The Projection Story

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Too often, a map is evaluated by the viewer on its aesthetic appeal to the virtual exclusion of more valuable though less obvious contributing factors. The present article relates one such factor, the framework of a map. It indicates the magnitude and nature of the research which is involved in the selection of a map projection for a new map of Australia required by the Division of National Mapping. The writer was Chief Cartographer of that Authority at the time.

It is generally asserted that a perfect map is an impossibility. If by "perfect map" is implied a presentation which is identical in all respects with that occurring in nature, then that assertion is absolutely correct. If, on the other hand, a lack of conformity with deliberately designed specifications is suggested, then it is far from the truth.

The field determination is related to a spheroidal surface of reference and not necessarily to a map. Hence it may be an end in itself. However, a map projection is conceived immediately the question of its plane presentation arises. Its delineation on to a mapping plane presupposes the existence of a projection, a fact that may not be obvious when the area in question is of such small extent as to render graticule construction and geographic control unnecessary. However, the need becomes increasingly evident with extension of the mapped area because of the increasing disparity between the spheroidal and corresponding plane surfaces that occurs. For example, terrain of 100 miles square or so would not differ significantly on plane or spheroid. However, the segment of the spheroid containing Australia arches about 180 miles above its chordal plane. That of a hemisphere arches about 3,960 miles. The disparity becomes even greater and more complex when larger segments of the spheroid are in question.

The reduction of nature to a mapping plane creates many distortions. A map projection permits the inclusion in the mapping of certain desired properties. Its title should convey to the informed map user their presence and their limitations. as well as the nature and magnitude of unavoidable distortions that may be expected in other properties of nature. Its graticule provides the means for positional and directional delineation and for some evaluation in the map reading of the various distortions that may be present.

If distances could be preserved true throughout the mapping, all other true-to-nature properties would automatically apply and a perfect map would result. However, preservation of correct linear scale is attainable only in certain defined positions or directions in the mapping, or without significant departure from true in mapping of small extent only.

Other desirable properties include correct linear delineation, azimuth, area scale and shape. Their inclusion in the mapping may rarely be achieved with completeness either singly or in association and the inclusion of any one or more is usually only at the expense of the other desirable properties and of increased deformations thereof.

Linear scale has been referred to above. Linear curvature of the great circle occurs in many positions and directions in the mapping and its track and azimuth are not always correct or apparent. The Gnomonic is the only projection that preserves the straight line orthodrome. Correct curvature of the small circle is rarely preserved.

Linear curvature usually varies in magnitude with position and direction in the mapping. Other deformations may also be present which may debar the use of normal methods of protracting bearings. However, true area scale may be preserved with completeness but the retention of correct shape applies only at points in the mapping and not in an areal sense, a fact that is often forgotten in this conformal conscious age.

As a projection cannot be created that will provide perfect mapping in all respects, one must be devised which will hold those properties most appropriate to the immediate purpose of the mapping. The special purpose of a map may in certain circumstances outweigh all other considerations even to their exclusion. For example, a chart required expressly for rhumb line navigation involves one solution only. On the other hand, a distributions map may permit a choice from several possibilities, each possessing a common equal area property. The less restrictive the purpose, the wider the range of possibilities.

The purpose of a map largely determines its property class. Its projection type (zenithal, conical, cylindrical or conventional) is governed by the extent of the area to be mapped, its geographical location, its predominant orientation and configuration, and by the relative importance to the purpose of the map of its component regions and its detached or protruding extremities.

A cartographer must ever be projection conscious. The choice of a projection is deliberate and not a matter of chance or convenience. Simplicity of computation or of construction, availability of published plotting tables, and the like, should not outweigh useful property considerations, but should be significant to the choice only when other aspects are equal. Each separate mapping project should be treated as unique. A projection should be selected on its degree of suitability for the terrain in question and for the particular purpose of the mapping and not on its known suitability in other circumstances.

During 1952 an investigation was made by the Commonwealth Division of National Mapping with a view to selecting a projection for a proposed new map of Australia, which was resolved substantially as follows:—

Object

To select a map projection suitable for use in a proposed new map of Australia designed for wall display, information and general reference purposes.

Scale

1:6,000,000 at publication.

Extent

Australia extends about 35 degrees of arc east to west (about 40 degrees of longitude) by about 28 degrees north to south on the mainland with a further 5 degrees to include Tasmania.

