The Role of Laplace Observations in Geodetic Survey

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ABSTRACT

The general utilisation of Laplace observations in geodetic survey is reviewed, the particular application of azimuth control is the design of first order traverse surveys is stressed. As a basis for this design, the effects of statistical summation of errors in traversing operations are considered in relation to accuracies currently being achieved. The conclusion is reached that the present planning and current operations will provide Australia with an extremely accurate National Geodetic Survey.

Introduction

1. This paper is based on the following definitions:—

> *A Laplace Observation* consists of a series of precise observations carried out at Geodetic Survey stations, for the purpose of determining astronomical latitudes, longitudes and azimuths which will subsequently be incorporated as an integral part of the survey calculations.

2. Admittedly only longitude and azimuth observations are necessary for azimuth control but latitude is necessary for geoid studies.

3. No attempt is made to deal with the theory of Laplace's Equation[1] or with relevant observations and computational methods[2]. The accent is on the design of first order traverses in order to give a maximum precision consistent with early utilisation of results.

General Utilisation of Laplace Observation data

4. In the process of making a geodetic survey it is normal, first of all, to select an arbitrary ellipsoid, a point of origin with arbitrary co-ordinate values and an arbitrary orientation; all for the purpose of mathematically computing initial co-ordinates of survey stations that have been inter-connected by continuous field measurements which emanate from the origin.

5. The arbitrary ellipsoid would usually be selected on the basis of the latest published determinations of other survey organisations.

6. For the Origin it would be normal to select a station in an extensive, fairly flat area on the assumption that the deflection of the vertical will not be large there. However, in practice it has been usual to accept some well established astronomical observatory.

7. Usually the values determined astronomically are accepted for the initial latitude, longitude and azimuth, and for the radius vector co-ordinate it is usual to adopt the corresponding ellipsoid radius vector modified by the height of the point above mean sea level.

8. The progressive accumulation of azimuth error throughout the survey would have to be corrected by the Laplace Equation.

(dA = d Long × Sin Lat).

9. The differences between the astronomical and geodetic values for latitude, and for longitude when multiplied by cos. latitude, respectively provide the meridian and prime vertical components of the deflection at each Laplace observation station and thereby define the slope of the surface of a small portion of the geoid surface around that point with respect to the reference ellipsoid.

10. Once fair converage has been achieved with the geodetic survey the astro/geodetic differences at Laplace observation stations should be analysed to determine more appropriate parameters for the mathematical reference ellipsoid and the latitude and longitude co-ordinates of the origin.

11. This analysis can be best done by a least squares fitting for the purpose of eliminating systematic effects introduced by the use of a wrong reference ellipsoid, wrong starting latitude, longitude and azimuth and also for the purpose of reducing the squares of the residual deflections to a minimum.

12. This process will need to be repeated in this instance, and again as more data becomes available, until stability is reached.

13. In order to provide the relative radius vector co-ordinates of the geodetic stations it will be necessary to establish a very large number of closely spaced astro/geodetic stations and by integrating the geoid slope components derive a mathematical surface expressed in elevation relative to the reference ellipsoid. (It should be noted that latitude and longitude data only are required for this purpose).

14. The surface so deduced is relative to the adopted origin and it may at this stage be thought desirable to raise or lower it to give a general mean value of zero elevation for the area under survey.

15. The writer believes that, even in very flat terrain, the geoid undulations conform to a statistical pattern of knolls and depressions at roughly 60 to 100 miles intervals and it is therefore necessary to establish a 10 to 15 mile grid pattern of astro/geodetic observations if sufficient geoid slopes are to be obtained for the purpose of deducing a reasonably accurate geoid contour pattern.

16. This intense network of astro/geodetic values is not likely to be available for some considerable time and it is possible that as an interim measure, selected geoid sections will first be observed. From these, elevations could be deduced and analysed to provide initial approximate radius vector co-ordinates.

17. If national geodetic co-ordinates are to be made as near absolute as practical it will be necessary to organise international surveys in order to directly connect all the isolated groups of co-ordinates over the surface of the earth, utilising special geodetic satellites to measure across spaces too large for the other forms of measurement, and then to undertake a large sized mathematical analysis of the results until a world geodetic datum and world wide continuous svstem of co-ordinates evolves.

18. It is likely that, in order to achieve greater scientific accuracy and to take advantage of more accurate survey equipment as it becomes available, processes of successive approximation will continue indefinitely and co-ordinate values will be progressively refined.

19. Fortunately, nowadays, the availability of electronic computing facilities makes this successive approximation technique a practical procedure.

20. The processes outlined above will take considerable time to finish. For immediate use in surveying and mapping activities, it is desirable to settle on sufficiently precise co-ordinate values as early as practical in the development of a national geodetic survey and to thereafter use those irrespective of subsequent refinements for scientific purposes.

Particular utilisation of azimuth control

21. Once a reasonably fitting geoid and accurate origin co-ordinates have been adopted, azimuth control can be used to improve and maintain the accuracy of the geodetic survey, by controlling the development of statistical effects on the summation of random errors. (Appendix 1 contains some notes on these effects).

22. With this in mind, an investigation was made into the linear, angular and azimuth accuracies currently being obtained in Australian geodetic survey operations (Appendix 2); another investigation (Appendix 3) was made into the pattern of errors likely to develop in horizontal control surveys and the way in which these surveys could be planned for optimum practical accuracy.

23. The accuracy investigations indicated that measurements are currently being made with the following external precision:—

(*a*) linear p.e. $\pm$ 2 parts in 10^6 on lines about 20 miles long;

(*b*) azimuth and angular p.e. $\pm$ 0.6″.

24. The design investigations were concentrated on first order traversing which has been adopted as the main method for first order horizontal surveys in Australia. These investigations showed that:—

(*a*) undoubtedly the most accurate surveys could be made by observing individual azimuths over every line of the traverse (this technique would require at least a longitude observation at every station);

(*b*) an accuracy meeting all immediate survey and mapping requirements could be obtained by;

(i) making basic observations of the above precision;

(ii) undertaking Laplace observations at every 6th station of a traverse in which sides average approximately 20 miles in length.

Utilisation in the Australian National Geodetic Suryey

25. Laplace observations have already been used for the purpose of arriving at provisional origin co-ordinates and a reasonable fitting ellipsoid but it will be many years before geoid sections, let along a complete geoid contour map, will be available. The " 165 " Figure has been adopted (a = 6 378 165 metres, f = 1/298·3).

26. At present Laplace stations have an average spacing of 6 stations and the National Mapping Council has specified that final spacing should be 4 to 6 stations.

27. The analyses of loop closures (Figures 6 and 7) indicate that actual misclosures are slightly greater than theoretical expectation. However, as the theoretical results are based on certain arbitrary assumptions and as the effects of geoid undulation are unknown, the agreement of better than 1 part in 10^6 between theoretical and actual values is surprisingly good.

28. Figure 8 shows the effects of progressive intensification of Azimuth Control on the calculated station values along the Wundinna (S.A.) to Moodina (W.A.) first order traverse. This traverse (about 500 miles long) closely follows the coastline and systematic refraction errors almost certainly exist in addition to those resulting from statistical double summation.

