations of stations at tide gauge sites and the results are tabulated below in metres.

	Goode 11	Hammond 12	Ince 14	East Strait 15	Twin 16	Frederick 19
Booby 10	.065 m	.068	.071	.074	.074	.074
Goode 11		.024	.033	.038	.039	.041
Hammond 12			.027	.035	.036	.042
Ince 14				.029	.029	.044
East Strait 15					.013	.044
Twin 16						.045

9. Other Work

Besides recovery of the old surveys mentioned in paragraph 1.2 other work included levelling down from the trigs to bench marks near sea level for later connection to tide gauges, improving the station marking in the area and establishment of photo control for some contouring at Turtle Head, on Hammond Island. All marks were targetted and vertical air photographs were taken of them; most bench marks were also coordinated horizontally. Astronomic latitude and longitude were observed at two stations and a second order Laplace azimuth was observed at East Strait.

10. Acknowledgements

While this report was being written three papers on the subject of heighting by vertical angles have been noted in The Australian Surveyor of September 1973. It was pleasing to see similarity of many ideas from independent sources.

Besides the joint authors the National Mapping team consisted of surveyors Mr B. Murphy, in charge for the latter part of the job, and Mr S. Bennett; technical officers Messrs W. Jeffery, D. Gray, J. Edmonstone and R. Bryant; technical assistants Messrs P. Walkley and A. Rodgers; and field assistant, Mr R. Small. Mr J. Clarke did much of the office computations. The field team generally consisted of four two-man parties, plus one man at Thursday Island and one surveyor visiting all stations to look after marking, levelling to adjacent bench marks and recovery of old marks.

The survey was undertaken for the Department of Transport. The complete and enthusiastic cooperation of the skipper, Mr L. Foxcroft, and crew of the Department's vessel M.V. Wallach; the Supervisor at Thursday Island, Mr L. Doricich; the Project Officer for the survey, Mr K. Hibbert; and Mr M. Strohfeldt and Mr B. Clarkson of headquarters in Melbourne, made the survey an interesting and pleasant task.

11. Conclusion

Heighting by vertical angles over lines of up to fifteen kilometres across water is practicable. Provided that topography is reasonable symmetrical and wind prevents layering effects in the atmosphere, the mean observed slopws can have standard errors of better than one second.

HP 9100B PROGRAM LISTING

Calculates Mean Standard Deviation and Standard Error of Simultaneous Reciprocal Vertical Angles

STEP		KEY	DISPLAY X	DISPLAY Y	DISPLAY Z	STORAGE f	e	d	c	b	a
0	0	Clear									
	1	1									
	2	$x \rightarrow ( )$									
	3	d									
	4	Stop	$\pm V_A''$	$\pm V_A'$	$\pm V_A^\circ$	Enter the Vertical Angle at one end of the line					
	5	If flag									
	6	3									
	7	7									
	8	Go to ()()									
	9	Sub/Ret									
	a	6									
	b	4									
	c	$x \rightarrow ( )$									
	d	a									
1	0	Stop	$\pm V_B''$	$\pm V_B'$	$\pm V_B^\circ$	Enter the Vertical Angle of the other end of the line					
	1	Go to ()()				SET FLAG after last entry					
	2	Sub/Ret									
	3	6									
	4	4									
	5	↑									
	6	a									
	7	$x \rightleftarrows y$									
	8	—	Calculates the mean of a pair of reciprocal angles			$mean = \frac{\pm V_A - \pm V_B}{2}$					
	9	2									
	a	÷									
	b	↓									
	c	Go to ()()									
	d	Sub/Ret									
2	0	7									
	1	5									
	2	Print	mean''	mean'	mean°	Prints the mean of V_A and V_B for each pair of reciprocal angles					
	3	Roll ↑									
	4	Clear x									
	5	Roll ↑									
	6	Clear x									
	7	Roll ↑									
	8	↑									
	9	X									
	a	Acc +									
	b	d									
	c	$x \rightleftarrows y$									
	d	1									

HP 9100 B PROGRAM LISTING

Calculates Mean Standard Deviation and Standard Error of Simultaneous Reciprocal Vertical Angles

