CHAPTER 3

Properties of Pairs of Photographs

Although single photographs can be used for plotting maps, a more practical procedure involves the use of pairs of photographs. Horizontal photographs can be used in this way, and as the principles will help to introduce certain aspects of mapping from the air, we will consider ground photogrammetry first.

Ground Photogrammetry

The map is plotted using methods similar to those used in plotting a theodolite survey. A theodolite is an instrument used for measuring horizontal angles. Basically it consists of a telescope mounted so that it can revolve in a horizontal plane. A pointer attached to the telescope moves over a horizontal disc graduated in a similar way to a protractor.

The instrument used in ground photogrammetry is called a photo-theodolite, but an ordinary theodolite and separate camera may also be used.

The photo-theodolite (see Fig. 3.1) is essentially a camera mounted in the same way as a theodolite so that angles may be read at the time of exposure. Immediately in front of the photographic plate a horizontal and a vertical hair are tautly mounted. These hairs intersect at the centre of the plate, and are so placed that their image is formed on the plate at the time of exposure. The image of their point of intersection is the principal point. When the camera is in proper adjustment the optical axis will meet the plate in the principal point. The image of the vertical hair is the *principal line* and the plane containing the principal line and the optical axis is the *principal plane.*

Figure 3.2 represents a diagrammatic plan view of the photo-theodolite at the moment of exposure. The camera plate can be thought of as the negative plane, on which the images of field objects A and B will appear at a and b respectively. A contact print of the negative would fit into this diagram in the position marked " picture trace," with the images of A and B falling at a' and b' respectively. Since the distances between image points on the print must be the same as those on the negative, then $a'b' = ab$. The camera plate and picture trace are both perpendicular to the optical axis and must

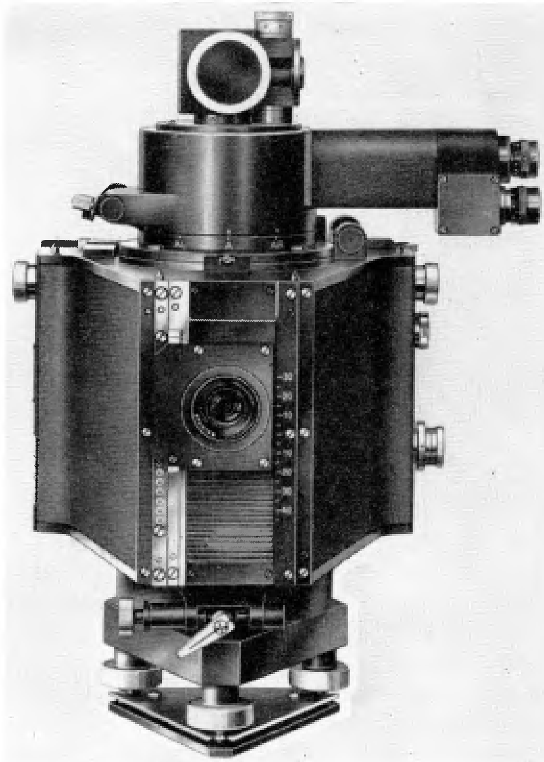


FIG. 3.1. PHOTO-THEODOLITE
(*Zeiss, Jena*)

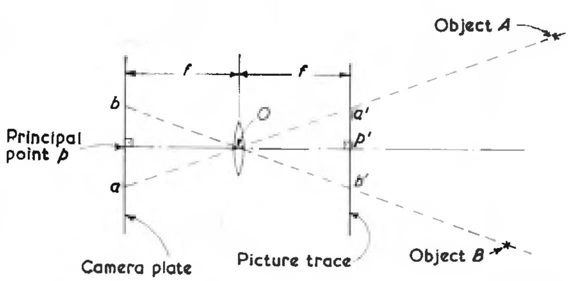


FIG. 3.2. DIAGRAMMATIC PLAN OF RAYS FOR HORIZONTAL PHOTCGRAPH

therefore be parallel. Therefore, by similar triangles, $p'O = pO = f$. That is, the perpendicular distance of O from the picture trace is equal to the focal length of the lens.

In Fig. 3.3, print 1 is the photograph taken from camera station O_1 (see Fig. 3.4); p_1 is the principal point, and a_1 the image of the object point A. Print 2 is the photograph taken of the same area

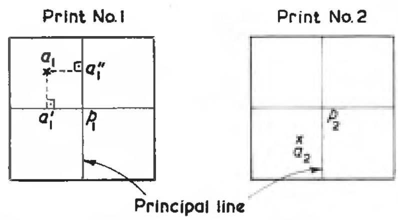


FIG. 3.3. SKETCHES SHOWING IMAGE OF A POINT ON TWO HORIZONTAL PHOTOGRAPHS

of land from the station O_2; p_2 is the principal point and a_2 the image of the object point A. Knowing the positions of O_1 and O_2 on the map, and having measured the appropriate angles at the moments of exposure, we can plot the position of A on the map. The procedure is as follows—

(i) Join O_1O_2 on map. Set out the angle $p_1O_1O_2$ equal to the angle measured for the first camera station, and the angle $p_2O_2O_1$ equal to the angle measured at the second camera station.

(ii) Set out O_1p_1 = focal length, and set out picture trace perpendicular to O_1p_1 and passing through p_1.

(iii) On Fig. 3.3 drop a perpendicular from a_1 to the horizontal axis, cutting that axis at a_1'.

(iv) On Fig. 3.4 set out $p_1a_1' = p_1a_1'$ on Fig. 3.3.

(v) Join O_1a_1' on Fig. 3.4 and produce.

(vi) Carry out the same procedure from O_2, using print 2 from Fig. 3.3.

(vii) O_1a_1' and O_2a_2' meet at A, which is then the intersected position of the point A on the map.

If we know the height of O_1, we can find the height of A. Figure 3.5 represents a vertical section along the O_1A of Fig. 3.4. As before a_1' is the point at which the picture trace intersects the vertical section, and $a_1'a_1$ represents the vertical distance of a_1 above the horizontal axis in Fig. 3.3. Draw AA' perpendicular to O_1A to cut

O_1a_1 produced in A'. A' is then the position in space of the object point A, while A in Fig. 3.5 is the map position. The length AA' then represents the height of station A above station O_1. Since the image point a_2 in Fig. 3.3 lies below the horizontal axis of print 2, A must be lower than O_2.

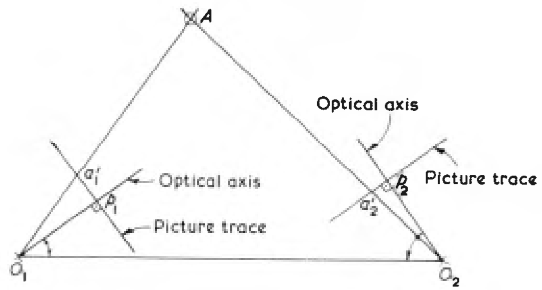


FIG. 3.4. GRAPHICAL PLOTTING FROM HORIZONTAL PHOTOGRAPHS

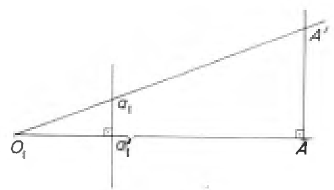


FIG. 3.5. HEIGHTING FROM GROUND PHOTOGRAPHS

Although it is more obvious when described as above, the length O_1p_1 in Fig. 3.4 need not be equal to f. If in Fig. 3.4, O_1p_1 were plotted equal to say $\frac{1}{2}f$, and p_1a_1' were also plotted at half scale, then the tangent of $\angle a_1'O_1p_1$ remains the same, i.e. $\angle a_1'O_1p_1$ remains unaltered if O_1p_1 and p_1a_1' are plotted to the same scale as one another. Similarly for O_2p_2 and p_2a_2'. All other distances will be plotted automatically at the same scale as O_1O_2, i.e. at map scale.

