SOME ASPECTS OF THE CONVERSION FROM IMPERIAL TO METRIC CO-ORDINATES IN THE AUSTRALIAN CAPITAL TERRITORY

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Introduction

1.1. Canberra is a city which is growing rapidly. Its development is planned by the National Capital Development Commission and all the necessary surveys are carried out by the Survey Branch of the Department of the Interior. The surveys required for development of broad acres to fully serviced subdivisional blocks on a scale such as is occurring in Canberra presents technical challenges which are probably unique in this country.

1.2. To gain some idea of the magnitude of the task and the rate at which surveys must be executed, consider the fact that in the 1971-72 financial year approximately 3,500 residential blocks were surveyed. It may be of interest to note that the survey of the 3,500 blocks includes not only the cadastral surveys, but also all major and minor topographic and engineering surveys. Maps required for land usage, town planning and engineering design are prepared initially. All suburban road centre lines, hydraulic services and kerb lines are located by survey for construction prior to the final subdivision of the area. In addition, major engineering surveys for water supply, freeway systems and trunk hydraulic facilities are effected. In these circumstances, the need for careful planning of urban survey control and co-ordination will be obvious. The purpose of this paper is to examine some aspects of a revision of the A.C.T. survey control system and the conversion to metric units.

1.3. The control of surveys for this development has been described elsewhere (1) (2) (3) and it is not the purpose of this paper to pursue this aspect. However, it is essential for my purpose that you appreciate that a very accurate control survey has been established covering most areas where development can be anticipated. The order of accuracy of the primary framework, which has a station approximately every six miles, is 1:250,000. For sectional control where this interval is approximately a half mile, the accuracy is 1:100,000.

1.4. Projects such as the Googong Dam on the Queanbeyan River in N.S.W. have caused us to carry this high order control survey up to 15 miles east of the central meridian of our local map projection. So far away from the central meridian, the point scale factor of the Transverse Mercator projection in use had fallen to 1:50,000. Using modern electronic distance measuring equipment on lines 10 miles long, this discrepancy is easily detected and provides a fine nuisance for a team of surveyors with a deadline to meet.

1.5. The introduction of metrication in the A.C.T. was a second factor which caused a review of the control network and its associated map projection to be carried out. If the co-ordinates of all the system had to be changed into metric units, then the time was surely ripe to introduce modifications to the projection to improve it. A committee within the Branch undertook this investigation and produced the following specification for the projection:

(a) The Transverse Mercator projection should be used as this is the most common projection in use in Australia. It is also suitable for the shape of the area to be mapped.

(b) The point scale factor should lie between 0.99999 and 1.00001, i.e., the projection correction should not exceed 1:100,000.

(c) Grid azimuths, which appear on all deposited plans and other documents, should not be affected unless a strong case could be built for doing so.

(d) To accord with normal surveying and mapping procedures, a false origin should be introduced so that all co-ordinates remain positive. The choice of a false origin should also be such as to differentiate Eastings from Northings by the leading digit.

(e) All dimensions of the projection should be in metric units.

1.6. Whilst these specifications are quite stringent and virtually spell out the projection before any investigation is made, it should be emphasised again that the projection co-ordinates are used to aid the surveyors in maintaining speed and accuracy in setting out and checking large scale projects. It is the intention of the system to generate co-ordinates which can be used as plane co-ordinates with plane trigonometry by the field parties. All corrections and refinements are applied at the control stage and thereafter, any small discrepancies can be distributed proportionally between control marks without serious error.

1.7. After a thorough examination of the problems, a projection was adopted with the following parameters:

(a) **Spheroid adopted** is the Australian National Spheroid.

(b) **Central meridian** is the meridian through Stromlo trigonometrical station — Longitude = E149° 00′ 33″.4614. This value is from the 1966 National Adjustment.

(c) **Central scale factor** = 1.000086 (see notes on Central Scale Factor).

(d) **True origin** is the intersection of the central meridian with the equator.

(e) **False origin** is such as to give Stromlo trigonometrical station the co-ordinates E200,000 N600,000.

(f) **Units** are in metres.