STEP	KEY	DISPLAY X	DISPLAY Y	DISPLAY Z	STORAGE f	e	d	c	b	a
3 0	+									
1	Y → ( )									
2	d									
3	↓									
4	Goto ( )( )									
5	0									
6	4									
7	d									
8	↑									
9	1									
a	−									
b	y → ( )									
c	d									
d	↑									
4 0	RCL									
1	Roll ↑									
2	÷									
3	y → ( )									
4	+									
5	x									
6	f									
7	x	Calculates the mean and Standard deviation of the mean reciprocal angles								
8	↓									
9	−									
a	d									
b	↑									
c	1									
d	−									
5 0	f									
1	Roll ↓									
2	÷									
3	d									
4	Roll ↓									
5	$\sqrt{x}$									
6	$x \rightleftarrows y$									
7	Roll ↑									
8	Print	n	mean seconds	std. deviation	prints number of reciprocal observations, the overall mean (seconds only) and the standard deviation in seconds					
9	Space									
a	$\sqrt{x}$									
b	Roll ↑	Calculates the Standard Error of the mean reciprocal angles								
c	$x \rightleftarrows y$									
d	÷									

HP 9100 B PROGRAM LISTING

Calculates Mean Standard Deviation and Standard Error of Simultaneous Reciprocal Vertical Angles

STEP		KEY	DISPLAY X	DISPLAY Y	DISPLAY Z	STORAGE f	e	d	c	b	a
6	0	↓									
	1	Print	Std Error	mean	mean	prints the Standard Error in seconds and the overall mean in seconds (twice)					
	2	Space									
	3	Stop									
	4	x → ( )									
	5	c									
	6	6									
	7	0									
	8	÷									
	9	Roll ↓									
	a	+									
	b	c	Sub routine	converts degrees,	minutes, seconds	to decimal degrees					
	c	Roll ↑									
	d	÷									
7	0	÷									
	1	Roll ↓									
	2	+									
	3	↓									
	4	Sub/Ret									
	5	↑									
	6	int x									
	7	−									
	8	Roll ↓									
	9	x ⇄ y									
	a	6									
	b	0									
	c	X									
	d	↓									
8	0	↑									
	1	int x									
	2	−	Sub routine	converts decimal	degrees to degrees,	minutes and seconds					
	3	x ⇄ y									
	4	y → ( )									
	5	c									
	6	x ⇄ y									
	7	6									
	8	0									
	9	X									
	a	x ⇄ y									
	b	y ⇄ ( )									
	c	c									
	d	Sub/Ret									

Reduction of Double Reading T3 Vertical Circle Readings to Elevations and Depressions

STEP		KEY	DISPLAY X	DISPLAY Y	DISPLAY Z	STORAGE f	e	d	c	b	a
0	0	Stop	½ Bubble Value			Enter half the bubble value (seconds / division)					
	1	x → ( )									
	2	c									
	3	clear									
	4	Stop	V°	0	0	Enter the vertical circle reading Degrees only					
	5	↑									
	6	2									
	7	x	doubles degrees								
	8	Acc +									
	9	Stop	V′			Enter the vertical circle reading Minutes only					
	a	↑									
	b	3									
	c	0	doubles minutes and adds them to 2 x degrees								
	d	÷									
1	0	Acc +									
	1	Rcl									
	2	y → ( )									
	3	a									
	4	Stop	V_1''			Enter the vertical circle reading 1st seconds reading					
	5	↑									
	6	Stop	V_2''			Enter the vertical circle reading 2nd seconds reading					
	7	+									
	8	2	Adds 1st and 2nd second readings								
	9	x									
	a	Stop	$bubble_1$			Enter 1st bubble reading					
	b	↑									
	c	Stop	$bubble_2$			Enter 2nd bubble reading					
	d	+									
2	0	c									
	1	x									
	2	↓									
	3	+									
	4	3	Calculates bubble correction								
	5	6									
	6	0									
	7	0									
	8	÷									
	9	Acc+									
	a	1									
	b	8									
	c	0									
	d	↑									

Reduction of Double Reading T3 Vertical Circle Readings to Elevations and Depressions

