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NEW DETERMINATIONS OF THE VELOCITY OF RADIO WAVES

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Abstract--The author deduces a new value (299,794.2 ± 1.4 km/sec) for the velocity of propagation of radio waves in a vacuum, as compared to the value of 299,792.4 ± 2.4 km/sec which he had previously reported. Whereas both values were determined by comparison of shoran and geodetic measurements of distances between widely separated points, the new value results from comparisons with improved shoran equipment. The improvements are discussed, with emphasis on the introduction of a gain-riding technique to reduce signal intensities to a constant level and eliminate the possible uncertainties in the signal-intensity corrections which were applied in the earlier determination. The methods of making shoran measurements are outlined and the results of a least-square adjustment of the new shoran data are discussed. In view of the corroborative evidence from other recent determinations, the author recommends abandonment of the Birge statistical value of the velocity of light and states that a value of 299,793 km/sec has been adopted for current shoran measurements.

In 1949 the writer [ASLAKSON, 1949] reported evidence that the BIRGE [1942] statistical value of the velocity of light was too low. That evidence was obtained from a comparison of conventional triangulation with shoran distances. The value derived was 299,792.4 ± 2.4 km/sec. One unsatisfactory feature of this earlier work was the necessity of applying an emperical signal-intensity correction. Although the writer had considerable confidence in the validity of the correction, it was noted that when the signal intensity was ignored, a velocity very close to the Birgevalue resulted. Since that time great efforts have been made to remove that source of error and those efforts have met with considerable success.

In February and March 1950, the United States Air Force completed an extensive project in Florida, wherein modified shoran equipment was tested. The shoran modifications and improvements were:

(1) Improvement of the goniometer or phase shifters

(2) Improvement of the zeroing of circuits

(3) Addition of a 0.2-mi sweep to permit more accurate pip alignment

(4) Photographic recording of the 0.2-mi oscilloscope, enabling pip alignment errors to be corrected

(5) Adoption of a "gain-riding" equipment to permit manual control of the gains, both in the airborne equipment and at the ground station

The last-named improvement was by far the most important one and it effectively removed the troublesome signal-intensity correction. On the bench, this gain control reduced the shoran readings to the same level over ranges far greater than are ever encountered on shoran missions. In the air an extra operator controlled the gains at all times during the line-crossing procedure. The ground-station operators likewise maintained the gain at a constant level. During rapid fluctuations of the signals considerable operator skill is required to "ride the gains," but nevertheless the effectiveness of the technique was reflected not only in the more uniform "sum-distance" curves but in the final computed results. These improvements will be discussed farther on in this report in greater detail.

Shoran method of distance measurement--For the benefit of those who are unfamiliar with the shoran methods as developed in the United States Air Force under the direction of the writer, the following description of the "line-flying" procedure is presented. The shoran instrument is well known and its technical description will not be included here. Basically, shoran is a system in which the travel time of a pulse from an airborne transmitter to a ground transponder and back is measured in the aircraft by means of a goniometer or phase-advancing system. The frequency is controlled by temperature-controlled crystals at the ground stations. The frequency of the crystal now being used is 93,109.87 cycles/sec, and is chosen because its 537th, 1074th, and 1611th harmonics are 5×10^7, 10×10^7, and 15×10^7 respectively, facilitating comparison with the standard frequencies of WWV. The principle of the shoran design is satisfied for distance when the timing frequency in cycles/sec is equal to one-half the velocity in mi/sec or

$$f_t = V_t/2 \qquad (1)$$

where f_t = the frequency of the timing crystal and V_t = the velocity of the impulse in mi/sec.

The velocity of propagation in air at a given instant is given by

$$V = cK_1^{-1/2} = c/\mu \qquad (2)$$

where V is the velocity of propagation in air, c is the velocity in a vacuum, K_1 is the dielectric constant of the air, and μ is the index of refraction. The values of K_1 and μ are dependent on experimental data and will be discussed later in this paper. The Birge statistical value of c = 299,776 km/sec corresponds to 186,271.8 mi/sec in units in the United States. The correct crystal frequency corresponding to the Birge velocity would have been 186,271.8/2 or 93,135.9. Therefore, it is necessary to apply a correction for the difference between these two frequencies or

$$\Delta T = (93{,}135.9 - 93{,}109.87)/93{,}109.87$$

where ΔT is the ratio by which the distances in miles were increased in the goniometer design to give the correct distance according to the Birge velocity. The timing-crystal frequency is not to be confused with the transmission frequencies. In shoran the airborne transmitter is usually on a frequency of about 220 to 240 mc while the ground stations reply on a common frequency of about 300 mc.