Conclusion

29. The investigations and the results set out in this paper indicate the extremely important role of Laplace observations in the making of geodetic surveys.

30. In the particular case of the Australian National Geodetic Survey it is hoped the utilisation of frequent Laplace azimuth control together with careful angular observation and the use of modern electronic distance measuring equipment will provide an extremely accurate result.

Appendix 1

STATISTICAL SUMMATION EFFECTS

Single Summation errors

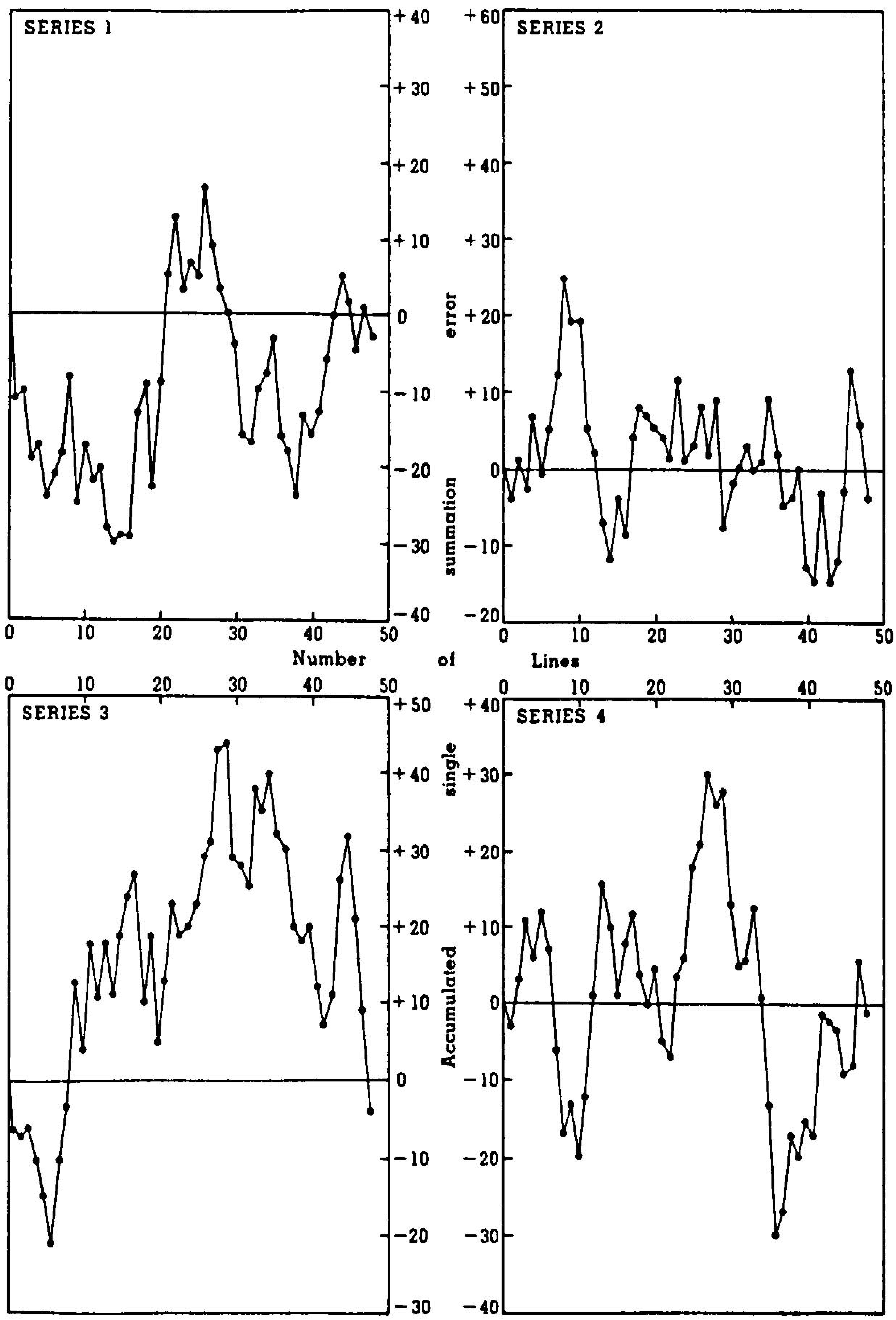
1. If the individual errors of a series of observations each having a random p.e. of $\pm$ e are added consecutively, the probable error of the nth summation theoretically is $\pm e\sqrt{n}$; provided the summation is not extensive.

2. However, it is not always realised that this consecutive type of summation produces error patterns that can have the appearance of systematic errors. Furthermore, if carried on to a considerable extent the error can develop oscillations in accordance with the statisticians " arc sin " law.[3]

3. The first phenomenon is shown in Figure 1A which consists of a group of diagrams illustrating the results of consecutive summation of 4 separately and randomly arranged series each consisting of the same 48 random numbers originally selected on the basis of the normal statistical distribution around a probable error of $\pm$ 5 units. In Figure 1B the consecutive summation has been extended to a group of 1,600 such numbers to show a definite oscillation pattern.

FIGURE 1A

SINGLE SUMMATION EFFECTS

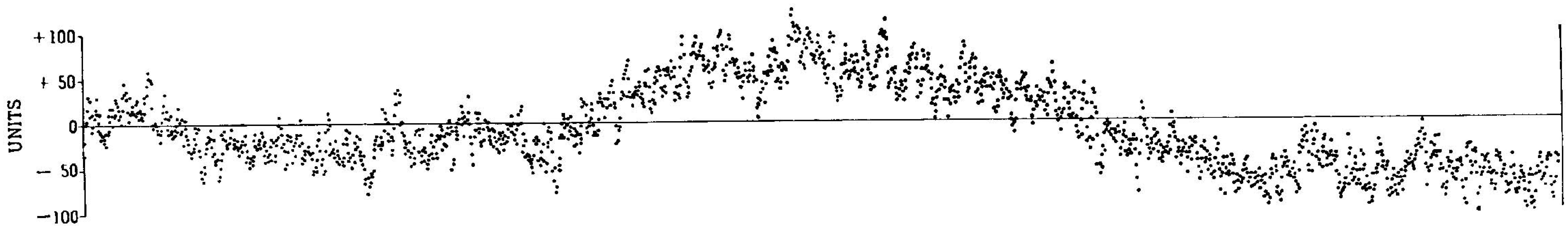


EXPLANATION

1. A p.e. ±5 units has been assumed for observations.

2. If the above graphs are used to represent azimuth summation errors then these will be in same units as angular observation errors e.g. for a p.e. ±1" each unit = 0.2"

3 If the graphs are used to represent offset errors accumulating from independent azimuths over equal lines the unit = (angular p.e./5) × (line length/206,000) e.g. for ± 1" and 20 mile lines; 1 unit = 0.1 ft.