(g) **Plane** on which the co-ordinates are expressed is 610 metres above M.S.L.

1.8. The problem of converting feet co-ordinates on the existing projection into metres on the new grid was then investigated.

Notes on the Central Scale Factor

2.1. Before tackling the problems associated with the co-ordinate conversion, an explanation is necessary for the paradox that the projection correction should not exceed 1:100,000 and the choice of the central scale factor is 1.000086. The average elevation of Canberra and environs is 2,000 feet above M.S.L. Therefore, to convert distances to sea level they should be multiplied by the factor 0.999904. In projecting the Australian National Spheroid using a central scale factor of 1.000096, the net result is to project the surface of that spheroid on to a cylinder which is 2,000 feet above the spheroid at the central meridian. The co-ordinates produced will then correspond very closely with the earth's surface in that vicinity. By introducing a negative correction of 1:100,000 at the central meridian, the scale factor becomes 1.000086. At the same time two useful by-products result. Firstly, the zone width can be increased to 50 minutes while maintaining a maximum distortion of 1:100,000. Secondly, rather fortuitously, the lower parts of Canberra lie fairly close to the central meridian and the elevation of the country rises towards the edge of the zone both in the east and west. Therefore, the country obligingly follows the rise in scale factor to some extent at least.

Conversion from Old to New Projection

3.1. In this day of fast, powerful computers, there is a simple method of converting from one system to another. This is to convert firstly from the original projection into geographical co-ordinates and then from these geographicals back on to the new projection. With modifications, to be described, this same process can be applied to handle changes of units and changes of spheroid where required.

The A.C.T. Problem

4.1. As Canberra developed, various standard feet have been used in survey work. Whilst these changes in standard are small in relation to a suburb, nevertheless, the investigation was carried out to ensure that significant errors were not propagated in the overall net.

4.2. The original work in Canberra was laid out around 1917 by Mr. Surveyor H. Mouat using invar wires standardised in British feet. Between 1950 and 1955, Mr. Surveyor K. Morgan re-measured much of this work using invar wires supplied by the Department of Crown Lands in N.S.W. The standard was again British feet and the conversion factor for this standard is 1 British foot = 0.30479947 metre.(4) Co-ordinates were generated from these surveys using plane trigonometry.

4.3. In 1960, as Canberra expanded into the Woden Valley (see Appendix 'A'), the control survey was extended by triangulation and trilateration using electronic distance measuring equipment. The machine used gave a result in metres which was converted to feet using the factor 1 "British A" foot = 0.304799433 metre. The term "British A" foot will be used throughout this paper to distinguish it from the British foot referred to in Paragraph 4.2. Plane co-ordinates were again generated.

4.4. Belconnen, to the north-west of the city, was the next area to be developed (see Appendix 'A' for location). Control work covering the whole of the area was carried out during the 1960s using the same techniques, equipment and conversion factor as for Woden Valley. However, at this stage, a map projection was introduced so that there would be no problems as survey moved away from the central area.

4.5. The projection selected was the Transverse Mercator and the following parameters were used:

(a) **Spheroid adopted** is Clarke's 1858 spheroid.
(b) **Central meridian** is the meridian through Stromlo trigonometrical station — Longitude adopted = E149° 00′ 33″.24.
(c) **Central scale factor** = 1.
(d) **True origin** is the intersection of the central meridian with the 34° S parallel of latitude.
(e) **False origin** is Stromlo trigonometrical station.
(f) **Units** are "British A" feet.
(g) **Plane** of co-ordinates is 2,000 feet above M.S.L.

4.6. It should be noted that the above figure of the earth was selected prior to the definition of the Australian National Spheroid in 1966. Clarke's 1858 figure was the basis of the Australian National grid and tables were readily available for computation on this grid. In fact, the projection used was a modification of the Australian National Grid.

4.7. Finally, the latest work including Tuggeranong has been computed on the Australian National Spheroid using a modification of the Australian Map Grid for the projection. This was done through the use of National Mapping's program "Varycord" without modifying the parameters of the spheroid but using observed distances in "'British A" feet. Small areas of Belconnen were adjusted holding the perimeter fixed from the earlier work and using the least square "Varycord" program to carry out the adjustment on the Australian National Spheroid.