Method of distance measurement--In the geodetic application the distances between shoran ground stations were determined in the following manner: The minimum distance between two stations was obtained by flying across the line between the stations. The counter readings, giving the distances to each ground station were photographically recorded at two-second intervals. The two ground stations involved in a particular line crossing are designated as "rate" and "drift" in accordance with the usual shoran terminology. The minimum sum distance was then determined analytically from a large number of simultaneous observations on each of the two distances, 30 to 40 sum distances being used in each computation. At least 12 and frequently 16 to 18 such measurements were made on each line. Each shoran distance was then corrected by a timing correction and a velocity correction.

The two corrected minimum shoran distances were next reduced to the geodetic distances by geometric corrections involving the altitude of the shoran aircraft and the altitude of the ground stations. The final result was the geodetic distance between stations which was then compared with the geodetic distance as determined by conventional triangulation.

The details of the correction system have been discussed in numerous papers [ASLAKSON and RICE, 1946; ASLAKSON, 1949; ASLAKSON and FICKEISSEN, 1950; ASLAKSON, 1951]. They have been proven in practice and will not be repeated here. However, inasmuch as certain physica measurements are involved in the velocity correction, it is desirable to mention that correction briefly.

The velocity correction--As the shoran ray travels between airplane and ground station it passes through an atmosphere of varying barometric pressure, temperature, and water-vapor pressure. The index of refraction changes continuously along the path, with consequent variations in the velocity of the ray. For this reason it is necessary to correct the measured distance for these changes in velocity from the standard velocity incorporated in the design by the timing-crystal frequency.

The changing index of refraction has a twofold effect on the measured distance: (a) a length change due to the different degrees of bending, and (b) a change in velocity along the path which is a function of the index of refraction in accordance with Eq. (2). In shoran work the lapse rates of barometric pressure, temperature, and water-vapor pressure are observed along the approximate ray path by a second airplane and the data are available for determining the correction. KROLL [1949] has shown that it is possible to combine both effects in a single velocity correction by integrating the basic differential equation of terrestrial refraction along the ray path. However, in shoran work a more practical and far less laborious method has been used and comparisons with the Kroll method show that the differences are generally insignificant. In shoran work the effect of the ray bending is separated from the velocity change by assuming a curved path and combining the bending effect with the geometrical correction. The effect due to change of velocity along the path can then easily be shown to be represented by the relationship

$$\Delta S = \int_0^S \Delta\mu \, ds$$

where ΔS is the velocity correction, $\Delta\mu$ is the index of refraction minus unity, computed at appropriate intervals along the ray path, and ds is the corresponding distance increment computed at appropriate intervals along the ray path. In practice the above expression is evaluated by numerical integration. It is combined with the frequency correction in a single expression of the form

$$\text{Velocity correction in miles} = \left\{[(V_0 - f)/f]^{10^6} - \sum_{h=k}^{h=H} \Delta\mu\right\} \bar{S}_a \times 10^{-6} \; \ldots\ldots\ldots \quad (3)$$

where V_0 = the value of c or the in-vacuo value of velocity adopted, f = the shoran timing-crystal frequency, k = the altitude of the ground station, H = the altitude of the shoran aircraft, $\Delta\mu$ = the increment of the index of refraction between the points selected for the numerical integration, and $\bar{S}_a$ = the computed minimum distance from the airplane to the ground station after correcting for the zero readings. Tables giving the distance increments on the various ray paths have been prepared for a large number of paths. Their use and the arrangement of calculations in a compact computing form make the entire integration process simple and straightforward.