In practice, if the point A is an important point, or is to be used as a control point for future work, its position would be trisected. That is, a third photograph would be taken from a known point O_3 and a further ray O_3A would be drawn. If this ray passes through the same point as the other two rays, then this is the correct position

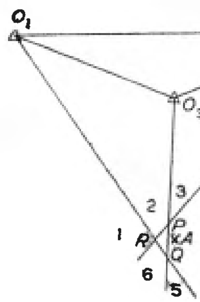


FIG. 3.6. ELIMINATING THE TRIANGLE OF ERROR
O_1, O_2 and O_3 are the three camera stations
PQR is the triangle of error
A is the correct position of the point

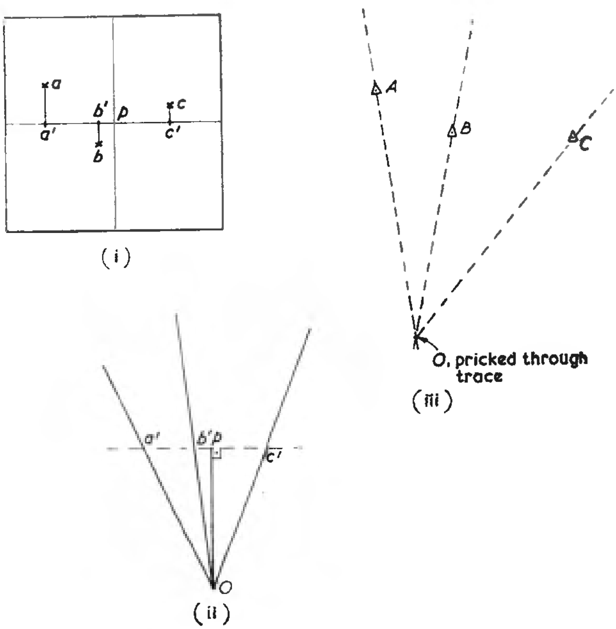


FIG. 3.7. RESECTING CAMERA STATION ON MAP
(i) PHOTOGRAPH
(ii) TRACE
(iii) MAP SHOWING TRACE RADIALS (BROKEN LINES) INTERSECTING THE CONTROL POINTS

of A, but if they form a triangle of error PQR, as in Fig. 3.6, then we must find the correct position of A by using the following rules—

1. If the triangle of error falls within the triangle $O_1O_2O_3$ then the correct position of A falls inside the triangle of error.
2. If, as in the figure, the triangle or error falls outside $O_1O_2O_3$, then A must be either to the left of all three rays or to the right of all three, i.e. in area 1 or 4 of the figure.
3. The exact position of A will be such that the perpendicular distance of A from each ray is proportional to the lengths of the respective rays.

If the camera stations are unknown they can be resected from three points of detail as follows—

On the photograph, drop perpendiculars from the images of each of the known points on to the horizontal axis. In Fig. 3.7 (i), a, b, and c are the three image points, and a', b', and c' are the feet of each perpendicular respectively.

Secure the print to the table. Superimpose a plastic transparency. On this trace, set off $Op = f$ and perpendicular to the horizontal axis. Join Oa', Ob', and Oc' and produce (see Fig. 3.7 (ii)).

Lift the trace and place it over the map (see Fig. 3.7 (iii)), so that Oa' intersects A, Ob' intersects B, and Oc' intersects C. This will be possible for only one position of the trace.

Prick through O on to the map: this is the map position of the camera station. If we prick through p at the same time, we can immediately establish the position of the optical axis.

Plotting may now proceed as before.

Other constructions can be evolved for special circumstances.

Ground photogrammetry is often used in mountainous areas such as Switzerland, and parts of Canada and Australia, but some of the photographs are taken with the optical axis pointing slightly downwards which involves a slightly more complicated procedure.

In practice, plotting machines may be used for making such maps.

Air Photographs

Vertical air photographs are also taken so that they may be used in pairs, and at least half the ground covered by one photograph must also appear on its fellow. To facilitate description we shall refer to the left-hand print as photo 1 and to the right-hand print as photo 2. Image points on photo 1 will be given the suffix 1, as a_1, and image points on photo 2 will bear the suffix 2. The principal points of photos 1 and 2 will be p_1 and p_2 respectively. If p_2 is the homologue of P_2, then the homologue of P_2 on photo 1 will be at p_2'.

The line p_1p_2' on photo 1 (see Fig. 3.8) is known as the *base line*, and it represents the apparent track of the aircraft between the two exposure stations.

If, exactly at p_2, there is a minute point of detail whose image can be found on photo 1, then p_1p_2' may be drawn in directly, but base-lining will normally entail a series of approximations as follows.

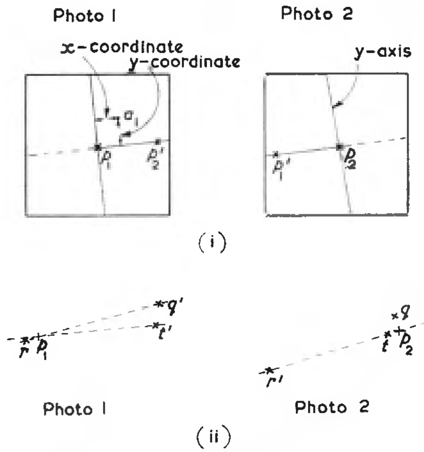


FIG. 3.8. BASE-LINING A PAIR OF PRINTS
(i) COORDINATE SYSTEMS ON A PAIR OF PRINTS
(ii) BASE-LINING BY SUCCESSIVE APPROXIMATIONS

In Fig. 3.8 (ii), q is the nearest easily identifiable point of detail to p_2. Using a magnifying glass mark the image q' of the same point of detail on photo 1. Now align a short steel straight-edge through p_1 and q' and pick a point of detail on this line lying as near to p_1 as possible. Call this point r and transfer it to r' on photo 2. Place the straight-edge to intersect both p_2 and r', and mark a suitable point of detail t on this line as near as possible to p_2. Transfer to t' on photo 1, and use this in place of q'. p_1t' is a second approximation to the correct position of the base line on photo 1. Continue finding successive approximations for each base line until the last approximation coincides with its predecessor.

When it is required to mark a point of detail, it should normally be pricked through with a very fine needle. To facilitate recognition it should then be ringed on the face of the print with a chinagraph pencil or preferably annotated on the back of the print. Lines should be drawn either with a very fine pencil or scribed with a fine needle, or drawn in red ink. All lines and pricks, however small, will obscure detail and they should therefore be used very sparingly.