The northern and southern portions of the surface to be mapped include a large amount of ocean intrusion leaving Tasmania, Cape York Peninsula and Arnhem Land as "isolations" to which reduced weight could possibly be given when designing the map structure. When thus interpreted, the mapping may be considered as being more or less concentrated between parallels of latitude 14° S. and 38° S. providing a compacted mapping equivalent extending 35 degrees of arc in longitude by 24 degrees of arc in latitude. With such a preponderance of longitude over latitude, and in view of its position in latitude, the map may be considered as a fitting subject for a conical projection in its normal setting.

If, on the other hand, equal weight is given to all land areas, as would be in closer accord with the prime purpose of the map, then zenithal mapping may provide a better answer. The land areas could be rather neatly contained within a circle of radius about 18 degrees of arc with only minor extrusion therefrom. An area equivalent to the total land mass would be contained within a circle of radius slightly more than 14 degrees of arc.

Possible Projections

The following projections were reviewed as to suitability:—

- Simple Conic with 1 standard parallel
- Simple Conic with 2 standard parallels
- Conformal Conic with 1 standard parallel
- Conformal Conic with 2 standard parallels (Lambert Conformal)
- Conical Equal Area with 1 standard parallel
- Conical Equal Area with 2 standard parallels (Albers)
- Bonne
- Polyconic
- Zenithal Equidistant
- Zenithal Conformal (Stereographic)
- Zenithal Equal Area.

A graticule of each projection was prepared at a nominal scale of 1:15,000,000 as a base for graphic illustration and to facilitate comparison. They were included in the Report as Annexures A to K respectively, and supplemented by Annexures L and M to show certain scale details in tabular and graphic form.

Centre of Mapping

Parallel of latitude 26° S. was selected as being most central to the land surface involved, with due consideration to its distribution. This parallel was accepted as standard in the various conics with one standard parallel.

In the cases of the conics with two standard parallels, the standards were selected at 18° S. and 34° S. with the object of keeping distortion within controlled limits over the bulk of the mapping rather than to balance the errors between the extremes and centre of the mapping. To effect the latter solution, it would be necessary to widen the gap between the standard parallels. Because of the acceleration of distortion that takes place with approach to the limiting latitudes on a conical projection, an equal distribution of scale error would bestow undue importance to those latitudes. In the case of Australia, this disproportion would be magnified by the uneven distribution of the land areas over the full range of latitude. Thus the combined effects of balancing the errors would be to burden the central latitudes with needlessly high projection errors to the mitigation of those at the extreme latitudes—a very doubtful benefit when measured in terms of square miles of land affected.

The middle longitude of the land lies somewhere between meridians of longitude 133° E. and 134° E. As the bulk of the land mass and the greatest intensity of the mapping occurs towards the eastern side of the map, 134° E. was selected as the central meridian. This arrangement would leave slightly more ocean width between shoreline and neatline at the east than at the west of the map and so provide a more aesthetic and utilitarian presentation.

For reasons stated above, the centre of the zenithal projections investigated was accepted as the intersection 26° S. 134° E.

Desirable Properties

The qualities most desired in any map are as follows:—

(a) True linear scale throughout, so that measured map distances may be accepted as correct irrespective of position or of orientation.

(b) Undistorted shape, both locally and extensively.

(c) Correct bearings and convenient map aids for plotting and measuring them.

(d) A great circle/straight line assumption, including the coincidence of the tracks, and correct small circle curvature.

(e) Correct area ratios throughout, so that equal map areas represent a common ground area irrespective of position in the mapping.

All of these properties are impossible of inclusion in the one map.

Linear Scale

Linear scale probably provides the best criterion of map perfection. The closer the actual scales throughout the map are to true, the better the map and the nearer it is to perfect. True scale is probably the property most sought after by the general map user. The distribution and intensity of true-to-scale lines (isoperimetric curves) provide a guide to the better utilisation of map measurement and to the better appreciation of inherent distortion generally.

Projections vary considerably as to the magnitude and distribution of scale distortion. Whilst linear scale may be preserved true in certain positions and orientations, it is not possible to construct a map of an extensive area over the whole surface of which the scale will be true. The best that may be hoped for is to keep distortion as low as possible over those portions of the mapping that are of most importance to the general map user.