The value of μ = 1.0002835 for standard dry air at 760 mm pressure and 0°C temperature was originally used in the computation. An extensive investigation of all work which had been

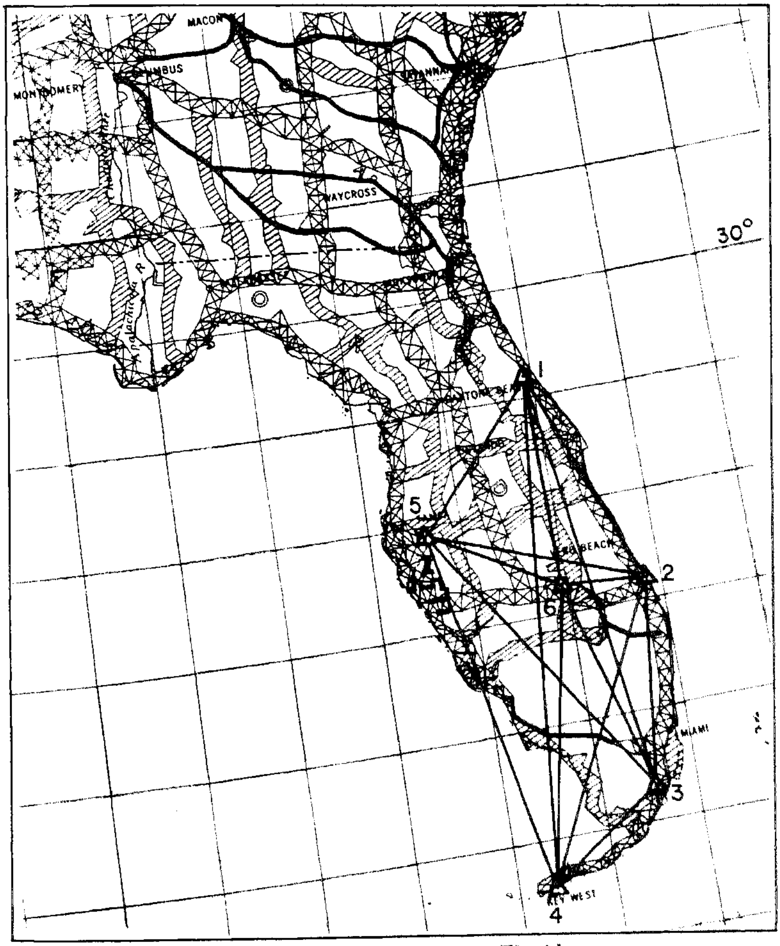

Fig. 1--Shoran scheme in Florida

Table 1--Comparison of geodetic distances with shoran determinations based on a velocity of 299,776 km/sec

Line	No. of crossings	Ground sta. no.		Distances		Shoran - geodetic
		Rate	Drift	Geodetic	Shoran	
				mi	mi	mi
2-6	13	2	3	40.6131	40.6041	-0.0090
3-4	12	1	3	96.7171	96.7049	-0.0122
5-6	11	1	2	100.3098	100.2988	-0.0110
5-1	12	1	3	118.9953	118.9840	-0.0113
1-6	12	1	2	133.0113	132.9985	-0.0128
2-3	12	2	3	134.9698	134.9546	-0.0152
5-2	12	1	3	139.1225	139.1127	-0.0098
3-6	17	1	2	145.8427	145.8276	-0.0151
1-2	18	3	2	145.8884	145.8768	-0.0116
4-6	11	1	2	190.5047	190.4889	-0.0158
2-4	12	1	2	199.1914	199.1735	-0.0179
5-3	14	2	3	226.9903	226.9698	-0.0205
5-4	11	1	2	235.5264	235.5042	-0.0222
1-3	12	2	3	277.0569	277.0347	-0.0222
1-4	13	1	2	320.1519	320.1271	-0.0248

Table 2--Summary of deviations from the mean

Deviation, mile	Individual line-crossings			Missions flown on different days		
	Number	Per cent	Cumulative per cent	Line no.	No. of missions	Average deviation, mile
0.0000 to 0.0010	97	54.2	54.2	3-4	9	0.0013
0.0011 to 0.0020	56	31.3	85.5	1-2	2	0.0025
0.0021 to 0.0030	21	11.7	97.2	3-6	2	0.0006
0.0031 to 0.0040	4	2.2	99.4	5-3	2	0.0012
0.0041 to 0.0050	0	0.0	99.4	5-4	4	0.0011
0.0051 to 0.0052	1	0.6	100.0		19	0.0013[a]
	179					

[a]Weighted mean.

done in the measurements of index of refraction and dielectric constant indicated that a statistical value of all preceding determinations would be about 1.0002875. A recent determination by BIRNBAUM, KRYDER, and LYONS [1951] gives a value for the dielectric constant of standard dry air as 1.0005763 or μ = 1.0002881. Inasmuch as the value of μ = 1.0002876, as determined from measurements in the optical band, lies between the above values, it was chosen and the originally computed lengths were changed to correspond with this value, which will be used in shoran work now in progress.