The base line of a photograph is usually taken as the line of reference, and is called the x-axis. The line through the photo principal point and perpendicular to the base line is the y-axis (see Fig. 3.8 (i)). x-coordinates are positive to the right and negative to the left. In this book, except where stated otherwise, y-coordinates are positive downwards and negative upwards.

Note that the base line relates to the overlap between two photographs. Thus in an overlapping series of say three photographs in a line, the first photo will have only one base line, whereas the second will have two base lines, one relative to photo 1 and the other to photo 3 (see Fig. 5.2).

Both the base line and the principal line pass through the principal point, but they are two entirely different lines. The direction of the principal line is described by the angle it makes with the positive direction of the x-axis.

Binocular Vision

Viewing normally with both eyes is known as binocular vision. Stereoscopy is a method of reconstructing the binocular view from a pair of photographs. It is only a method of viewing the photographs using both eyes in order to be able to see the ground in three dimensions, but the ability to see a three-dimensional model of the ground from air photographs is a great advantage both in recognizing ground objects and in facilitating heighting and plotting operations. The stereoscopic " model " plays no direct part in the actual plotting of maps from air photographs either in the simple methods of Chapter 7 or in the major instruments, and stereoscopic theory is therefore not essential to a proper understanding of photogrammetry. However this " model " is perhaps the most dramatic phenomenon associated with the subject, and because students are always eager to understand its peculiarities, and because it seems to fascinate both examiners and teachers, it is considered advisable to enter into a brief but approximate discussion of stereoscopy.

When our eyes are functioning normally, each acts in a similar way to that of a camera at the time of exposure. Light rays pass through the lens which causes the bundle of rays from each light source to converge to a focus in the same way as in the case of the camera lens (see Fig. 1.5). The image is formed on the retina which is the inner skin of the wall of the eye. The brain is closely connected with the retina, and thus receives the impression of sight.

By changing the curvature of the lens, each eye automatically changes its focus to accommodate objects at different distances from the eye.

The apparent size of any object is dependent on the angle subtended by the object at the eye. In Fig. 3.9, tree *A* subtends an angle

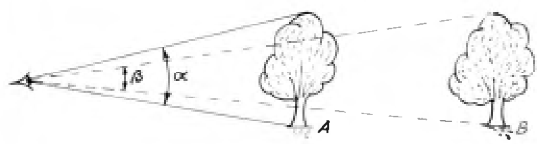


FIG. 3.9. APPARENT SIZE OF OBJECTS

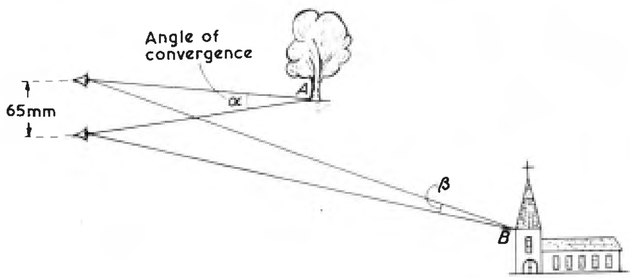


FIG. 3.10. BINOCULAR VISION—ANGLE OF CONVERGENCE

α and so appears bigger than tree *B*, which subtends the smaller angle β. A normal eye can discern objects which subtend angles as small as thirty seconds of arc.

The distance separating our eyes is known as the eye base, or inter-ocular distance, and averages about 65 mm in length. We therefore obtain a slightly different view of any object with our left eye from that obtained by our right eye.

The direction of the line of sight of each eye is automatically adjusted to obtain a particular field of view. When looking at one object we may say that the closer it approaches to our eyes the greater the angle through which each eye must turn towards the other; this action is known as convergence. The sum of the angles through which each eye is turned to view a particular object (e.g. *A* in Fig. 3.10) is equal to the angle between the lines of sight from each eye to that object.

This angle is, for convenience, sometimes referred to as the angle of convergence, or even as the parallactic angle. Because it decreases as the viewing distance increases, the angle of convergence enables our eyes to convey the idea of distance to the brain, which then builds up a three-dimensional impression of the view. In Fig. 3.10, *A* appears nearer than *B* because α is greater than β.

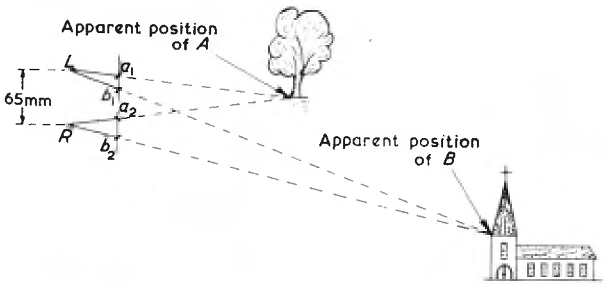


FIG. 3.11. RECONSTRUCTION OF THREE-DIMENSIONAL MODEL

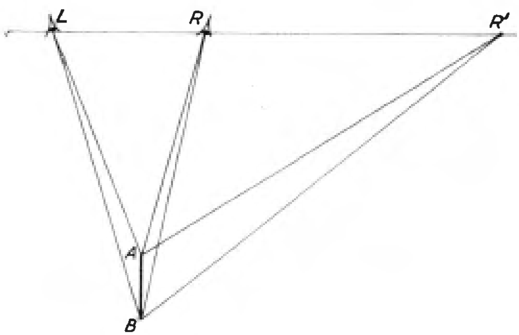


FIG. 3.12. EXTENSION OF THE EYE BASE
AB is a vertical ground object
LR is the normal eye base
LR' is the extended eye base

The minimum change in the angle of convergence which our eyes can appreciate is here taken to be about thirty seconds, and there is therefore a limit to the distance at which our eyes can obtain a three-dimensional impression. For normal views this limit will be reached well within three hundred metres distance, and a person in an aircraft several thousand metres above ground level will have no appreciation of normal ground relief. However, if we could somehow increase our inter-ocular distance we should at the same time increase the angle of convergence, and also the variations in convergence. In Fig. 3.12, *LR* represents the normal eye-base, and *LR′* the eye-base of a giant. In this figure (∠ *LAR′* − ∠ *LBR′*) is greater than (∠ *LAR* − ∠ *LBR*), and the height *AB* appears greater when viewed by the giant.

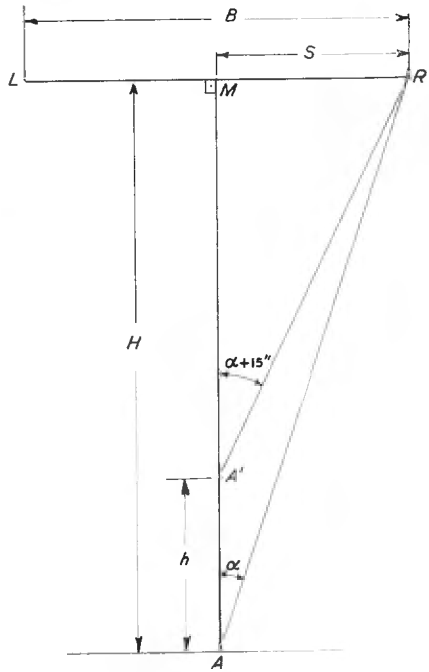


FIG. 3.13. LIMIT OF BINOCULAR DEPTH PERCEPTION
LR is eye base
M is mid-point of *LR*

We can illustrate this mathematically by reference to Fig. 3.13, in which *LR* represents the eye base, of length *B*. We can assume that we are looking vertically downwards at a vertical pole *AA′*, of height *h*, where the bottom of the pole *A* is at a distance *H* below our eyes.