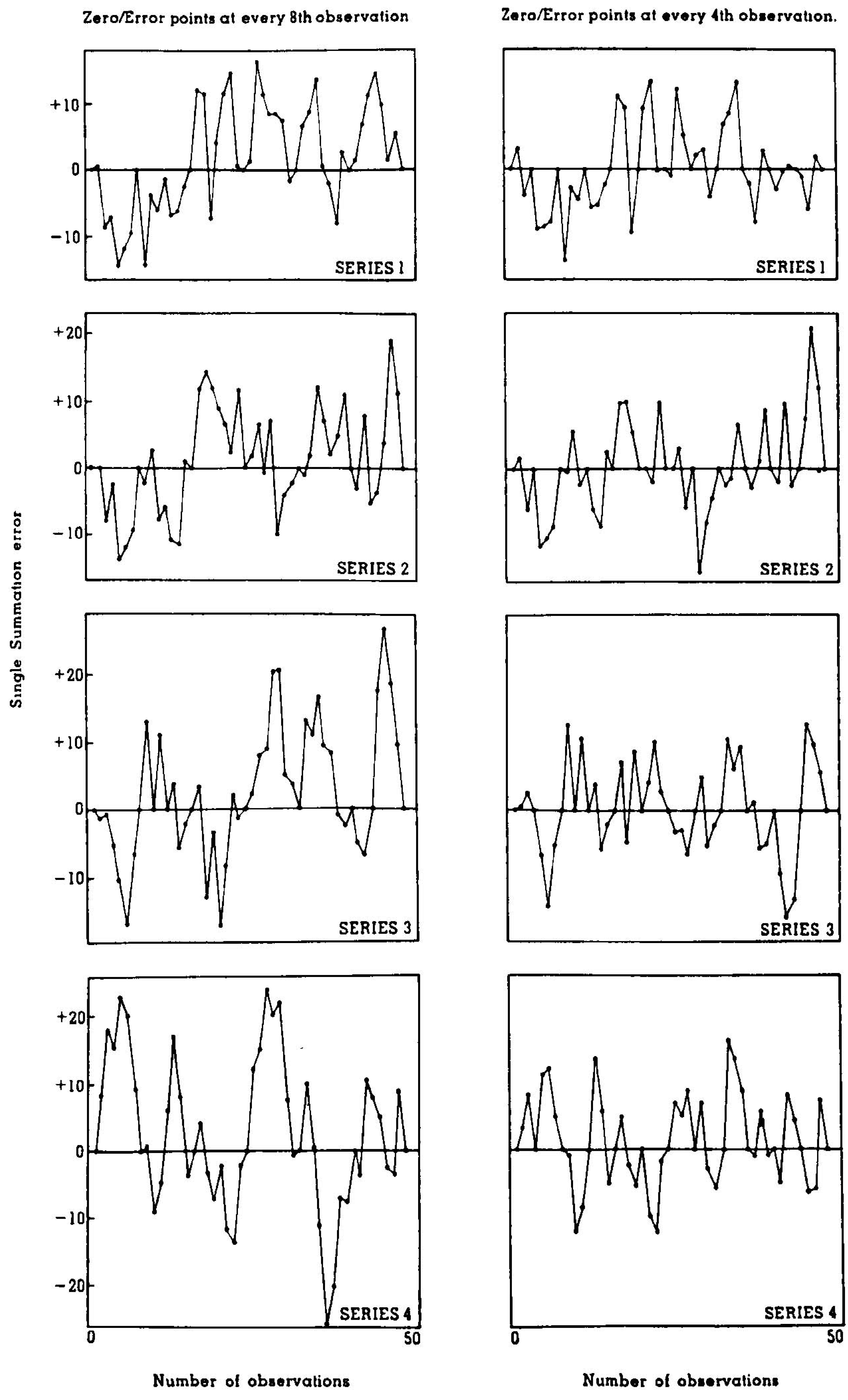


CONSECUTIVE SINGLE SUMMATION OF 1600 RANDOM NUMBERS
HAVING VALUES STATISTICALLY CONSISTENT WITH AN
INDIVIDUAL PROBABLE ERROR OF ± 5 UNITS

FIGURE 1B

FIGURE 2

CONTROL OF SINGLE SUMMATION ERROR BY LINEAR INTERPOLATION OF MISCLOSE BETWEEN "ZERO ERROR" POINTS.



4. Differential levelling provides an almost perfect example of the development of single summation errors and the " arc sin " law may well explain the " systematic errors " in levelling that have baffled surveyors for many years.[4]

5. It should be noted that if a levelling operation is carried out twice under similar conditions then varied ideas of the accuracy of the survey could arise according to the point at which the two sets of results were compared.

6. This type of error occurs in the accumulation of longitudual errors along a traverse and also in the summation of offset errors when independent azimuths are observed for each line of the traverse.

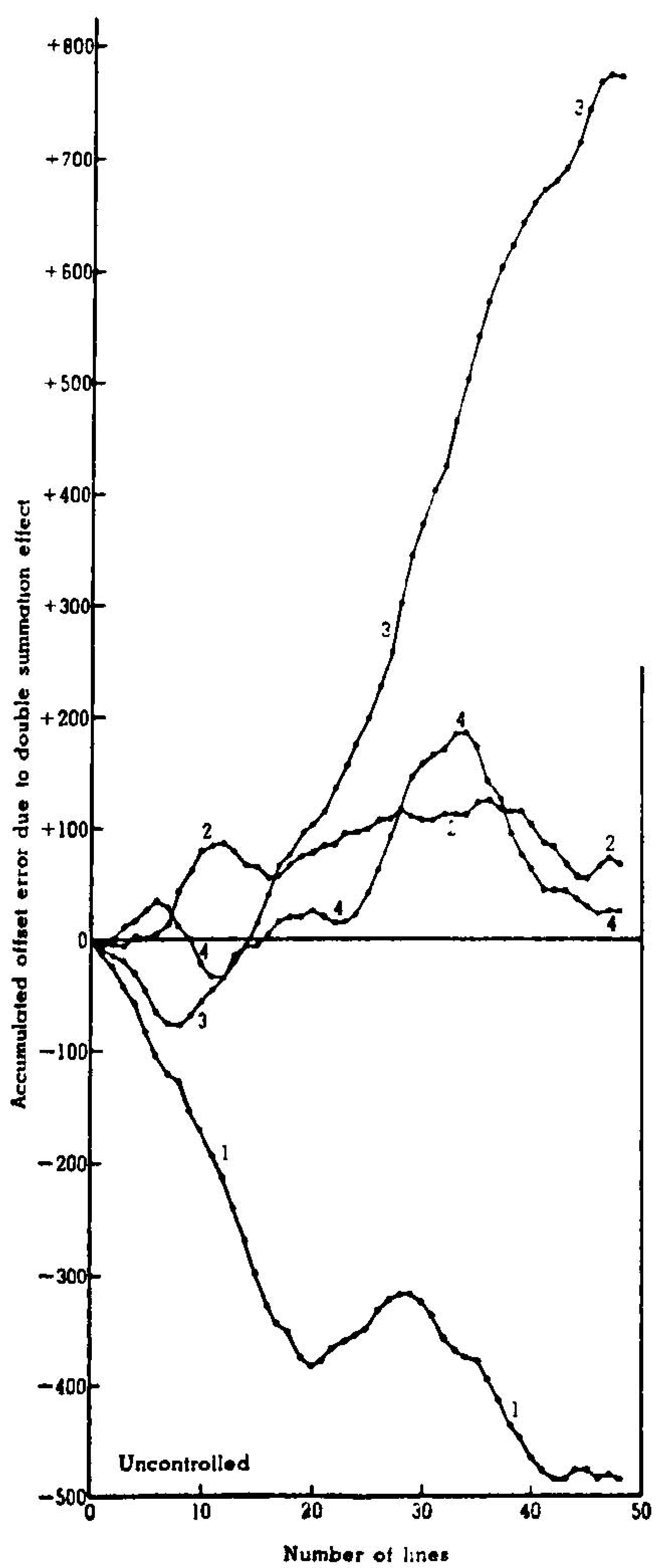
7. It occurs in the summation of angles along a traverse to provide the azimuths of lines.