The interpolation equation currently in use to correct to the accepted velocity is

$$\Delta\mu \times 10^6 = 287.6 - 77.54\ P/T + 9.66\ e/T - 37.84\ e/(T/100)^2 \quad \ldots\ldots\ldots\ldots (4)$$

where $\Delta\mu$ = the change from the standard dry-air condition to the observed condition, P = the total pressure in mb, e = the water-vapor pressure in mb, and T = the temperature in °K.

The equation used in computing the values of $\Delta\mu$ in this paper differed very slightly from the above, the principal change being the change of the index of refraction for standard conditions, and corrections to distances were made for this change.

The distances measured--The project as a whole is shown in Figure 1. All stations were at or near triangulation stations of the U. S. Coast and Geodetic Survey. The inversed geodetic distances were computed using a modified Helmert method and are numerically exact to 0.0001 mile. The results of these measurements are tabulated in Table 1. Examination of the last column clearly shows the increase of the discrepancy with distance, which the writer attributes to the basic velocity error. However, the discrepancy in this column also includes a small constant error which was known to be about -0.0050 mile. In other words, each shoran ground station was known to measure about 0.0025 mile too short. The determination of these errors will be discussed later.

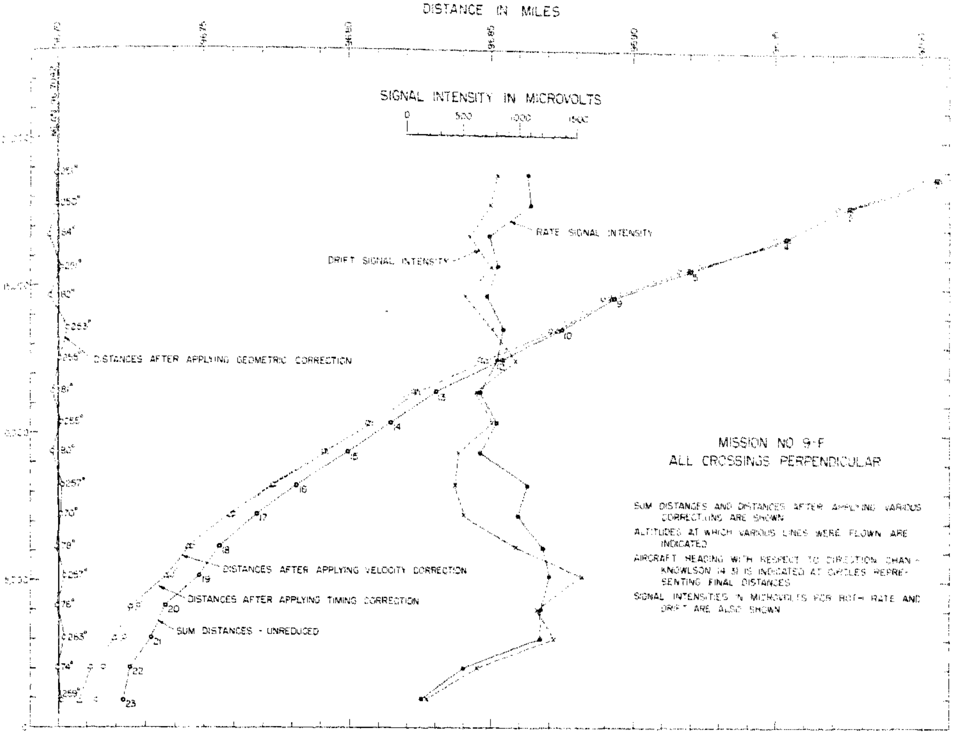

Fig. 2--Observed and corrected distances on a test mission

The internal consistency of the individual shoran line crossings was very high as shown by Table 2, which also illustrates that the repetitional accuracy of missions flown on different days is approximately the same as the internal consistency of a single mission. Numerous test missions which were not included in the record missions were flown. One of the most interesting ones was Mission 9-F, illustrated in Figure 2,which brought out three facts: (a) the validity of the gain-riding technique for removing the signal-intensity error, (b) the validity of the timing, velocity, and geometrical corrections, and (c) the first evidence of a multipath propagation effect.

Regarding (a) it is noted in Figure 2 that Line 3-4 was flown at 18 elevations at approximately 1000-ft intervals. In flying in this manner wide variations of signal intensity were encountered as shown in the figure. Yet there is no evidence that the absolute measurement was changed by this signal-intensity range.