Notes on the Modified Australian National Spheroid

5.1. Whilst it is stated in the previous paragraph that the parameters of the Australian National Spheroid were not modified, this remark applies to the units in which these parameters are expressed. In order to compute on the surface of the spheroid 2,000 feet above the true A.N.S., it is convenient to add the 2,000 feet to the "a" radius of the spheroid at the equator. If the flattening of the spheroid is maintained as 1:298.25, this will give a separation of the A.N.S. and the modified A.N.S. of 1993.3 feet at the pole. At the latitude of Stromlo trigonometrical station, the separation will be of the order of 1998 feet. That is, the computational surface will be within a few feet of the nominated plane and will lead to insignificant errors when distances are reduced to the "2000 foot plane".

5.2. Because this modified spheroid is not exactly parallel to the Australian National Spheroid, there will be a slight but insignificant change in the directions of the normals at a point. This can be neglected as it is beyond the observational limits over an area the size of the A.C.T. The two normals are assumed co-incident at Stromlo.

5.3. Due to the fact that reduced levels in the A.C.T., even those based on the Australian Height Datum, refer to the geoid rather than to the spheroid, there is an implied shift in the spatial centre of the Australian National Spheroid. The implied shift arises from the assumption that the geoid and spheroid coincide at Stromlo. All distances are reduced to the 2,000 foot plane and computed as though they are on the spheroid.

Conversion Factor Feet to Metres

6.1. It will be noted that only two "foot to metre" conversion factors have been used. These are:

1 British foot = 0.30479947 metre

1 "British A" foot = 0.304799433 metre

6.2. Since the difference is only 0.03 parts per million and the first factor would only apply to the area designated "city" in Appendix 'A', the second factor has been used exclusively to convert feet to metres.

Conversion of Co-ordinates in the Area using Clarke's 1858 Spheroid

7.1. Here again it is necessary to look at the historical growth of the control net and the underlying assumptions implicit in the computations carried out. When the Transverse Mercator projection was originally introduced in the A.C.T., Stromlo trig. was adopted as a false origin in a modified zone of the Australian National Grid. Astronomical co-ordinates of:

Latitude S35° 18′ 59″.46

Longitude E149° 00′ 33″.24

were adopted as the Clarke 1858 values of this station. Computations then proceeded on the projection from this point again using a surface approximately 2,000 feet above the Clarke spheroid. Once more there is an assumption that the geoid and computation surface are tangential at this station together with the assumption that the gravity and spheroidal normals are coincident.

7.2. There is an added complication in that the surface measurements were in "British A" feet, while the "a" radius of the spheroid used was in "Clarke" feet.(5) One Clarke foot = 0.3047972656 metre. This aspect of the problem will be discussed more fully under the heading "Standard Errors Resulting from Mixed Units".

7.3. To convert co-ordinates on this grid to the new grid, the following steps are necessary:

(a) Convert the Easting and Northing into Clarke feet.
(b) Reduce T-M co-ordinates to latitude and longitude on the Clarke 1858 spheroid.
(c) Transpose Clarke 1858 latitude and longitude into Australian National Spheroid latitude and longitude.(6)
(d) Re-project the latitude and longitude from (c) on the new grid.

7.4. Step (c) of this transformation is the most difficult of the four steps and it would be convenient to avoid it if possible. Remembering that the Clarke 1858 spheroid was used over a limited area, let us look at the result of a transformation of Majura trig., which is the most remote station from Stromlo based on this projection. The location of Majura can be seen in the plan at Appendix 'A'.

7.5. Its A.C.T. grid co-ordinates in "British A" feet are E51025.78 N28439.45. Converting these into "Clarke" feet gives E51026.144 N28439.653. These co-ordinates can then be reduced by the usual Transverse Mercator projection formulae for the Australian National Grid to give latitude and longitude as follows:

ϕ = S35° 14′ 17″.7743
λ = E149° 10′ 48″.2712

7.6. A spheroid-to-spheroid transformation can now be made using the following data:

$\phi^o = 35° 18' 59''.46$
$\lambda^o = 149° 00' 33''.24$
} ϕ^o and λ^o are the Clarke latitude and longitude of Stromlo trig. and are also known values on the Australian National Spheroid.