STEP		KEY	DISPLAY X	DISPLAY Y	DISPLAY Z	STORAGE f	e	d	c	b	a
3	0	Acc −									
	1	y ⇄ ( )									
	2	e									
	3	Pause									
	4	Pause									
	5	| y |									
	6	↓									
	7	↑									
	8	Int x									
	9	−									
	a	Roll ↓									
	b	x ⇄ y									
	c	6									
	d	0									
4	0	X	Converts decimal degrees to degrees, minutes, seconds								
	1	↓									
	2	↑									
	3	Int x									
	4	−									
	5	x → ( )									
	6	d									
	7	6									
	8	0									
	9	X									
	a	d									
	b	x ⇄ y									
	c	STOP	Angle "	Angle '	Angle °	Read off the Vertical Angle (Elevation or depression)					
	d	Clear				IF degrees and minutes change,					
5	0	a				CLEAR, END and enter ½ bubble value once again					
	1	x → ( )									
	2	e				IF degrees and minutes unchanged enter 1st seconds reading and continue from step 1, 4					
	3	Go To ( )( )									
	4	1									
	5	4									
	6	END									
	7										
	8										
	9										
	a										
	b										
	c										
	d										

STEP	KEY	COMPUCORP 322 G Double Reading T/3 Vertical Circle Readings Converted to Vertical Angles Pages 1 of 2.	STORE
		Enter ½ the bubble value, (seconds/division),	
		into Store 9	Bubval/2
		Enter the 1st seconds(") reading	
1	×		
2	Exp		
3	chg Sgn		
4	4		
5	D.MS→Dec		
6	+		
7	(		
8	Stop/start	Enter the 2nd seconds reading	
9	×		
10	Exp		
11	chg Sgn		
12	4		
13	D.MS→Dec		
14	)		
15	=		
16	+		
17	(		
18	Stop/Start	Enter Degrees . minutes	
19	D.MS→Dec		
20	)		
21	=		
22	×		
23	2		
24	=		
25	St_n		
26	0		Uncorrected Angle
27	Stop/start	Enter the 1st bubble reading	
28	+		
29	Stop/Start	Enter the 2nd bubble reading	
30	×		
31	RCL_n		
32	9		Bubval/2
33	=		
34	×		
35	Exp		
36	chg Sgn		
37	4		
38	=		
39	D.MS→Dec		

STEP	KEY	COMPUCORP 322.G Double Reading T/3 Vertical Circle Readings Converted to Vertical Angles Page 2 of 2	STORE
40	+		
41	RCLn		
42	0		uncorrected Angle
43	=		
44	−		
45	1		
46	8		
47	0		
48	=		
49	Stn		
50	1		Vertical Angle
51	×		
52	3		
53	6		
54	0		
55	0		
56	=		
57	Stop/Start	Read off the vertical angle (seconds and decimals)	
58	RCLn		
59	1		Vertical Angle
60	Dec→D.MS		
61	Stop/Start	Read off the vertical angle in degrees · minutes and seconds. Exact seconds and decimals of seconds must be noted from step 57.	

Heighting by Vertical Angles

Alidade Bubble Calibration - Wild T3 Double Reading

Level the instrument on a firm setup with the alidade bubble about central.

Set the micrometer on 30 seconds, to avoid a change of minutes.

Clamp the telescope and bring the circle divisions into coincidence with the vertical tangent screw.

Book the bubble and circle readings

Move the micrometer say 0.5 seconds

Bring the circle divisions to coincidence with the alidade bubble screw, always making the final motion clockwise.

Book the circle reading

Wait for the bubble to settle and book the reading

Continue from the initial bubble reading to one end of the desired range, proceed to the other end and return to the initial setting.

If the bubble reading at the final point is not in reasonable coincidence with the initial reading at that point, continue in the same direction until they converge. The difference is due to slight movement of the instrument.

Separate the readings across the central portion of the bubble by say one second on the micrometer, that is four seconds of arc or about 2/3 of a 2 mm bubble division. Towards the end of the range reduce the spacing to 0.5 micrometer seconds or a little less.

As mentioned earlier the desired calibrated value is over the range -2 to +2. There is no difficulty in keeping the bubble within this range during observations, using a footscrew.

The sign of bubble readings is illustrated on the observation form.

The seconds per division obtained here have to be multiplied by four.