Regarding (b) the successive corrections are applied to the original sum-distances with the final reduced distance resulting for each altitude. Again no correlation of reduced distance with altitude is encountered.

However, regarding (c) it is clearly evident that there is a correlation of aircraft heading with measured distance. The headings are shown for each reduced distance. It was clearly evident that this was not related to the signal intensity, and after much study it was demonstrated that this was a multipath propagation effect due to a combination of circumstances seldom encountered in practice. On this mission, the altitude of the airplane was such that a third re-radiated ray from a portion of the B-29 wing was responsible for this effect. The combination of circumstances resulting in such a condition is that, first, the ray re-radiated from the wing must be stronger in signal intensity throughout the crossing than the signal resulting from the phase combination of the direct ray and the ray reflected from the surface of the Earth. Secondly, this ray, which has a slightly longer path length, cannot exceed that path length by more than approximately 0.002 mile or its signal will be obliterated by the signal from the direct ray. Clearly, a mean of all crossings will differ little from a true value. However, a study of the antennae wiring and their placement was made, and before the record missions certain modifications were made in the equipment such that there is no evidence of this effect in the record missions. Indeed, a portion of the directional effect may be attributable to the fact that these crossings were all nearly perpendicular crossings rather than the accepted figure-8 technique. In perpendicular crossing, a slight slackness in the gear trains of the goniometer might cause an apparent directional effect. This is eliminated in the figure-8 technique. An observer's personal error may also contribute to the effect.

Multipath propagation effects from surfaces other than those on the aircraft and the surface of the Earth can cause no concern when shoran methods are used. The path length is so much greater that it is obscured by the direct ray or appears only momentarily on the oscilloscope as a stray, whereas the resultant of the direct and reflected rays registers continuously on the oscilloscope.

Computation of the velocity--The results in Table 1 were analyzed to determine the velocity compatible with the distances obtained. It is also possible in this computation to make an analytical determination of the constant errors of each ground station. It is noted that only three shoran ground stations were used in the work. Inasmuch as 15 distances were measured, there are 15 observation equations in all. These equations are of the form

$$FS + K_1 + K_2 + E = 0$$

where F is the factor by which the shoran distances S [or the design velocity of 299,776 km/sec] must be increased to obtain the correct distance; K_1, K_2, and K_3 are the respective constant errors of ground stations 1, 2, and 3; and E represents the value of S-G, or the shoran reduced distance minus the inversed geodetic distance.

A least-square solution of the above 15 observations results in the following:

$$F = 1.0000606 \pm 0.0000046 \text{ or}$$
$$c = 299{,}776\ F = 299{,}794.2 \pm 1.4 \text{ km/sec}$$
$$K_1 = -0.00193 \pm 0.00058 \text{ mile}$$
$$K_2 = -0.00369 \pm 0.00073 \text{ mile}$$
$$K_3 = -0.00200 \pm 0.00053 \text{ mile}$$

Thus

$$K_1 + K_2 = -0.0056 \text{ mile}$$
$$K_1 + K_3 = -0.0039 \text{ mile}$$
$$K_2 + K_3 = -0.0057 \text{ mile}$$

Probable error of a shoran measurement = $\pm$ 0.00118 mile

The values of the combinations of the constant ground-station errors must be applied to the shoran measured distances. The values obtained in the solution are negative. The errors are of opposite sign and the shoran distances must be increased by the amount of the factor F and the constant errors to give the correct distances. The velocity designed into the instrument must also be increased by the factor F.

Determination of local survey error by shoran measurements--An interesting example of the confidence of the writer in the shoran measurements occurred during the course of the project. Examination of Figure 1 shows that five distances are measured to each point. When the shoran measurements were first compared with the geodetic distances, the results at Station (4) near Key West were not compatible with the results of the remaining 10 lines. This discrepancy is best illustrated by reference to Figure 3. Therefore, a preliminary adjustment was made assuming a single constant error for all combination ground stations and the amount of movement of Station (4) necessary to bring it into agreement with the remaining shoran distances was computed. The following observation equations were used.