Those users who possess the knowledge and desire to make allowance for the varying distortion in the linear scale of the map are few and far between. Whilst adequate information should appear on the published map to cater for this need, the projection selected should permit of the acceptance, by the less critical user, of scaled distances as near enough. Although the results obtained by the use of scale factors are closer to correct under normal conditions, they are approximations nevertheless. Refinements in measuring the mapped line may even be nullified by paper distortion brought about by changing atmospheric conditions or by other causes. The user who seeks correct values must resort to computation independent of mapped values.

When a map user is required to utilise scale adjustment in map measuring, it would probably be determined as the arithmetic mean of the actual scales at the extremities of the line in question, or of component intervals of that line. The conformal conics have a distinct advantage in this regard, for the actual scale is directly related to latitude at all points in the mapping. The zenithal conformal scales may also be readily, though not so conveniently determined. The application of scale adjustment factors to the other projections under consideration would be less sure. However, any relative advantage in this regard was considered to be of minor importance and outweighed by other considerations.

In determining the elements of the various projections under review, scale factors other than those associated with the normal development of the projection were not introduced into the computations in the first instance. Extraprojection scale factors are designed to bring actual scales closer to

PERCENTAGE SCALES

Name of Map Projection	Linear Scale				Area Scale			
	At Map Centre 26°S. 134° E.	At Land Limit	Scale Range	Maximum Scale Departure	At Map Centre 26° S. 134° E.	At Land Limit	Scale Range	Maximum Scale Departure
Simple Conic 1 standard parallel	100.00	* 100.00 105.58	5.58	5.58	100.00	* 105.58	5.58	5.58
Simple Conic 2 standard parallels	100.00 99.03	* 100.00 104.43	5.40	4.43	99.03	* 104.43	5.40	4.43
Conformal Conic 1 standard parellel	100.00	* 105.28	5.28	5.28	100.00	* 110.83	10.83	10.83
Conformal Conic 2 standard parallels (Lambert)	99.03	* 104.32	5.30	4.32	98.06	* 108.83	10.77	8.83
Conical Equal Area 1 standard parallel	100.00	* 94.42 105.91	11.49	5.91	100.00	* 100.00	nil	nil
Conical Equal Area 2 standard parallels (Albers)	99.03 100.98	* 95.52 104.69	9.17	4.69	100.00	* 100.00	nil	nil
Bonne	100.00	** 99.83 100.17	0.35	0.17	100.00	** 100.00	nil	nil
Polyconic	100.00	*** 105.34	5.34	5.34	100.00	*** 105.34	5.34	5.34
Zenithal Equidistant Normal case	100.00	** 100.00 102.22	2.22	2.22	100.00	** 102.22	2.22	2.22
Zenithal Equidistant Minimum error (Scale factor .996307)	99.63	** 99.63 101.84	2.21	1.84	99.26	** 101.46	2.20	1.46
Zenithal Conformal (Stereographic)	100.00	** 103.32	3.32	3.32	100.00	** 106.76	6.76	6.76
Zenithal Equal Area	100.00	** 98.38 101.65	3.27	1.65	100.00	** 100.00	nil	nil

* South-East Cape, Tasmania. ** Tasman Peninsula, Tasmania. *** Cape Cuvier, Western Australia.