8. The accumulation of error due to single summation can be limited by the introduction of appropriate control points of superior accuracy.

9. Figure 2 illustrates the effect of the introduction of control points (assumed free of error) into typical single summation curves at intervals of 8 and 4 observations and followed by linear interpolation for correction of error between these control points.

DOUBLE SUMMATION EFFECTS

FIGURE 3



EXPLANATION FOR FIG 3

1 Angular accuracy is assumed to be ± 5 units

2 All lines are of equal length

3 Therefore the unit of offset = (Actual angular pe/5) × (L/206000)

4. For example if actual pe = ± 1 0" and L = 20 Miles then unit of offset = 0 2 × 20 × 5280/206000 = 0 1 ft

5 Therefore at 960 miles

series 1 would have an error of − 48'
" 2 " " " " " + 7'
3 " " " " " + 78'
4 " + 3'

FIGURE 4A

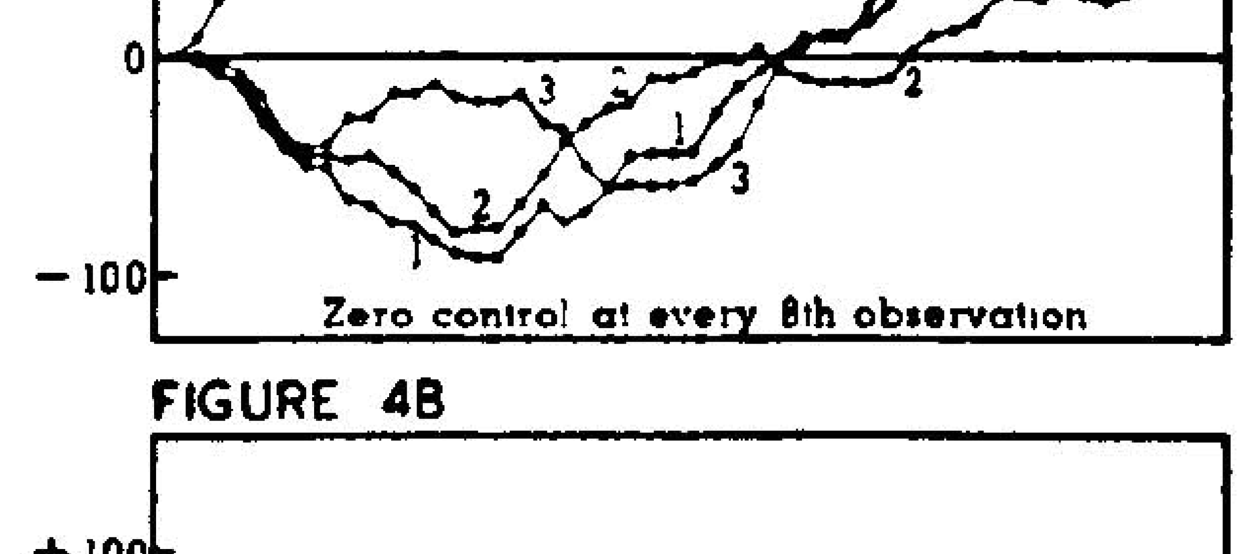
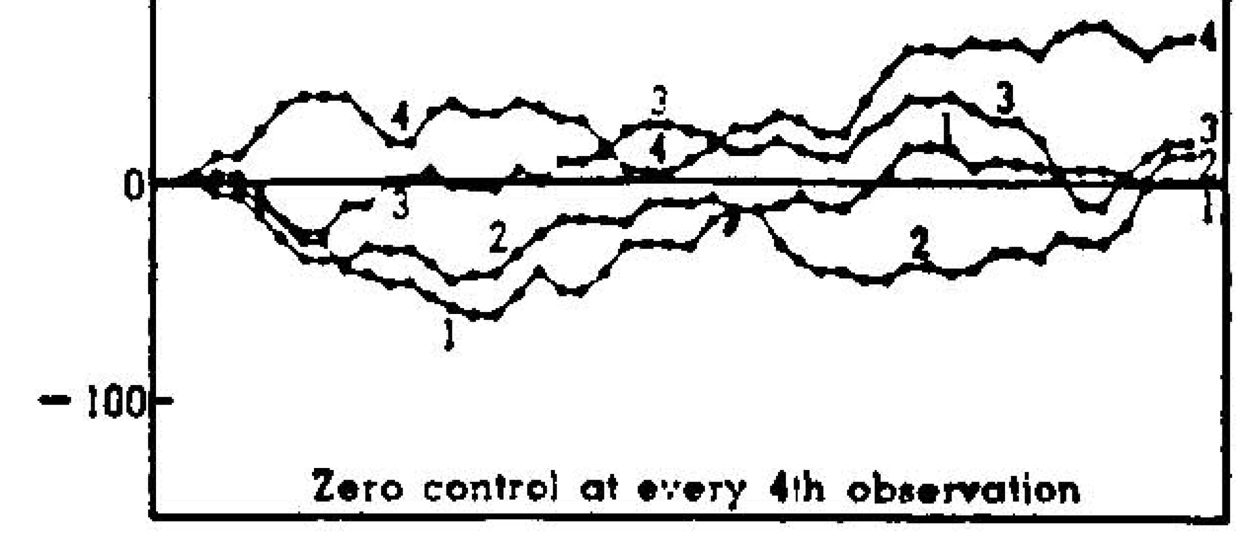


FIGURE 4B



Double Summation

10. In many surveying processes quantities that have been subjected to single summation effects are used to give a second summation, e.g., in a traverse, if a start is made with an accurately known azimuth and then angle observations are added consecutively to form a single summation for azimuth, the calculated offsets are subject to double summation.

11. Assuming observations of " n " angles the first azimuth has an effect over " n " traverse lines the second azimuth over " n — 1 " lines and so on. Moreover, the early summation pattern of error can develop so as to control the whole pattern. This type of effect is shown in two of the curves of Figure 3 where quite large offset errors are developed.

12. If " Zero error " points are introduced at regular intervals then each section of the traverse between these controls becomes an independent statistical unit. While there remains a limited form of double summation internally within each section, the summed errors of consecutive sections are only subject to single summation effects. Furthermore, linear interpolation of the azimuth misclose reduces the double summation effect within the sections themselves.

13. The final effects of such control on the development of offset error is shown quite spectacularly in Figures 4A and 4B.