Our eyes automatically tend to centralize the object of our attention, so that we may assume that *AA′* lies on the right bisector *AM* of the eye base.

Let $S = \frac{1}{2}B$ and $\angle MAR = \alpha$

Then if h is the shortest distinguishable depth at a distance H (i.e. if h is the height of the shortest pole that can be seen to have any height)—

$$\angle MA'R = \alpha + \frac{30''}{2} = \alpha + 15''$$

$$\text{Let } d = \tan 15'' = 0{\cdot}000075$$

$$\text{Now } \tan \alpha = \frac{S}{H}$$

$$\text{and} \quad \frac{S}{H - h} = \tan(\alpha + 15'') = \frac{\tan \alpha + \tan 15''}{1 - \tan \alpha \tan 15''} = \frac{S + H.d}{H - S.d}$$

$$\therefore \quad H - h = \frac{S(H - S.d)}{S + H.d}$$

$$\text{i.e.} \quad h = \frac{S.H + H^2.d - S.H + S^2.d}{S + H.d}$$

$$\therefore \quad h = \frac{H^2 + S^2}{\dfrac{S}{0{\cdot}000075} + H} = \frac{H^2 + \frac{1}{4}B^2}{H + B \times \dfrac{10^5}{15}}$$

Viewing A from 30 m, $H = 30$ m, $B = 65$ mm, $\quad \therefore \quad h \simeq 2$ m

Thus normal binocular vision will enable appreciation of a change of distance of 2 m, at a distance of 30 m.

If H is increased to 3,000 m, then $h \simeq 2{,}600$ m, which means that at a height of 3,000 m, variations in height are virtually unnoticed. But if we were giants with say a 2,000 m eye base, then from 3,000 m we could distinguish changes of height as small as 0·75 m, and with a 4,000 m eye base it would be only 487 mm.

The Reconstructed View

Let us consider that we are standing in the field looking at a view as in Fig. 3.10. Take a photograph with a pinhole camera in the place of the left eye, and a second photograph with the camera in the place of the right eye. In both cases the optical axis of the camera can, for convenience, be considered to be perpendicular to the eye base. If we use our eye base LR to represent the line O_1O_2 in Fig. 3.4 we can use the idea of the picture trace to plot from the photographs. This idea can be carried a step further by actually setting up the photographs themselves so that they take up the three-dimensional positions of their respective picture traces.

If we could place our eyes at L and R respectively so that the left eye saw only the left-hand photograph and the right eye only the right-hand print, then we could actually reconstruct the lines of sight of the original field view, as in Fig. 3.11.

The Exaggerated Impression of Depth

We have seen that with a normal eye base it would be impossible to obtain a reasonable perception of ground height from an aeroplane. However, exposures are usually separated by a distance of a thousand feet or more. The position from which each exposure is

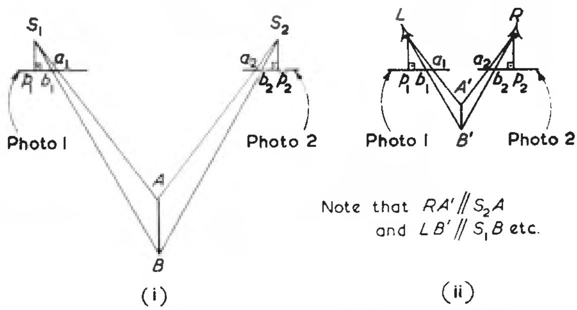


FIG. 3.14. APPARENT EXAGGERATION IN STEREOSCOPIC HEIGHTS
(i) THE FORMATION OF THE IMAGES ON THE PRINTS
(ii) VIEWING THE SAME PRINTS STEREOSCOPICALLY

made is called an air station (or camera station) and the distance between two successive stations is known as the air base: S_1S_2 in Fig. 3.14 (i) where S_1 is the perspective centre at the time of the first exposure, and S_2 the perspective centre of the second photograph.

If two successive photographs could be viewed simply (without lenses) as in Fig. 3.14 (ii), then a three-dimensional model would be seen, in which a giant's impression of depth would be maintained. In this model the horizontal scale is reduced in the proportion $\frac{\text{eye base}}{\text{air base}}$ so that the apparent increase in scale in depth of model is

$$\frac{\text{length of air base}}{\text{length of eye base}} \text{ or } \frac{B}{b}$$

This ratio is sometimes referred to as the *specific plastic*; the term is presumably derived from the habit of referring to the stereoscopic image as the plastic model.

It must be remembered that we have been considering a purely theoretical case; in practice the photographs cannot be taken with a pin-hole camera, nor are the ideal viewing conditions of Fig. 3.14 likely to be satisfied. In addition, the use of lenses will alter the angular relationships. Nevertheless the ratio does suggest a reason why gradients appear to be so much steeper in a stereoscopic model than they do on the ground, and this is the only abnormality of the stereoscopic model of which we are usually aware.

The effect of increased viewing distance is illustrated in Fig. 3.15, in which the apparent height of AB depends on the difference

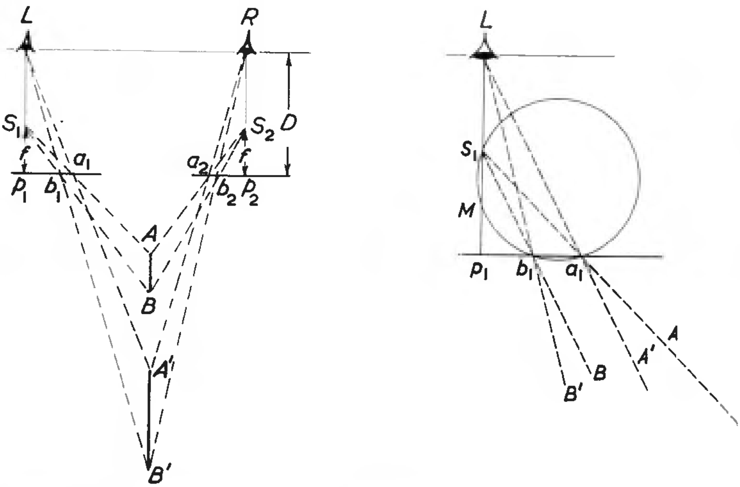
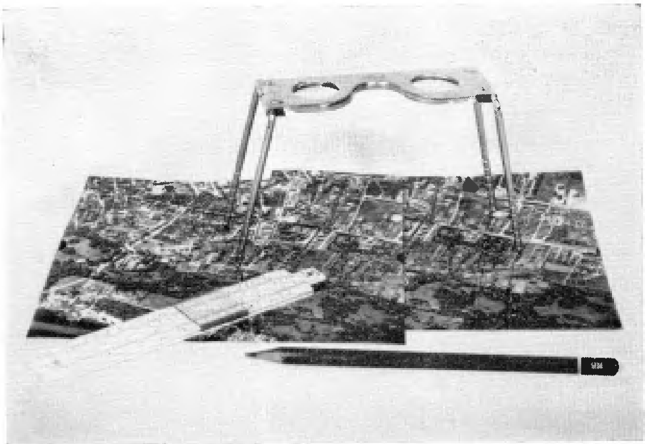