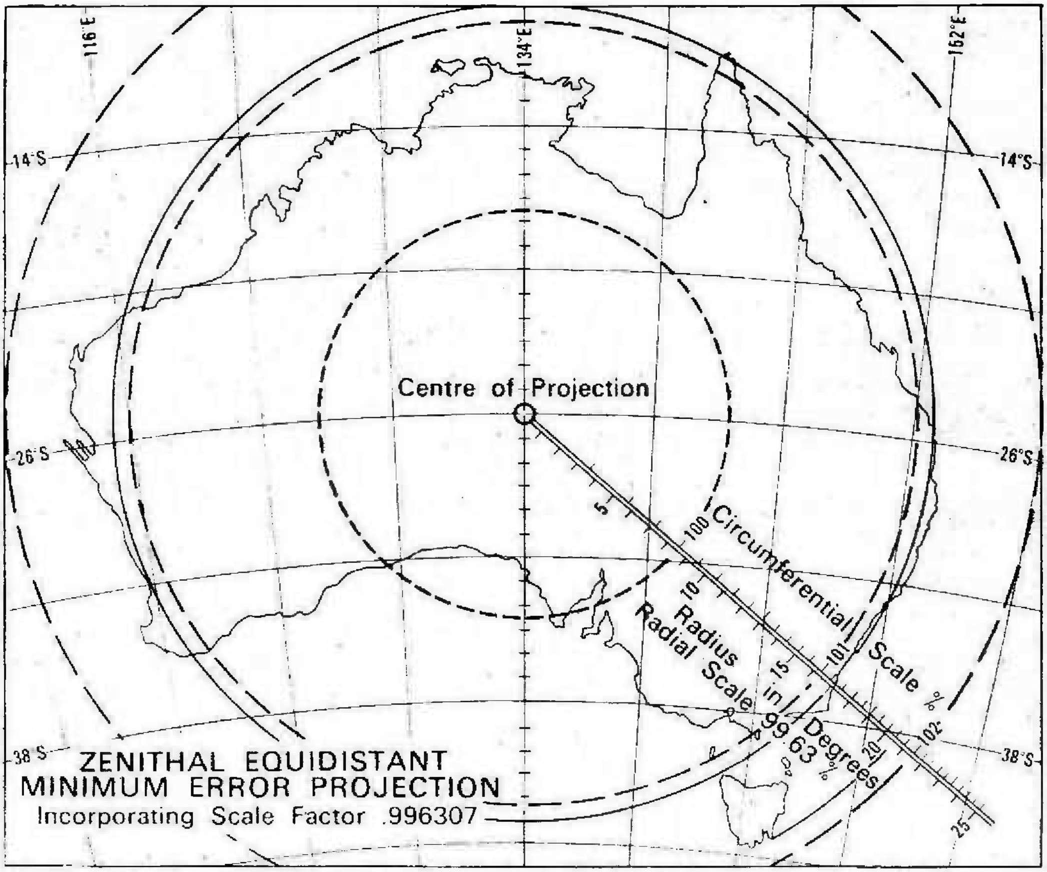
the nominal scale of the map, and in this regard they have some merit. However, in general, they do not reduce the range of scale departure from true, for in bringing the distorted scale in certain parts of the map closer to true, they magnify the errors in other parts. A scale factor designed to limit distortion in certain particulars only, as in the Lallemand Projection, may have considerable merit, though its use may destroy the basic properties of the projection. The use of two standard parallels in the conics in lieu of one may have effects similar to the introduction of either general or limited scale factors into the computations.

In the conformal conic projections, only one and two lines respectively are true-to-scale. The latter (Lambert) was preferred because it balances the actual scales closer to true even though it does not reduce the range of linear scale change which remains at 5.3 per cent at land limit of the mapping.

In the simple conic projections all meridians are true-to-scale as well as one or two parallels respectively. The latter was preferred as it brings the scale along the parallels closer to true with maximum departure 4.4 per cent against 5.6 per cent for the former, without alteration to the true-to-scale meridians.

In the conical equal area projections, the linear scales in orthogonal directions at any point in the mapping are in inverse proportion. In addition to the one or two standard parallels, true-to-scale lines occur throughout the mapping in directions diagonal to the graticule. Albers Projection with its two standard parallels reduces the maximum scale range from 11.5 per cent to 9.2 per cent and the maximum scale departure from true from 5.9 per cent to 4.7 per cent.

On the Bonne and Polyconic Projections the Central Meridian and all parallels of latitude are true. Scale elsewhere varies with



distance in longitude from the Central Meridian. It is very much superior in the former than in the Polyconic, the respective maximum scale ranges being about 0.3 per cent and 5.3 per cent and the maximum departures from true about 0.2 per cent and 5.3 per cent.

Of the zenithal projections, the conformal case is true-to-scale at the centre of the mapping only. Scale elsewhere is too large with maximum range and departure each 3.3 per cent. Some reduction of the departure could be effected by the use of a general scale factor. The equidistant and equal area cases have close distributions of isoperimetric lines, those on the former being radial from the centre of the mapping and those on the latter being cross oblique to such radials.

The respective maximum scale ranges are 2.2 per cent and 3.3 per cent with departures 2.2 per cent and 1.6 per cent. The scale varies with radial distance from the centre of projection.