14. Double summation effects develop in triangulation surveys both in respect of scale and azimuth while in photogrammetric stereotriangulation they develop in respect of scale, azimuth and both lateral and longitudinal levelling.[5]

Appendix 2

ACCURACIES BEING ACHIEVED IN THE COURSE OF AUSTRALIAN GEODETIC SURVEY OPERATIONS

Accuracy of Length Measurement

1. This analysis is based on the following lines measured by the Division of National Mapping during 1962:

Total number of lines measured	157
Total number of miles measured	2,770 Miles
Average length of lines	18 Miles
Longest line	40 Miles
Shortest line	0·8 Mile

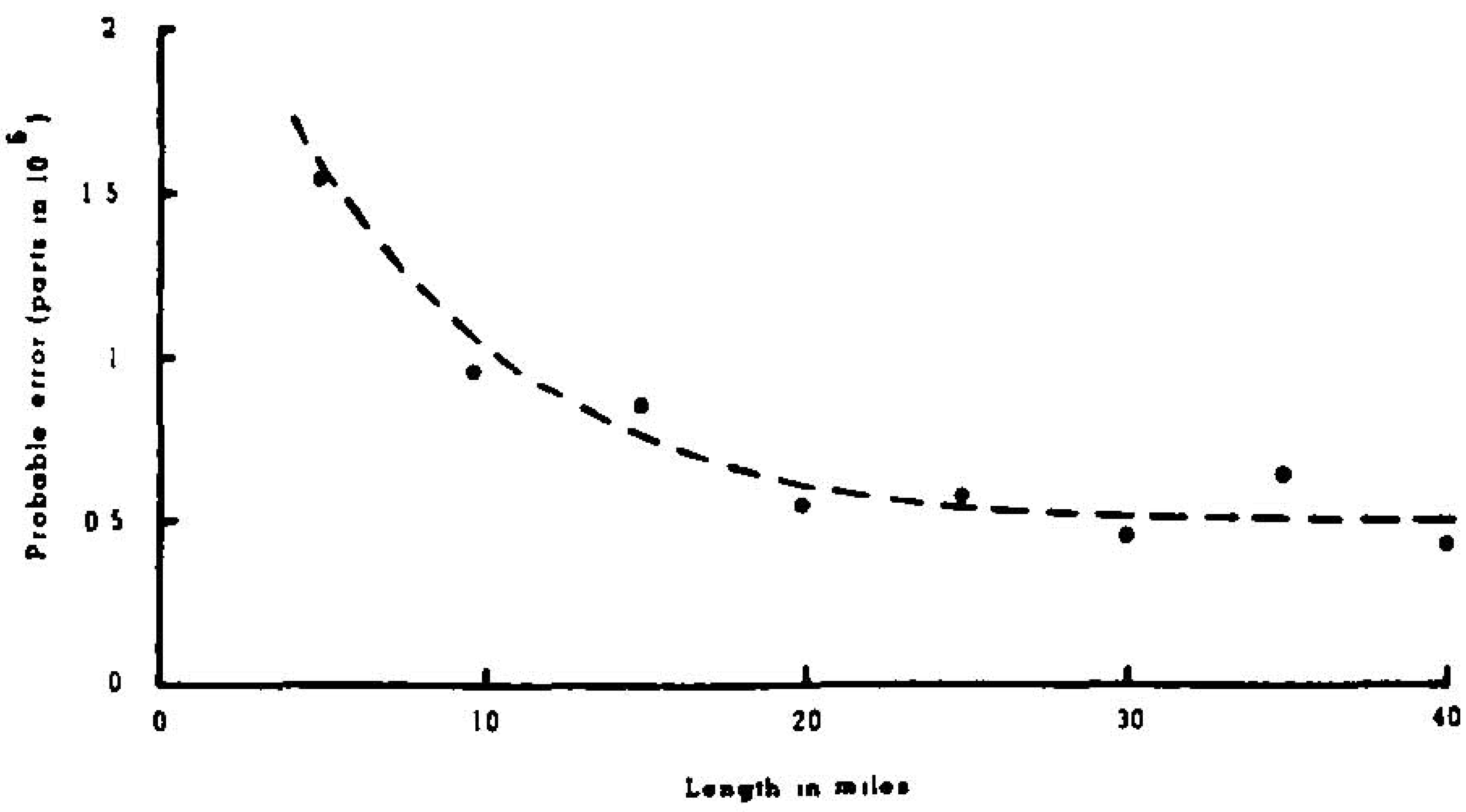
SUMMARY OF LINES AND INTERNAL ACCURACIES OF MEASURED LINES

	Errors (Part in 10^6)	
2. Grouping	Probable	Standard
5 mile lines (14)	± 1·6	± 2·4
10 mile lines (39)	± 1·0	± 1·5
15 mile lines (35)	± 0·8	± 1·3
20 mile lines (24)	± 0·6	± 0·9
25 mile lines (20)	± 0·6	± 0·9
30 mile lines (8)	± 0·5	± 0·7
35 mile lines (9)	± 0·6	± 1·0
40 mile lines (6)	± 0·5	± 0·7

These internal accuracies are shown graphically in Figure 5.

FIGURE 5

INTERNAL ACCURACY
OF
TELLUROMETER DISTANCE
MEASUREMENT



RESULTS OF ANALYSIS

3. *Internal accuracies*—All lines were measured twice or more in the same direction along each line. The values for internal accuracies (probable and standard errors) were derived by taking the average ratio dL/L for the number of lines considered and multiplying by 0·42 and 0·63 respectively. (dL = difference in pair of measurements and L = length of line).

4. *External accuracies*—When estimating external (actual) accuracies it is desirable to multiply the internal accuracy ratios by a factor of about 3.

5. *Accepted accuracy*—On this basis the external p.e. of the mean of a pair of Tellurometer measurements of any line about 20 miles long considered as a unit in a group of such measurements is ± about 1·8 parts in 10^6. A figure of $2{\cdot}0/10^6$ has been adopted.

Angular and Azimuth Accuracy

6. An analysis was made of 147 sections of traverse with Laplace azimuths at either end, on the following basis. In each section the number of angles (n) to which adjustments were made was increased by two on the assumption that the angles at each end to Sigma Octantis were of equal value to the traverse angles. The misclosures of each section were divided by $\sqrt{n + 2}$ and the results tabulated. A good Statistical pattern evolved with the 50 per cent. probability at ± 0·6″.

7. Special azimuth tests were carried out by the Division for the purpose of assessing the accuracies that could be obtained with astronomical and first order geodetic theodolites. These consisted of a series of observations at Khancoban (N.S.W.) over the one line in 1961 and 1962 (6) which were analysed in 1963 to give the following results:—

Type of Theodolite	Internal Probable Error	External Probable Error
Astronomical	0·2″	± 0·4″ to ± 0·6″
First order geodetic	± 0·2″	± 0·5″ to ± 0·7″

8. Internal probable errors were deduced from the variation of observations in separate sets of observations. The external accuracy was deduced from the variation of the means of individual sets of observations with respect to the overall mean value.