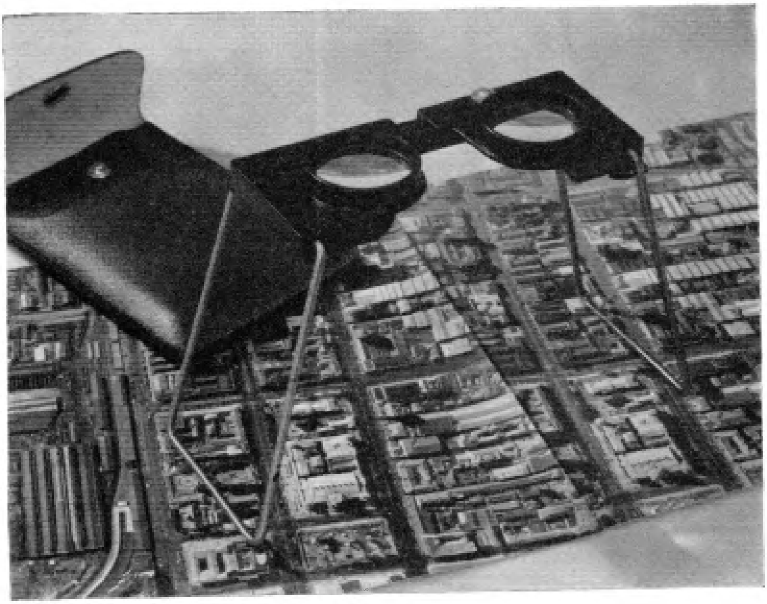
FIG. 3.15. STEREOSCOPIC MODEL—VIEWING DISTANCE GREATER THAN f
D = viewing distance
f = focal length of camera lens

between $\angle\ LA'R$ and $\angle\ LB'R$, i.e. on ($\angle\ A'LB'$ + $\angle\ A'RB'$). In the circle which passes through S and contains the chord b_1a_1, $\angle\ b_1S_1a_1 = \angle\ b_1Ma_1$; thus, ignoring the effect of the right eye, when L lies between S_1 and M then $A'B'$ will appear taller than AB would have done, whereas as L is removed beyond S_1 the apparent height of the model will begin to diminish.

There will be a corresponding, but different, circle for the right-hand photograph. It can therefore be seen that for long viewing distances ($\angle\ b_1La_1$ + $\angle\ b_2Ra_2$) is less than ($\angle\ b_1S_1a_1$ + $\angle\ b_2S_2a_2$), but that in other cases the position is not clear-cut. Thus we cannot say that there is any simple relationship between distance and



(*Zeiss Aerotopo*)



(*C. F. Casella & Co. Ltd.*)

FIG. 3.16. THE POCKET STEREOSCOPE

apparent heights, though an increase in viewing distance is often accompanied by a decrease in apparent gradients.

Magnification has various complicated effects upon the angular relationships, but as it increases the horizontal scale it might be expected to cause some decrease in apparent gradients (in accordance with general experience in stereoscopic viewing).

The combined effect of extending the eye base and magnification is sometimes called the *total plastic*, but the term has little value.

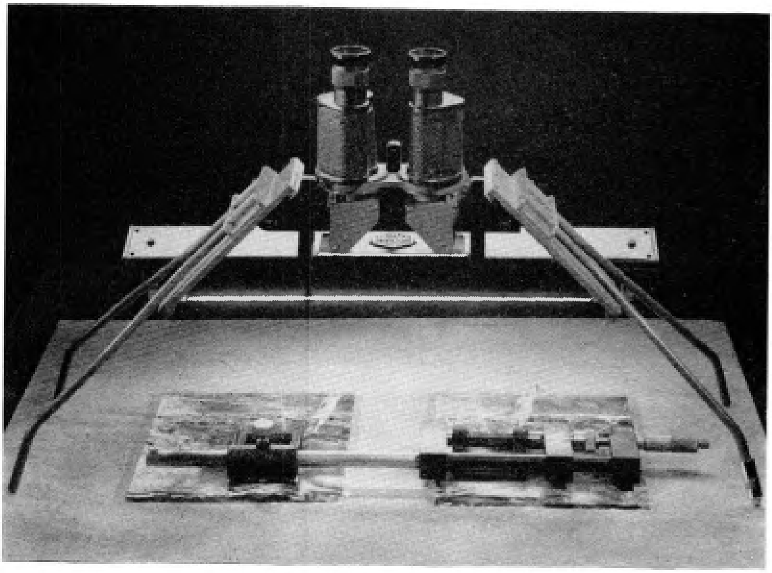


FIG. 3.17. MIRROR STEREOSCOPE WITH MAGNIFIERS AND PARALLAX BAR
(*C. F. Casella & Co. Ltd.*)

In Fig. 3.14 (ii), the eye base has been assumed to be greater than the width of one photograph. This can only be true for photographs less than 65 mm wide. To overcome this difficulty specially adapted viewing instruments known as stereoscopes are used.

STEREOSCOPES

The first stereoscope was invented by Wheatstone in 1838. Improvements were added by Pulfrich, and the stereoscope became very popular in Edwardian households for viewing family groups in three dimensions.

There are two main types of simple stereoscopes used at present in photogrammetry—

1. The Pocket Stereoscope. This is sometimes known as the lens stereoscope, and consists essentially of a pair of convex lenses (see Fig. 3.16).

The lenses give slight magnification and the refraction of the light rays enables a slight increase in the spacing of the photos for viewing. The main drawback of this type of instrument is that the width of stereoscopic model is small, so that the whole of the overlap of two 230 mm × 230 mm prints cannot be used unless the prints are cut or folded. It is very useful for stereoscopic examination of air photographs in the field.

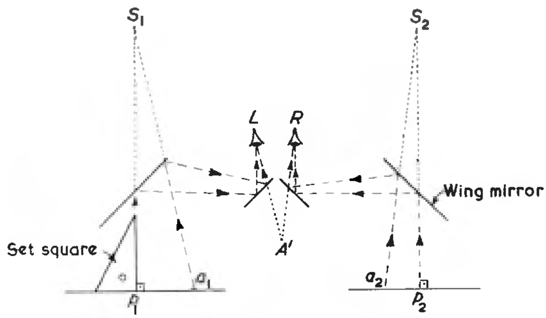


Fig. 3.18. Stereoscopic Fusion with Mirror Stereoscope Ignoring the Effect of Lenses and Magnification

2. The Mirror Stereoscope (see Fig. 3.17). This is the basic instrument in manual methods of heighting, and the idea is used in many of the automatic plotting instruments. Such an instrument enables the photographs to be spaced well apart, and yet to be viewed simultaneously, so that the largest of air photographs can be seen stereoscopically.