Actual scales at key points, including land limit, appropriate to each of the projections reviewed, were computed and illustrated graphically on the several annexures to the Report. A diagram showing the true-to-scale pattern was included in each case. A separate tabulation was prepared for ease of comparison.

Shape

The conformal class of projection is usually considered unique for the portrayal of correct shape, but as conformality is a property existing at a point only, it becomes less significant as a general shape preserver when the area to be mapped is extensive. A projection designed for the mapping of a large area as a whole, should possess properties or advantages less restrictive in their application than that of conformality. For example, the conformality of the normal Mercator would bestow no advantage whatsoever in the use of that projection for a general purpose map of Australia. In mapping of the geographic expanse envisaged, this property was considered to be less useful than either true equal area or true equilinear scales.

In the conformal conic projections there occurs an expansion in size at the extreme parallels relative to the central latitudes. Again, although meridians and parallels are everywhere perpendicular to each other, there is incorrect convergence of meridians and incorrect curvature of the parallels other than at the mid latitude. Hence the overall shape is distorted and bending of the great circle occurs.

The simple conics and the conical equal area projections present local compression of shape, for the diverging straight line meridians create a longitudinal stretch which becomes more pronounced with distance from the central latitude. The stretch is limited in these two classes of projection to the one direction only, due to an unchanging meridional scale in the former case and to a reducing meridional scale in the latter. These two classes projection also suffer from the "averaging" of convergence of the meridians, from incorrect curvature of the parallels and from linear curvature of varying magnitude, all resulting in some degree of shape distortion.

In addition to deformations in curvature of great and small circles, a shearing effect develops in the Bonne Projection. In the Polyconic Projection, the preservation of correct scale and curvature of the parallel results in changing meridional scales which creates differential stretching accompanied by some shearing and linear curvature. All such defects are inimical to good shape presentation.

The zenithal projections incorporated shape distortions which vary in magnitude with size and distance from the centre of projection. They are not free from curvature and scale defects but in the case of the projected map they are closer to truth in convergence of the meridians and in curvature of the parallels. They probably present the best general shape of the country. The meridians are curved lines concave to a straight line central meridian. However, in a general purpose map of Australia, the curved line meridian is not a serious disability from the aspect of shape. Its greatest adverse effect would be at the extremities of the mapping where the connections of the "isolations" to the main land mass are generally meridional and the graticule may not be adequate to provide the visual orientation and contact.

It was considered that any of the projections under review did not have significant disabilities in shape inasfar as their use for a general purpose map of Australia is concerned. The lay map user may not appreciate such defects, nor be inconvenienced by their existence. The better informed has learned to recognise shape distortion by the manifestation of projection characteristics associated with the projection name and its plotted graticule, and to make allowance for it. He has learned to tolerate "compression", which is more prevalent than is generally supposed, but may be less happy when shearing exists.

Bearings and Linear Curvature

The ease of protracting and of measuring map bearings is largely dependent upon the degree of straightness of the meridians. The straight line meridian has a universal appeal both for the practical purpose of protracting bearings and because it is "straight" in nature.

The true conics have straight meridians. The other projections under review have straight central meridians but other meridians are curved. In such cases the protraction of bearings must be effected by alignment of the protractor to the unmapped tangent of the mapped meridian. At other positions in the mapping the correct alignment of the protractor is far from simple or certain.

A sheared and/or compressed graticule renders the protraction of bearings a very approximate procedure. A normally divided protractor may be unsuitable for use in such cases and assuredly so where extreme angular distortion occurs, as it may do and with wide variations dependent upon position and direction in the map.

Even in conformal mapping, the map bearing may not be free from error due largely to the bending of the great circle on the map. Its track may not be apparent. This characteristic is common to all of the projections under review. It varies in complexity with projection and in magnitude with position and direction. Any detailed analysis would prove very laborious.

In general, ease in protracting bearings is not a very critical requirement for the majority of map users. Most of the projections considered, and certainly all of those with straight meridians, would give results sufficiently close for most practical purposes using the great circle/straight line assumption. Those who require more accurate results are usually sufficiently well informed to recognise the limitations of the mapping and to obtain a more accurate result by other means.

It is ever desirable that meridians and parallels intersect at right angles, for great circles to be straight, for meridians to be correctly converged, for parallels to be at their true curvature. As all these characteristics are impossible of combination on the one map, the straight line meridian is possibly the most desirable for retention provided that distortions in the others are held within tolerable limits—always a matter of informed personal opinion.