For lines not including Station (4) the equations took the form

$$FS + K + E = 0$$

For lines including (4), two additional terms were added to allow for shift of Station (4) and the equations were of the form

$$FS + K + d\,[\cos(\alpha - \beta)] + E = 0$$

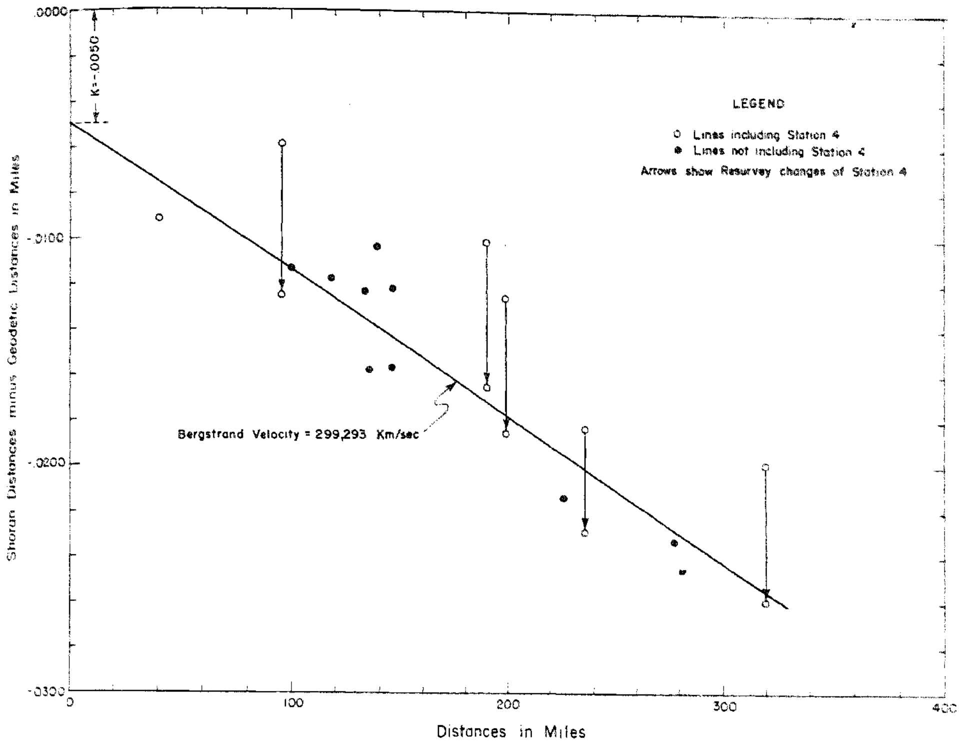

Fig. 3--Shoran distances minus geodetic distances prior to resurvey of Station 4 at Key West

F, S, K, and E have been defined, d is the distance station (4) should be moved, α is the azimuth of the line from Station (4) to the remote ground station, and β is the azimuth of the short distance joining the new position of (4) to the original position.

This adjustment resulted in the following determinations: d = 0.0067 mile and β = 219 3/4°, or β - 180° = 39 3/4°.

As a result of the above computation, the writer in a written report recommended that a resurvey be made at Key West. This resurvey was made several months later by the U. S. Coast and Geodetic Survey. The error predicted by the writer in the above adjustment was verified and the results were: d = 0.0069 mi and β = 37.0°.

Thus a local survey error was predicted within two feet by five shoran measurements which varied from 96 to 320 miles. This instance supports the value of the velocity of radio waves derived from shoran measurements.

In fairness to the local survey party it should be stated here that the error was not in their field work but was caused by the fact that they tied to a single reference mark which had evidently been disturbed; the original station mark was lost.

Results--The results computed from the final values are shown graphically in Figure 4. In that figure the analytically determined values of the constant error have been applied. The final tabulated results of the adjustment are shown in Table 3. The mean proportional error of about 9 parts in 10^6 is clearly compatible with the figure of $\pm$ 1.4 km/sec derived as the probable error of the velocity determination from the computation, when it is considered that 15 measurements were made.

Recent published determinations of c--ESSEN [1951] tabulated the values shown in Table 4. The writer believes that the value of 299,794.2 $\pm$ 1.4 as published in this paper is far better than the previous value obtained from the earlier shoran work and that it should also be included in

Table 3--K and F corrections and residual errors (v's) derived from the adjustment by least squares