$\triangle \phi^o = 4''.3906$
$\triangle \lambda^o = 0''.2214$
} Where $\triangle\phi^o$ and $\triangle\lambda^o$ are the differences in known latitudes and longitudes of Stromlo trig. on the Clarke 1858 and Australian National Spheroids. The difference is taken in the sense "A.N.S. — Clarke".

7.7. The "a" radius of the Clarke 1858 spheroid is 6378339.005 metre.

$\triangle a = -179.005$ metres is the difference in "a" radii of the two spheroids in the same sense as for $\triangle\phi^o$ and $\triangle\lambda^o$.

$\triangle f = -0.000045463$ is the difference in flattening of the two spheroids in the same sense as above.

$\triangle N^o = 0$ metres is the height separation of the spheroids at Stromlo trig.

7.8. After carrying out the transformation, the latitude and longitude of Majura trig. on the Australian National Spheroid are found to be:

$\phi = S35° 14' 22''.1715$
$\lambda = E149° 10' 48''.5238$

The height separation of the two spheroids is 0.26 m at this station. Projecting this latitude and longitude on to the new A.C.T. grid, gives the station co-ordinates of:

E215 552.467 m N608 668.253 m

7.9. To determine whether or not the spheroid-to-spheroid transformation is worthwhile, it can be assumed that the co-ordinates quoted at the station are a projection of the A.N.S. on to the old Canberra grid. Using the appropriate parameters for that spheroid, and the A.N.S. values for Stromlo trig., the original co-ordinates of Majura trig. can be converted into latitude and longitude. The reason for the intermediate step of converting to metres will be apparent from the section headed "Standard Errors Resulting from Mixed Units".

7.10. A.C.G. grid co-ordinates in "British A" feet are:

E51025.78 N28439.45

Converted to metres using 1 foot = 0.304799433 metre results in:

E15552.629 N8668.328

7.11. Reducing to latitude and longitude (approximately on the A.N.S.):

$\phi = S35° 14' 22''.1719$
$\lambda = E149° 10' 48''.5240$

7.12. Projecting this approximate latitude and longitude on to the new grid gives station co-ordinates of:

E215 552.472 m N608 668.241 m

7.13. Comparing this value with that obtained from the rigorous method, it is seen that the error is only 5 mm x 12 mm.

7.14. This error is well within the observational errors for the station and can be safely ignored. It is also the maximum error to be expected in the area where the Clarke spheroid was used since the two spheroidal surfaces become closer as we move back towards Stromlo where they are coincident.

Sectional Control for the Belconnen Valley

8.1. Approximately half of the control net in the Belconnen Valley was developed using the grid described in the previous section. The rest of the area was fitted using the "Varycord" adjustment assuming the co-ordinates and observations were on the surface of the A.N.S.

8.2. Although the application of "Varycord", as it is used in the A.C.T., has a different effect on the standard from that described above, the scaling of the work was controlled by holding perimeter stations fixed. Since it is such a small area and the two spheroids are very close together in the vicinity, as demonstrated by the comparison at Majura, the same approximate conversion method as described in Section 7 can be used.

Standard Errors Resulting from Mixed Units

9.1. In 1970 a re-adjustment of the primary framework was carried out in conjunction with an extension of the net to cover approximately half of the A.C.T. (For the extent of this area, see Appendix 'A'). Once again, "Varycord" was used but, in this instance, there was only one fixed station and the survey covered a relatively large area. The net was scaled by having all lines measured in metres and converted into "British A" feet using the factor 1 foot = 0.304 799 433 metre.