Area

Equivalence of area is ever a desirable property in small scale mapping. Published maps are frequently accepted and used for the measurement of area when they are not suitable for the purpose. The Conical Equal Area, the Bonne, and the Zenithal Equal Area projections preserve true area ratios throughout. The remainder of the projections under review do not possess this property. The Simple Conic with two standard parallels is considerably superior to Lambert Conformal in this regard, the respective maximum area scale errors being 4.4 per cent and 8.8 per cent. The maximum errors in the corresponding cases with one standard parallel are 5.6 per cent and 10.8 per cent respectively. The Zenithal Equidistant projection is superior to all the other non-equivalent projects under review holding a maximum scale departure from true of 2.2 per cent over the land surface of the mapping, which may be reduced by incorporating a "minimum error" adjustment.

Whenever possible, a map should bear a clear indication of the distribution and magnitude of its scales and of the scale factors necessary to obtain both true lengths from mapped lengths and true lengths from mapped areas. By this means a map may be made to satisfy the basic requirements of both length and area. This is a simple matter in the case of the true conics but would be more involved in the other cases reviewed. Although such aids are both useful and desirable, they nevertheless are admission of weakness in the map. It would be entirely fallacious to suppose that the provision of such aids on a map would eliminate its disabilities and place any number of projections similarly treated on a parity.

Selection

Of the conical type projections, it was considered that the Simple Conic with 2 standard parallels is the most acceptable for the proposed map because:—

(a) It is intermediate in size between the corresponding conformal and equal area cases.

(b) All the parallels are correctly spaced. Hence it has true linear scale along all the meridians (mapped and unmapped) and along two selected parallels. In the incidence of true-to-scale lines, it is superior to Lambert and almost the equal of Albers.

(c) Its lack of conformality has no practical significance and any theoretical advantages bestowed by that property are outweighed by a generally truer linear scale.

(d) Its maximum linear scale departure from true and range of scale are approximately equal to those of Lambert and superior to those of Albers.

(e) It creates no greater difficulty in the adjustment of linear measurement than does Lambert and less difficulty than Albers.

(f) It creates no greater difficulty in area measurement adjustment than does Lambert and less difficulty than does linear adjustment in Albers.

(g) It is superior to Lambert in all aspects of area scale.

(h) In the practical use to which the map will be put, there would be no appreciable difference in the protraction of bearings between any of the true conical projections.

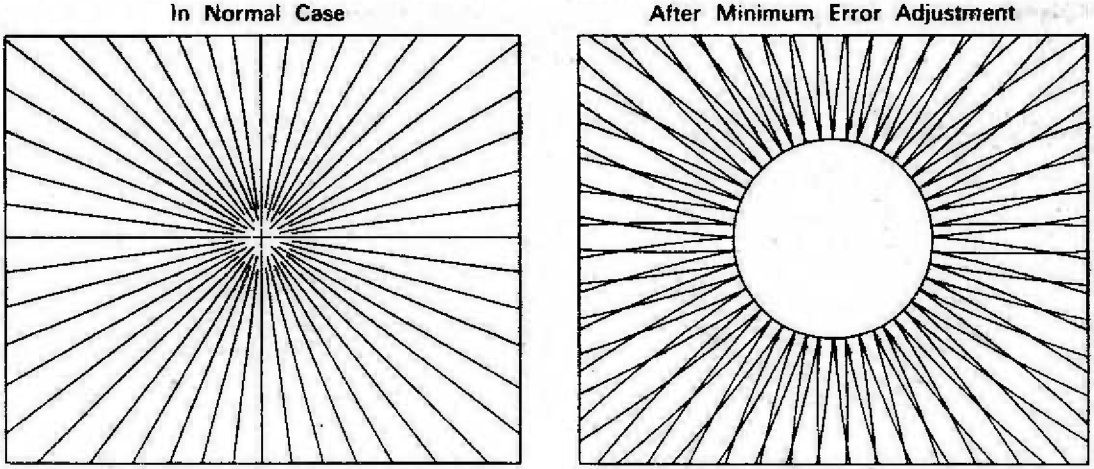
(i) The conics with two standard parallels are superior to those with one standard parallel in scale and not inferior in other respects.