Line	Ground station numbers	Shoran distance (s)	Shoran minus geodetic distance	Correction for K	Correction for F (F-1)s	v's from adjustment	Proportional error
		mi	mi	mi	mi	mi	(parts in 10^6)
2-6	2 and 3	40.6041	-0.0090	+0.0057	+0.0025	+0.0008	19.7
3-4	1 and 3	96.7049	-0.0122	+0.0039	+0.0059	+0.0024	24.8
5-6	1 and 2	100.2988	-0.0110	+0.0056	+0.0061	-0.0007	7.0
5-1	1 and 3	118.9840	-0.0113	+0.0039	+0.0072	+0.0002	1.7
1-6	1 and 2	132.9985	-0.0128	+0.0056	+0.0081	-0.0009	6.8
2-3	2 and 3	134.9546	-0.0152	+0.0057	+0.0082	+0.0013	9.6
5-2	1 and 3	139.1127	-0.0098	+0.0039	+0.0084	-0.0025	18.0
3-6	1 and 2	145.8276	-0.0151	+0.0056	+0.0088	+0.0007	4.8
1-2	3 and 2	145.8768	-0.0116	+0.0057	+0.0088	-0.0029	19.9
4-6	1 and 2	190.4889	-0.0158	+0.0056	+0.0115	-0.0013	6.8
2-4	1 and 2	199.1735	-0.0179	+0.0056	+0.0121	+0.0002	1.0
5-3	2 and 3	226.9698	-0.0205	+0.0057	+0.0138	+0.0010	4.4
5-4	1 and 2	235.5042	-0.0222	+0.0056	+0.0143	+0.0023	9.8
1-3	2 and 3	277.0347	-0.0222	+0.0057	+0.0168	-0.0003	1.1
1-4	1 and 2	320.1271	-0.0248	+0.0056	+0.0194	-0.0002	0.6
				Means		0.00118	9.1

Table 4--Velocity of waves in vacuo (c)

Author	Method	Velocity in vacuo	Weight
		km/sec	
Birge	Optical	299,776 ± 4	1
Essen and Gordon-Smith	Cavity resonator	299,792 ± 9	1
Bergstrand	Optical	299,793.1 ± 0.25	2
Aslakson	Shoran	299,792 ± 2.4	2
Essen	Cavity resonator	299,792.5 ± 3	2
Bol	Cavity resonator	299,789.3 ± 0.4	2
	Resultant mean value	299,790.2	

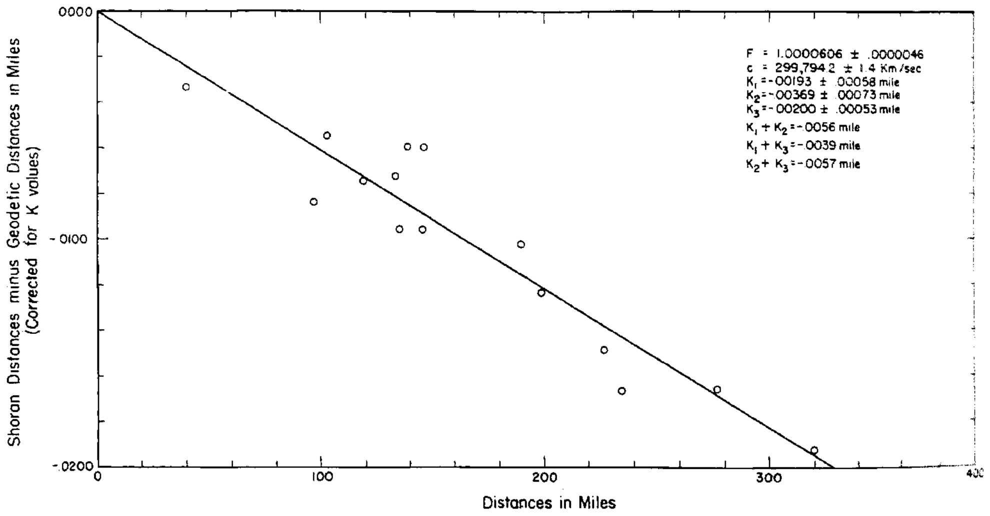

Fig. 4--Adjusted shoran results assuming a separate K for each ground station

any proposed value for c. In fact, the evidence now is very strong that the Birge statistical value should be abandoned entirely. A detailed examination of all of the results of the individual observers included in the determination of that value reveals a lack of consistency far below that obtained by recent observers. An average of the latest observations results in a new value of 299,792.2 which, in view of the better consistency of recent work, seems to be a reasonable figure to adopt for the present.

It can now be stated that, in shoran work in progress, a value of 299,793 km/sec is being used in the computations and to date excellent checks are being obtained in comparison with surveyed distances. During the present work at least 9 to 11 new distance comparisons will be made.

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