The Magnifier Unit

The magnifier unit (which in Fig. 3.17 looks like a pair of binoculars) is normally detachable, and some older mirror stereoscopes were suppiled without lenses. Fig. 3.18 shows a diagrammatic section through the eye base of such an instrument. It will be seen that the total length of the line of sight from the right eye R to the photographic image a_2 is equal to S_2a_2, so that S_2p_2 can be regarded as the viewing distance (similarly for S_1p_1). The effect of extending the distance between wing mirrors is much the same as increasing the viewing distance (Fig. 3.19), and mainly for this reason the apparent exaggeration of gradients is normally less under a mirror stereoscope than under a pocket instrument. It will be seen that this might also be explained by regarding the increased separation of the photographs as an increase in eye base, and therefore as a decrease in the B/b ratio.

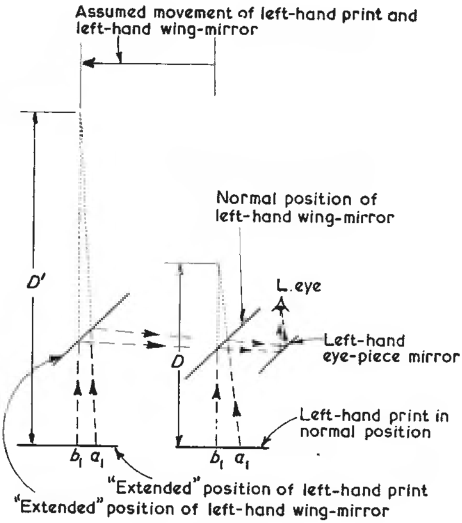


FIG. 3.19. MIRROR STEREOSCOPE—EFFECT OF INCREASING MIRROR SEPARATION
D=normal viewing distance
D'=increased viewing distance

The Effect of Lenses

Eye-piece lenses are incorporated in most modern instruments. Apart from producing some magnification, their main effect is to bend the light rays so that they enter the eyes along more or less parallel paths. Thus the conditions of viewing distant objects are simulated, and the eyes operate under the most comfortable conditions. Such eye-pieces obviously invalidate much of the earlier argument, especially that relating to viewing distance.

TRAINING THE EYES IN FUSION

Let us try to build up a stereoscopic model using a purely theoretical reconstruction.

Take two small sheets of paper and make two identical dots of about 3 mm diameter, one near the edge of each sheet. Place the dots opposite one another and about 60 mm apart as in Fig. 3.20. Place the head about 250 mm above the table on which the pieces of paper are resting. Take a suitably sized piece of cardboard or stiff opaque paper and place it in a vertical plane between your nose and the mid-point between the two dots. Make sure that your eye base is parallel with the line joining the dots. You should now be able to see only the left-hand dot with your left eye, and only the right-hand dot with your right eye. Now view with both eyes open: the dots should appear to fuse into one dot. Persevere until only

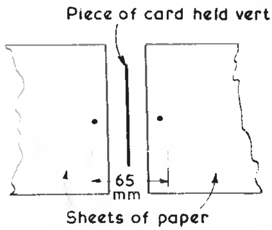


FIG. 3.20. STEREOSCOPIC FUSION OF TWO DOTS

one dot is seen: push the pair of dots slightly closer together until fusion is obtained.

Practise alternatively pushing the two dots closer together and drawing them further apart. You should find that the dots will remain fused at gradually wider and wider separations. This is known as increasing the " tolerance " of fusion. After much practice you may be able to obtain fusion without using the piece of cardboard to separate your binocular vision.

The foregoing is a useful exercise in preparing for the use of stereoscopic instruments.

Now add a second pair of dots to the two sheets of paper, as in Fig. 3.21 (i) and repeat the exercise (b_1b_2 must be parallel with a_1a_2). You should be able to fuse both pairs of dots simultaneously, but the new fused dot will appear to be lower down than the original dot, i.e. dots b_1 and b_2 will appear to fuse at a greater distance from the eyes than dots a_1 and a_2 (see Fig. 3.21 (ii)).

Thus we can appreciate the relative depth of the fused points, and if we could measure the stereoscopic depth, we should actually have measured the difference in height between the two points in the stereoscopic model. We shall see in Chapter 6 that this is the basis of an instrument used for measuring heights from air photographs.

Figure 3.22 shows a reconstruction of the stereoscopic model from two points on a pair of photographs. A comparison of Figs. 3.21 and 3.22 shows the similarity of the two reconstructions (see Fig. 5.5).

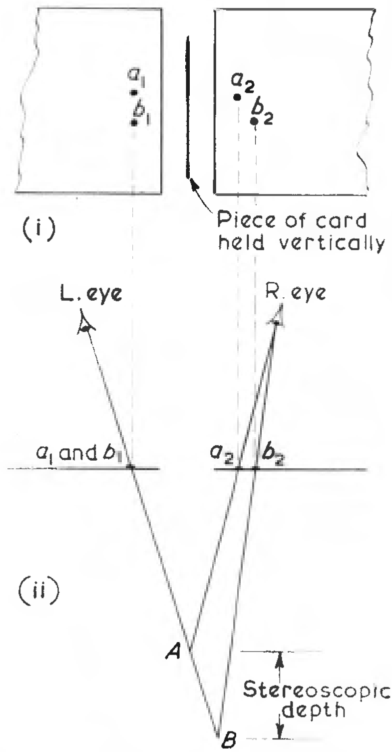


FIG. 3.21. SYNTHESIS OF STEREOSCOPIC DEPTH
(i) PLAN VIEW OF THE TWO PAIRS OF DOTS
(ii) SECTION SHOWING FUSION AND STEREOSCOPIC DEPTH

From Fig. 3.21 it seems that the apparent depth of a particular point depends on the distance apart of the two dots in a direction parallel with the eye base. In Fig. 3.22 the x-axis of both photographs has been aligned parallel with the eye base. In reconstructing the " solid " model we have seen how the eye base takes the place of the air base S_1S_2, so that for correct orientation the image of the air base should be parallel with the eye base. a_1 and a_2 in Fig. 3.22

are the two images of *A* and are sometimes known as a pair of conjugate points.

The fact that variation in distance causes an apparent relative movements of points only in a direction parallel with the eye base can be very simply illustrated. Hold a pencil vertically and at arm's

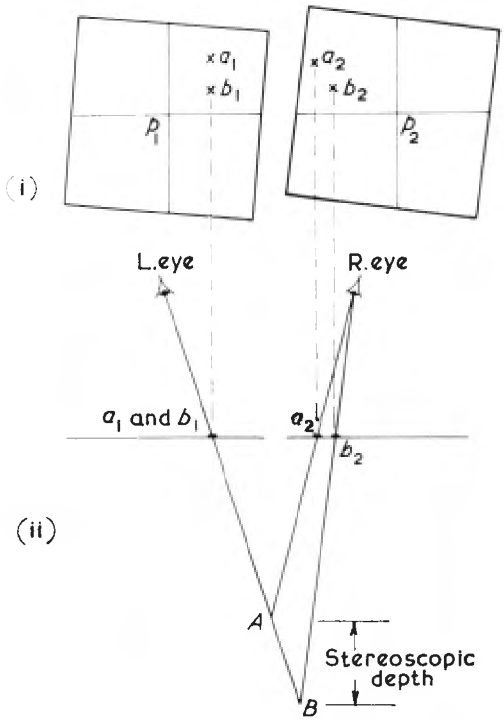


FIG. 3.22. ANALYSIS OF STEREOSCOPIC DEPTH
(i) PLAN VIEW OF TWO PAIRS OF IMAGE POINTS
(ii) SECTION SHOWING FUSION AND STEREOSCOPIC DEPTH

length in front of you, close your right eye and note the alignment of the pencil with respect to the corner of the room. Now close your left eye and open your right, and note that the pencil appears to move rapidly to the left, but that the apparent movement is wholly horizontal and can be shown to be solely a relative motion in which the nearer object appears to move more than the further object. Repeat the experiment with your head tilted sideways, and note that the apparent movement is now wholly parallel with your eye base.