RESULT OF ANALYSES

9. For convenience the p.e. of both angular and azimuth observations was accepted as ± 0·6″.

Closures of Traverse Loops

10. Statistics of closures of traverse loops and combinations of loops:—

No. of Lines	Total length in miles	Misclosures (parts in 10^6 of total length) Latitude	Longitude	Vector
8	171	0·6	0·9	1·0
9	209	3·2	3·4	4·6
15	279	0·3	0·0	0·3
11	279	2·2	0·3	2·2
13	305	1·8	2·8	3·3
14	393	3·0	1·3	3·2
24	414	1·2	1·9	2·3
22	567	0·0	1·1	1·1
26	669	0·5	0·7	0·9
33	722	0·1	0·4	0·4
32	737	2·8	0·2	2·8
35	738	2·2	0·9	2·4
**42	786	1·4	2·8	3·1
**46	863	4·4	1·8	4·8
39	871	2·6	0·4	2·6
46	921	2·3	0·2	2·3
59	1,028	0·6	0·4	0·7
72	1,201	3·6	0·3	3·6
42	1,233	0·4	0·8	0·9
*72	1,249	2·3	3·6	4·3
136	1,528	0·3	0·6	0·7
113	1,568	1·2	0·3	1·3
112	1,913	3·3	1·6	3·6
*96	2,074	0·9	1·5	1·7
117	2,448	0·1	1·0	1·0
*98	2,636	0·3	1·5	1·5
138	2,790	1·4	1·3	1·9
154	2,801	0·3	2·3	2·3
135	2,850	0·0	0·6	0·6
149	2,884	0·1	2·6	2·6
*133	2,909	1·5	0·3	1·5
*175	3,090	3·2	0·5	3·2
205	3,798	1·4	0·4	1·5
*187	4,025	2·0	1·1	2·3
*371	5,926	1·3	1·2	1·7
*363	6,660	1·8	0·1	1·8
*539	9,572	0·4	0·8	0·9

* Some triangulation distances used.

** Some sides to be re-measured to bring up to acceptable N.M.C. accuracy specification.

RESULT OF ANALYSES

11. The above figures are shown graphically in Figures 6 and 7, where an attempt has been made to locate lines of 50 per cent. probability, both theoretical and actual.

AUSTRALIAN NATIONAL GEODETIC SURVEY
ANALYSIS OF LOOP CLOSURES

FIGURE 6 COORDINATE MISCLOSURES

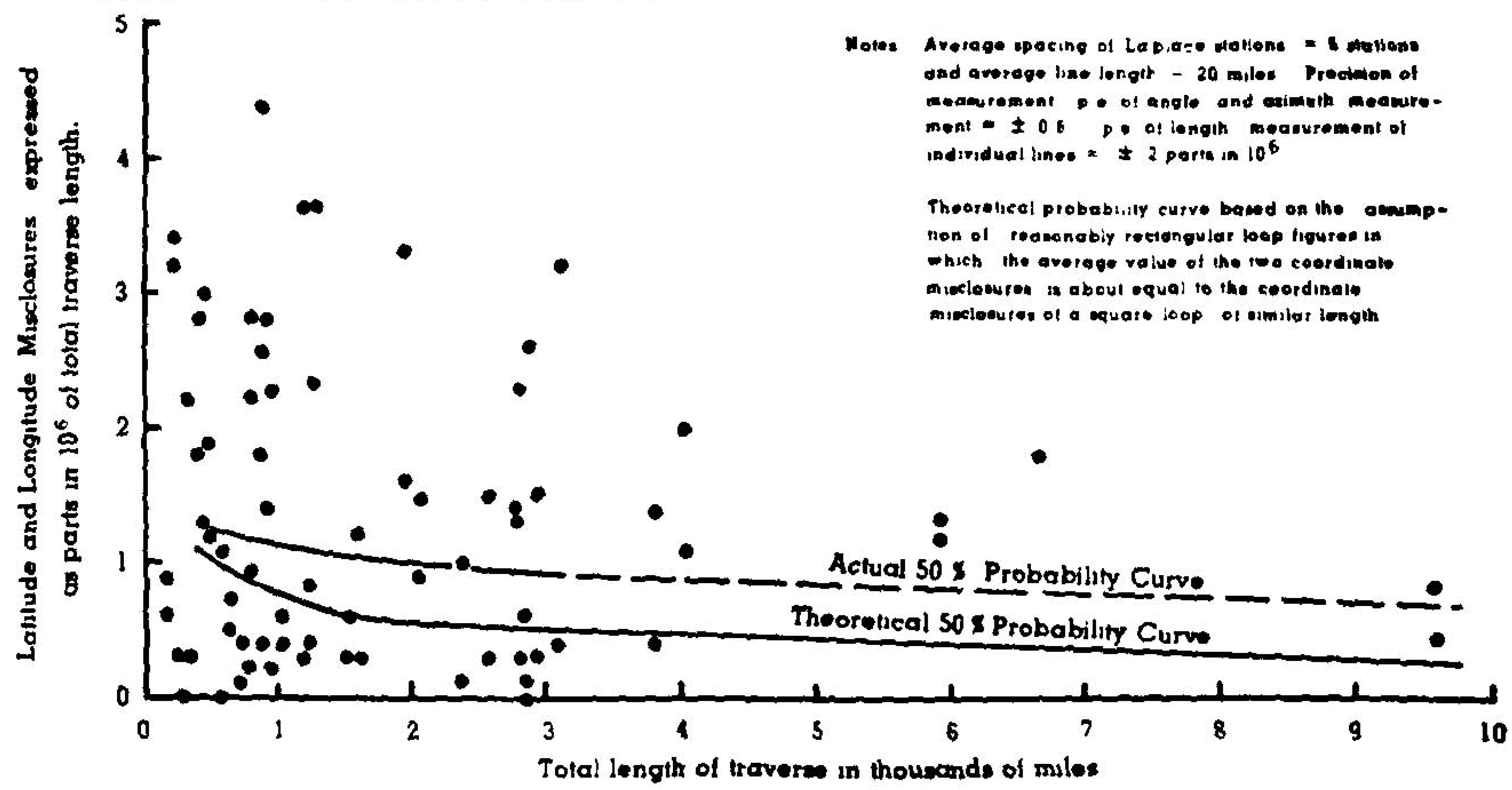
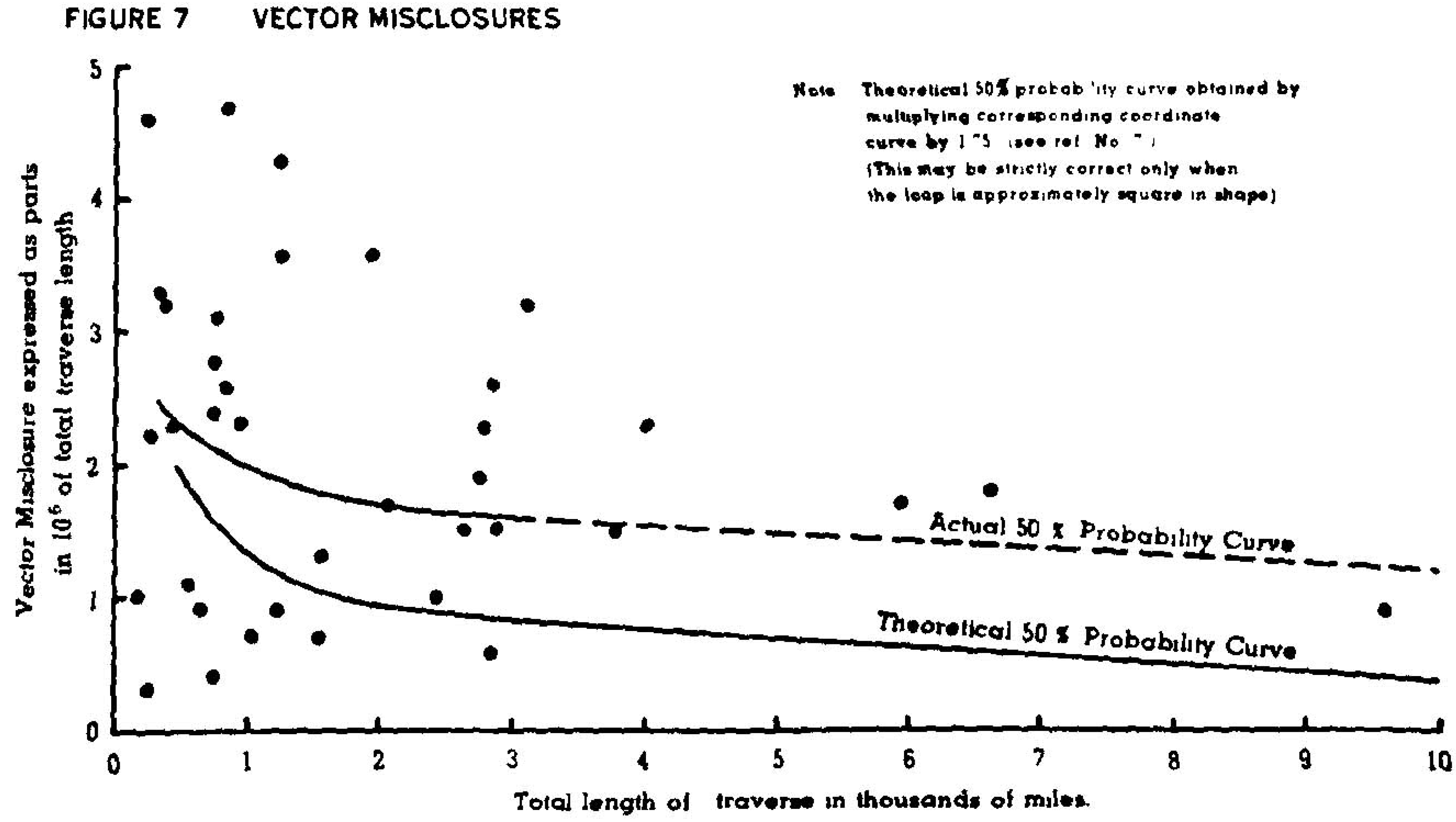