9.2. However, the "a" radius of the A.N.S. in International feet was used in conjunction with the flattening of 1:298.25 to define the spheroid. The result is equivalent to a standard error in the survey. This can be looked at from two points of view. Either the latitudes and longitudes quoted on the A.N.S. are incorrect; or the latitudes and longitudes quoted are correct but the adopted spheroid is NOT the A.N.S.(7)

9.3. The equivalence of the two points of view can be demonstrated for a random station such as Cunningham trig., as follows.

9.4. If a standard error is assumed, all measurements can be converted into one common unit, metres, by using the appropriate conversion factors. Thus, surface measurements should be converted using 1 foot = 0.304799433 metre and the "a" radius using 1 foot = 0.3048 metre.

9.5. A.C.T. grid co-ordinates of Cunningham are E37975.156 N-101 347.655 (in "British A" feet). Converted into metres, this gives E11574.806 N-30890.708. When this is transformed into latitude and longitude using the normal parameters of the A.N.S. (a = 6378160 m and f = 1:298.25), the results are:

$\phi = S35° 35' 45''.8288$
$\lambda = E149° 08' 13''.2337$
} .....(I)

9.6. Using the other approach, the values are assumed to be correct, but not on the A.N.S., then the argument is as follows:

A.C.T. grid co-ordinates of Cunningham are —
E37975.156 N-101 347.655 (in "British A" feet).

9.7. The parameters of the modified spheroid and projection are:

a = 20925 721.786 "British A" feet
f = 1:298.25
Central scale factor = 1.000096

9.8. Converting the projection co-ordinates into latitude and longitude gives:

$\phi = S35° 35' 45''.8307$
$\lambda = E149° 08' 13''.2346$

9.9. A spheroid-to-spheroid transformation can now be carried out from the modified spheroid to the A.N.S. using the following data:

$\phi^o = S35° 19' 03''.8506$
$\lambda^o = E149° 00' 33''.4614$
} Common co-ordinates of Stromlo on both spheroids

$\triangle a = 11.87$ m

$a = 6378\ 148.13$ m

$\triangle\phi^o = \triangle\lambda^o = \triangle N^o = \triangle f = 0$ where the variable are as defined in Paragraphs 7.6 and 7.7.

9.10. Such a transformation gives the co-ordinates of Cunningham as:

$\phi = S35° 35' 45''.8288$
$\lambda = E149° 08' 13''.2337$
} .....(II)

9.11. The results of (I) and (II) are identical, a result which is linked to the fact that the flattening of both spheroids is the same.(8) Therefore, all work from the 1970 adjustment can be reduced to correct geographical co-ordinates simply by converting feet into metres, using the factor 1 foot = 0.304 799433 metre, and then using the normal parameters of the A.N.S. with the local Transverse Mercator projection formulae.

The New Grid

10.1. As stated earlier, once the geographical co-ordinates of a station are known, it is a simple matter to re-project the point on the new grid. The only difference between the new grid and the old is the change

of 1:100,000 in the central scale factor. All the effort has been directed towards obtaining the best possible values for the geographical co-ordinates to be projected.

Conclusion

11.1. Throughout the whole of the A.C.T. the factor to be used to convert feet into metres is 1 foot = 0.304 799 433 metre.

11.2. All existing co-ordinates can be treated as a Transverse Mercator projection of the Australian National Spheroid on a cylinder 2,000 feet above the surface of the spheroid. The maximum error introduced by this assumption will be less than 10 mm x 15 mm and will occur approximately 20 km north-east of Stromlo trig. station.

11.3. To convert from the old grid to the new grid, the original grid co-ordinates should be converted into metres. They can then be reduced to geographical co-ordinates on the A.N.S. and reprojected on to the new grid. This method will be quite rigorous in all areas except Belconnen where the Clarke 1858 spheroid was used. Here the maximum error will be less than 10 mm x 15 mm.

11.4. The final projection will give a result which will be capable of being used as a plane projection with a distortion of less than 1:100,000 for a distance of 25 miles either side of the central meridian. It should not be necessary to apply a height reduction to measured distances except at the control stage. Some discretion is necessary where long lines are involved or the elevation of the terrain exceeds 2,500 feet above the Australian Height Datum.

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AUSTRALIAN CAPITAL TERRITORY

HORIZONTAL CONTROL