Viewing under the Stereoscope

When it is desired to view a pair of air photographs stereoscopically, it will be found convenient to observe the following rules.

The photographs should first be base-lined relative to one another. The operator should then choose a position so that the light is falling towards him. Place the photographs on the table so that the base lines are collinear, and parallel with the eye base. The shadows on the photographs should fall towards the operator. The shadows of solid objects on the table now fall in approximately the same direction as the shadows on the photos. Make sure that the overlapping parts of the photographs are adjacent to one another. If the base lines are made collinear and the stereoscope is placed in position with the eye base parallel with the base lines, then one of the photographs should be moved laterally until comfortable fusion is achieved. Provided that the base lines remain truly collinear, this method of setting is adequate for all practical purposes, though, to obtain an angularly correct model, the ray of light from the principal point of each photograph should be perpendicular to the plane of the photograph. This could be checked by erecting a set square so that one edge rises perpendicularly from the principal point, then moving the photograph so that this edge of the set square appears as a single point (see Fig. 3.18).

If the lighting in the room does not fall in roughly the same direction as the photographic shadows, there may be a tendency to see the hills as valleys, and vice versa. This phenomenon is known as *pseudoscopic vision*, and may be obtained by the beginner even under correct conditions of lighting.

A pair of photos mounted together for stereoscopic viewing is known as a stereogram. The term may also apply to a pair of stereo drawings correctly mounted, or even to the model itself.

THE REAL MODEL

The image seen in a stereoscope is a figment of the brain, i.e. it is not real and measurements could not be made directly from the model which is therefore termed a virtual model. If light were projected through transparent positive prints (known as diapositives), in such a way that the light rays themselves set up a three-dimensional model of the ground, then the model would be real, and linear measurements could be made on the model itself.

If the model in Fig. 3.15 were real, as it is in the multiplex type of plotting instrument, then measurements of height could be made on the actual model, and an increase in the focal length of the projector would cause an increase in measurable model depth.

Parallax

Figure 3.24 shows a section along the air base, and illustrates the perspective rays set up by a pair of overlapping vertical photographs at the moment of exposure. In this figure not only are both

FIG. 3.23. EFFECT OF OUTWARD DISPLACEMENT OF A PAIR OF PHOTOGRAPHS ON THE RECONSTRUCTED "MODEL"

a_1 = image of A on photo 1 a_2 = image of A on photo 2
b_1 = image of B on photo 1 b_2 = image of B on photo 2
a_1', b_1', a_2', b_2' are the positions of the image points after the prints have been displaced outwards.

photographs free of tilt, but they are taken with the lens, i.e. the perspective centres, at exactly the same height. A and B are any two points of ground detail. In Fig. 3.24 (ii) p_1a_1 represents the x-coordinate of the image of A, whatever its y-coordinate may be.

The lens positions are S_1 and S_2. On photo 2, p_2 is the principal point, and a_2 and b_2 the two image points. Suppose that the positive plane is carried along with the camera, so that a_1 is carried forward to

a_1', and b_1 to b_1'. p_2 is still the principal point and, relative to the camera lens, is the same point as p_1. The ray from A now strikes the positive plane at a_2, and there has been an apparent movement of the

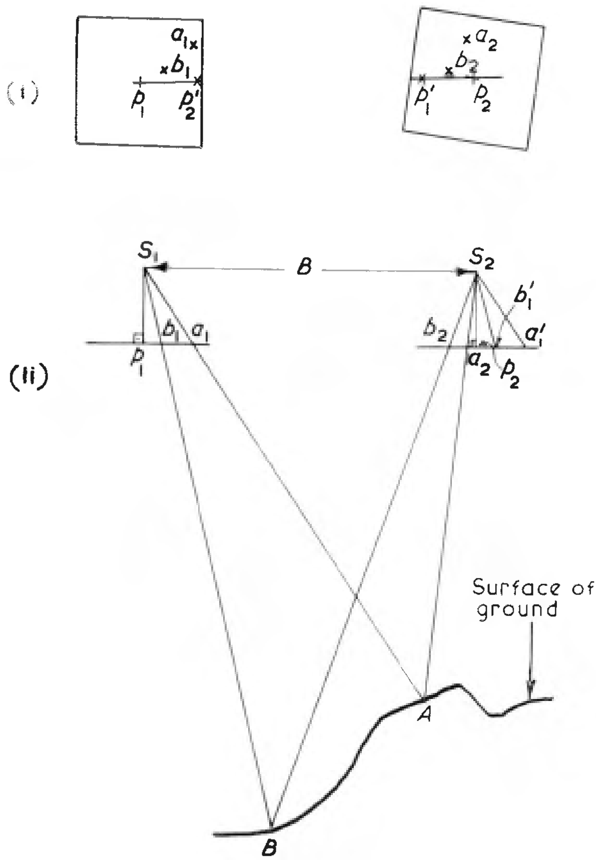


FIG. 3.24. PARALLAX OF A POINT
(i) PLAN
(ii) SECTIONAL ELEVATION

image of A relative to the camera, from a_1' to a_2. This length $a_1'a_2$ is known as the *parallax* of A (sometimes as the absolute parallax of A), and $b_1'b_2$ as the parallax of B. We see that

$$a_1'a_2 = p_1a_1 + p_2a_2$$

but p_1a_1 is the x-coordinate of A on photo 1 and $-p_2a_2$ is the x-coordinate of A on photo 2.

$$\therefore \quad \text{parallax of } A = a_1'a_2 = p_1a_1 - (-p_2a_2)$$

= the algebraic difference in the x-coordinates of A on the pair of photographs.

($a_1'a_2$ could be measured with a millimetre diagonal scale.)

We shall see in Chapter 6 that the difference in parallax of two points is a measure of their difference in height. The parallax of B, a lower point than A, is b_2b_1' which is demonstrably less than a_2a_1'.