FIGURE 7 VECTOR MISCLOSURES



Appendix 3

DESIGN OF HORIZONTAL CONTROL SURVEYS FOR A SPECIFIED ACCURACY

Traversing

1. The National Mapping Council of Australia has specified, in respect of first order control, that the p.e. of linear measurement shall not exceed ± 5 parts in 10^6 for individual lines and that the azimuth of lines shall not have a p.e. greater than ± 1" of arc.

2. This latter is the equivalent of an offset error of ± 5 parts in 10^6.

3. Accepting 20 miles as the average length of line the p.e. of vector displacement at the end of each line would be $\pm$ 9 parts in 10^6 and as the traverse proceeds the ratio of error would decrease directly with the square roots of the number of lines measured; at 480 miles it would be $\pm$ 1·8 parts in 10^6 and at 1,920 miles $\pm$ 0·9 parts in 10^6. (7)

4. However, if individual azimuths are to be observed a Laplace observation must be made at each station.

5. In order to avoid the present expense and ultimate delay from such a procedure, an investigation was made into the errors likely to develop if use was made of the greater linear and angular measurement accuracies actually being achieved and if the Laplace observations were spread further apart.

6. These investigations were empirical, being based on a series of artificially created traverses in which the p.e. of angle and azimuth observation was $\pm$ 5 units and the lines were uniformly 20 miles long.

7. This arrangement had the convenience that the offset error due to an angular error of $\pm$ 1″ of arc was $\pm$ 0·5 feet i.e., 0·1 foot per unit.

8. A p.e. of $\pm$ 0·6″ was assumed for both azimuth and angular observations and appropriate conversions were made to the results of the investigation.

9. In the first instance the accumulation of azimuth error was investigated and it was found that where Laplace observations were made at every 8th station and at every 4th station respectively, on long traverses then the probable azimuth errors were $\pm$ 1·0″ and $\pm$ 0·8″ respectively.

10. Both these figures are within the specified limits, however, there was a strong tendency for the errors within individual sections between azimuth control to be either all + or all — as " n " decreased.

11. The development of offset was then investigated on the assumption that there would be a linear adjustment of the misclose of azimuth.

12. In this investigation it was assumed that independent azimuths were available at the beginning and end of each group of " n " lines. In practice the azimuth at the end of one group is normally the starting azimuth for the next group and offset errors would be slightly greater than those calculated. However, terminal Laplace azimuths are usually observed with astronomical theodolites and have an accuracy slightly better than $\pm$ 0·6″. This should tend to compensate for any error due to the assumption of independent terminal azimuths.

13. It was further assumed that traverse loops were reasonably rectangular in shape.

14. The following results were obtained for an assumed traverse of 480 miles (24 $\times$ 20 mile lengths) with Laplace observations at multiples of " n " stations.

" n "	p.e. in feet for total length	p.e.—parts/10^6 of total length
1	$\pm$ 1·5	$\pm$ 0·6
4	$\pm$ 2·9	$\pm$ 1·2
6	$\pm$ 3·5	$\pm$ 1·5
8	$\pm$ 4·8	$\pm$ 2·0

15. The preceding figures were then modified to incorporate the effects of a concurrent linear p.i. of $\pm$ 2 parts in 10^6 for each 20 mile line and gave the following results:—

" n "	p.e. in feet for total length	p.e.—parts in 10^6 of total length
1	$\pm$ 2·1	$\pm$ 0·9
4	$\pm$ 3·1*	$\pm$ 1·3*
6	$\pm$ 3·6*	$\pm$ 1·5*
8	$\pm$ 4·9*	$\pm$ 2·0*

* p.e. of these values estimated to be $\pm$ 10 per cent.

16. It would therefore seem that if " n " = 6, then something very close to the desired accuracy should be maintained; i.e., an accumulated error ratio not greater than that likely to accumulate from individual observations of the specified accuracy (see paragraphs 1 and 3).

17. In Figures 6 and 7 the 50 per cent. theoretical probability curves are plotted, on this basis for the range of distances likely to be encountered in Australian surveys. *When examining these figures it should be clearly understood that they are based on the restrictive assumptions set out in the preceding paragraphs and take no account of errors arising from unknown geoid elevations.*

Triangulation Chains

18. Since the development of the Bergstrand Geodimeter, the Wadley Tellurometer and its various copies, no worthwhile geodetic surveyor would undertake a triangulation chain without measuring at least one continuous series of lengths through the chain and thereby eliminating double summation effects in scale.