(j) The wide range of longitude involved creates distortions generally that render the modified conics (Bonne and Polyconic) less acceptable than the true conics. However, Bonne is particularly good in scale value.

Regarding the zenithal projections, it was considered that the equidistant case is superior to either the conformal or equal area cases for the purpose of the mapping on the following grounds:

(a) It has the smallest range of linear scale over the land area, being 2.2 per cent against 3.3 per cent for each of the other two cases. Although this range is not balanced about the nominal scale as in the equal area case, the greater degree of "compression" in the latter places it at a comparative disadvantage. The three projections are very similar, the only apparent difference being the overall size, the equidistant case being intermediate between the smaller equal area case and the larger conformal case.

ZENITHAL EQUIDISTANT PROJECTION
SYSTEM OF ISOPERIMETRIC CURVES

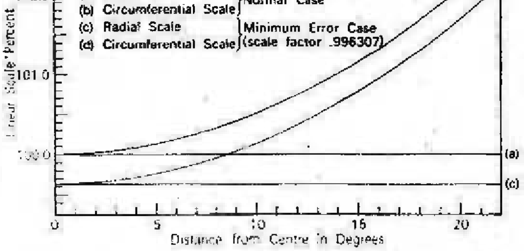


In Normal Case

After Minimum Error Adjustment

(b) The conformal case is considered to be inferior to the other two in linear scale which is everywhere larger than true. The introduction of a scale factor would bring the maximum scale departure closer to the nominal scale of the map but even then only along one circle would the scale be true.

(c) The equidistant and equal area cases have many true-to-scale lines. The introduction of a scale factor into the former would provide an increased and better distribution of such lines, the radial isoperimetric lines being displaced by a system of crossed curves oblique to the radials. On the assumption that Australia has a land area of about 3 million square miles, the mapping may be considered as being concentrated within a circle on the sphere of about 14.2 degrees. Using the formulae developed by A. E. Young for the minimum error projection of this class (which provides that the squares of the two scale errors integrated over the surface of the sphere represented by the map is a minimum) the appropriate scale factor to be introduced into the computations would be .997436. However, as important parts of Australia are located beyond this circle, better mapping would result by assuming, for purposes of the formulae, a map area of greater radius. A radius of 16 or 17 degrees of arc would appear to be more appropriate, the respective scale factors being .996731 and .996307. The use of the latter factor reduces maximum linear scale departure from 2.2 per cent to 1.8 per cent and maximum area scale departure from 2.2 per cent to 1.5 per cent. In each case the scale range remains unchanged.



(d) The radial scale of the equal area case is better than those of the latter two minimum error equidistant cases up to 9½ and 10 degrees respectively from centre of mapping but is inferior to them beyond. The circumferential scale of the equal area case is better than those of the minimum error equidistant cases up to 6 and 6½ degrees from centre of mapping and beyond about 16 and 17 degrees from centre but is inferior to them in the intervening interval.

For a map of Australia which is to be utilised for charting purposes or as a base for thermatic illustration, the Simple Conic with two standard parallels was considered the most suitable. For a map that is designed for reference purposes only, the Zenithal Equidistant Minimum Error Projection was considered to have considerable merit.

Recommendation

Zenithal Equidistant Minimum Error Projection based on assumed mapping of radius 17 degrees of arc from centre of projection located at 134° E. 26° S.

POSTSCRIPT

The foregoing recommendation was accepted and production proceeded along those lines.

Graticule construction and compilation control involved the computation of spheroidal bearings and distances of about 120 lines radiating from the centre of the mapping to selected geographical intersections. The International Figure (Hayford, 1910) was used. The "Minimum error" adjustment was then applied and rectangular plotting co-ordinates calculated at a nominal map compilation scale of 1:3,500,000, for final publication at 1:6,000,000 nominal.

Originally published in 1956, the map has been revised and reprinted from time to time to meet an ever growing civilian and service demand.

The Division of National Mapping has produced a new map of Australia, at a scale of 1:2,500,000 published in 4 sheets. This map was exhibited at the time of the Fifth United Nations Regional Cartographic Conference for Asia and The Far East in Canberra in March, 1967. The simple Conic Projection, with two standard parallels is used. A companion map in one sheet, at 1:5,000,000 is on programme. Present indications are that it will supersede and displace the Zenithal Equidistant map now current.

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