19. It would seem reasonable to assume in respect of angular measurements that a triangulation chain consists of about three or four individual traverses in which case the overall carrying on of azimuth would be about twice as accurate as a simple traverse and the spacing between Laplace stations could be about twice as much as that planned for traversing.

Trilateration Chains

20. Trilateration is naturally weaker than triangulation unless a considerable number of redundant measurements is incorporated in the scheme.

21. However, there is some indication in Appendix 2 that the external accuracy of distance measurement with the Tellurometer may be higher than that of angular measurement.

22. It may therefore be practical to design a trilateration scheme of comparable accuracy to a triangulation chain, in which case, a similar number of Laplace observations could be required.

23. Nevertheless, the writer would prefer to see such a scheme strengthened by at least one continuous line of angular measurements.

Block Surveys

24. If the traditional procedure of laying down a primary grid is followed and carried out to a design and performance that will ensure precisions of the order considered in this paper then it is thought most likely that a complete " block survey " infill of adequate accuracy, whether traverse, triangulation or trilateration, could be adjusted as one whole without further requirement for azimuth control.

25. This has not yet been proved in Australian surveys but will probably be attempted.

NOTE: If the probable errors set out in paragraph 39 of the " Report of I.A.G. Special Study Group No. 19 (1960) " are adopted then the computed p.e. of a pair of measurements over a 20 mile line would be 3 parts in 10^6. This would increase the figures in the last column of paragraph 13 of this Appendix by not more than 0·1 part in 10^6.

FIRST ORDER TELLUROMETER TRAVERSE

MOODINI BLUFF (W.A.) TO WUDINNA (S.A.)

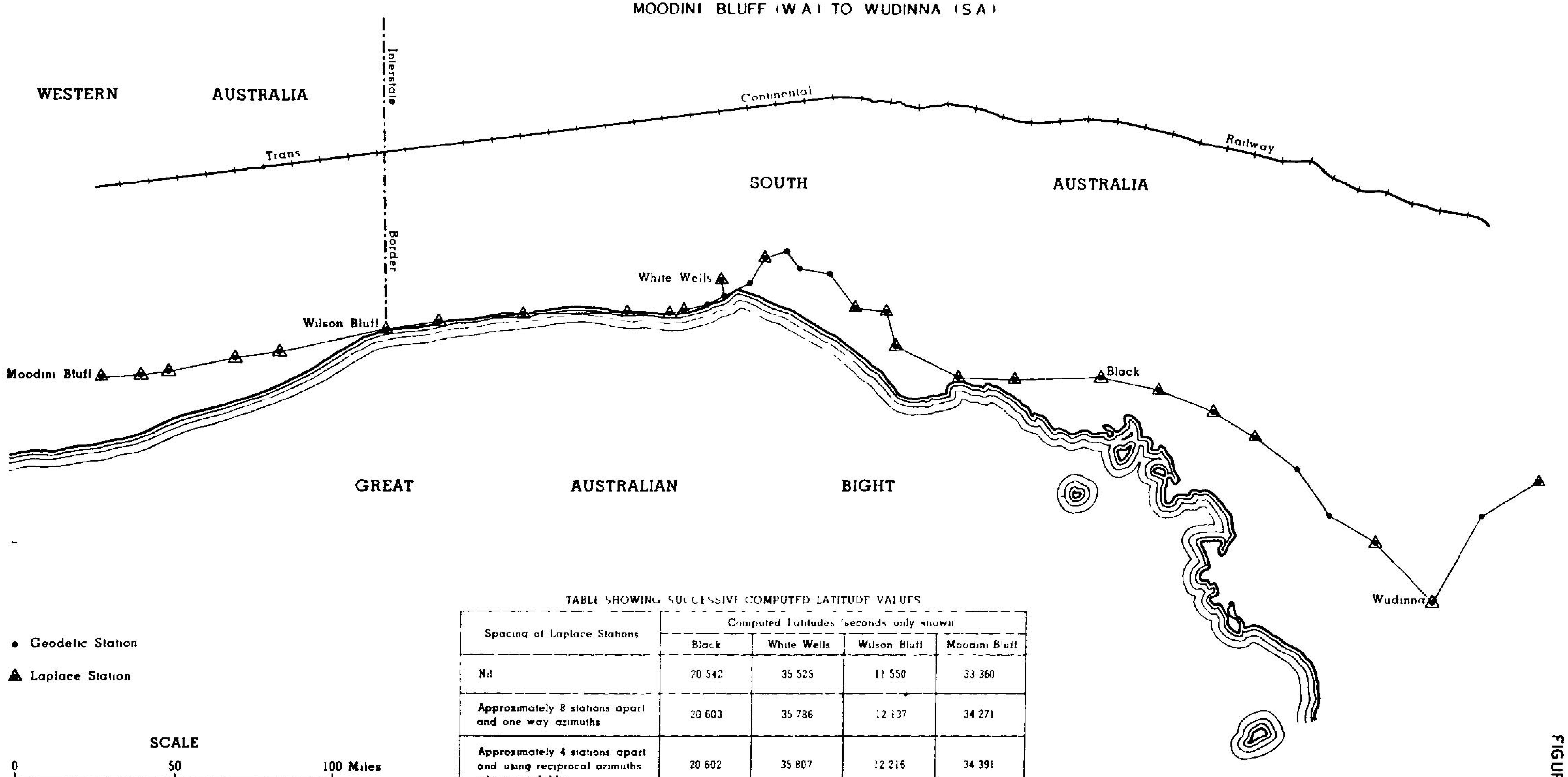


TABLE SHOWING SUCCESSIVE COMPUTED LATITUDE VALUES

Spacing of Laplace Stations	Computed Latitudes (seconds only shown)			
	Black	White Wells	Wilson Bluff	Moodini Bluff
Nil	20 542	35 525	11 550	33 360
Approximately 8 stations apart and one way azimuths	20 603	35 786	12 137	34 271
Approximately 4 stations apart and using reciprocal azimuths where available	20 602	35 807	12 216	34 391
All L.P. Stations used and all azimuths	20 604	35 804	12 226	34 361

FIGURE 8

References

[1]. (a) *Bulletin Geodesique* No. 52 (1936)—Rapport sur la répartition et l'utilisation practique des points de Laplace—Professor C. F. Baeschlin.

(b) *Geodesy*, Second Edition (pp. 88-90)—Brig. G. Bomford—Clarendon Press.

[2]. (a) *U.S. Coast & Geodetic Survey, Special Publication* No. 237—Manual of Geodetic Astronomy—Determination of Longitude, Latitude and Azimuth—Cdr. A. J. Hoskinson and J. A. Duerksen.

(b) Publication by Henry Wild Surveying Instruments Supply Co. Ltd.—*Instructions for the Determination of Geographic Positions*—H. Odermatt.

(c) *Division of National Mapping Technical Report* No. 1—Small Corrections to Astronomic Observations—A. G. Bomford.

(d) Division of National Mapping Technical Report yet to be published on the subject of Laplace observations (practical procedures and computations).

(e) *Survey Review* Vol. VXII No. 129, July, 1963—Precise Astronomical Longitude Determinations in Australia—G. G. Bennett.

(f) *Australian Surveyor* Vol. 19, No. 7, September, 1963—Some aspects of Precise Latitude Determination—G. G. Bennett.

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