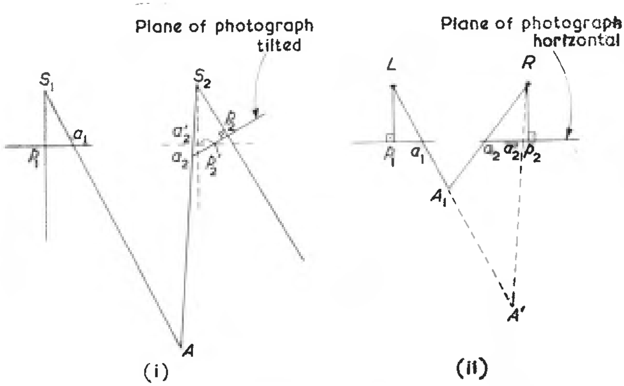


FIG. 3.25. ERROR IN RECONSTRUCTED MODEL DUE TO TILT
(i) LINE DIAGRAM AT EXPOSURE
(ii) PROJECTED MODEL

Errors in Parallax Due to Tilt

If one of the photographs is tilted, then the x-coordinates of points on that photograph are incorrect. For example see Fig. 3.25 (i), where the second photograph was tilted so that the image of A fell at a_2, whereas in the untilted photograph it fell at a_2'. Thus $p_2'a_2'$ is the correct x-coordinate, but p_2a_2 is the actual coordinate. Figure 3.25 (ii) shows how the reconstructed position of A will be at A_1, whereas the correct reconstructed position would have been at A'.

The more expensive plotting machines generally aim at recreating a correct spatial image or model by orienting each photograph at the actual angle of tilt. The manual methods and cheaper instruments generally produce an erroneous model or measure the photographs lying in a horizontal plane; corrections are then made mathematically or graphically.

Even when both photographs are truly vertical, errors in the x-coordinates may occur through a change in height of the aircraft between the two exposures as in Fig. 3.26. This situation is known as inclination of the air base. From the figure it can be seen that the angle $a_2S_2p_2$ is less than the correct angle $a_2'S_2'p_2'$. Therefore a_2p_2 is less than the correct x-coordinate $a_2'p_2'$.

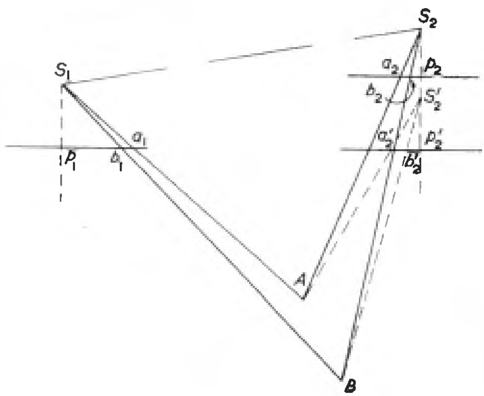


FIG. 3.26. EFFECT OF INCLINATION OF AIR BASE ON x-COORDINATES
S_2' is position which S_2 should have occupied if height of aircraft had remained constant. A and B are ground objects.

This is of course to be expected, since S_2 is too high, making the scale of the photograph too small. Thus if we regard the height of the first exposure as correct, the x-coordinates of points on the second photograph will be too small if S_2 is too high, or too great if S_2 is too low.

Want of Correspondence

So far we have considered only changes in the x-coordinate, because it is this coordinate which is used in heighting measurements; but since height displacement will have the same effect on the y-coordinate of one photo as it has on the y-coordinate of the next, a difference in y-coordinates is often regarded as a measure of the relative tilt between two photographs of an overlapping pair.

Figure 3.24 shows a section containing the air base of a pair of untilted photographs whose perspective centres are at the same height.

If we were to draw a sectional elevation of the same conditions but looking along the air base, the result would be as in Fig. 3.27. Thus the y-coordinate of A on photo 1 is p_1a_1 and on photo 2

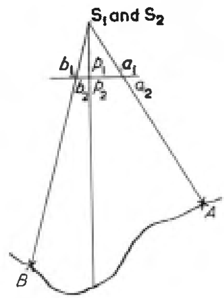


FIG. 3.27. SECTIONAL ELEVATION LOOKING ALONG AIR BASE
A and B are ground points. All rays to S_2 overlie corresponding rays to S_1.

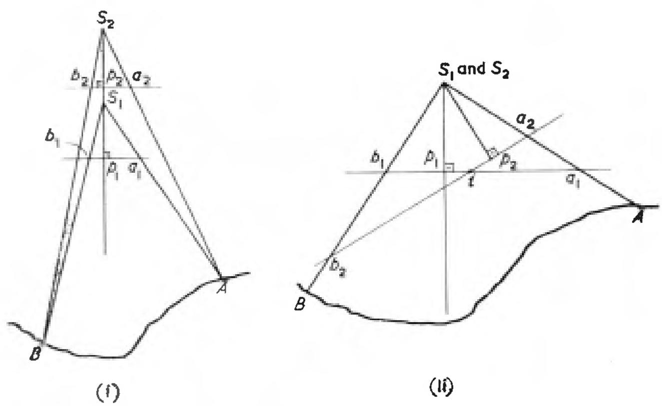


FIG. 3.28. WANT OF CORRESPONDENCE DUE TO—
(i) INCLINED AIR BASE
(ii) LATERAL TILT
Both diagrams are sectional elevations looking along the air base.

p_2a_2. These lengths are obviously the same so the y-coordinates of the pair of photographs are the same, and the photographs are said to be in correspondence.

Figure 3.28 (i) shows the sectional elevation looking along the air base of the conditions depicted in Fig. 3.26. The scale of photo

2 is again less than that of photo 1, and the y-coordinates no longer correspond.

In Fig. 3.28 (ii) the air base is no longer inclined, but photo 2 is tilted in the y-direction (i.e. the principal line is assumed to lie along the y-axis). p_2a_2 is much less than p_1a_1, and there is " want of correspondence " between the pair of photographs.

Tilt purely in the x-direction of photo 2 will cause a scale error in the y-direction. This is because the principal line lies along the x-axis, and the lines parallel with the y-axis are all plate parallels along which there is no tilt. Thus except for images lying along the isometric parallel of photo 2 there will again be want of correspondence.

Tilt of one photograph relative to its pair will always give rise to want of correspondence, though tilts in the y-direction have a much bigger effect than those in the x-direction. If both photographs are tilted by the same amount and in the same direction correspondence will remain. Want of correspondence can also be due to inclination of the air base. In practice there will usually be relative tilts in both the x- and y-directions and also inclination of the air base.

Want of correspondence is sometimes referred to as y-parallax, vertical parallax, or more simply as K.

Check on Base-lining

Set up the photographs for viewing under a stereoscope. Turn each photograph through a right angle, both in the same angular direction. Move photo 2 so that p_1' falls on the extension of the y-axis of photo 1. If want of correspondence is present, then the differences of y-parallax will now occur in a direction parallel with the eye base, and will cause apparent variations in depth of image points below the eyes. Thus an appearance of hills and valleys is generated, giving another type of pseudoscopic vision. This gives a method of checking the accuracy of base-lining. If both base lines have been accurately drawn then the fused image should appear to be in contact with the ground throughout its length.

Base-lining under a Stereoscope

We have described a purely graphical method of drawing in the base-lines on a pair of photographs, and the beginner has been advised to use these for setting up his photographs accurately to form a stereogram, so that he may more readily obtain a stereoscopic view. However, in practice the experienced photogrammetrist will be able to obtain fusion from an approximately oriented pair of photographs, and can then proceed to a much quicker method of base-lining.

The pair of prints are placed under the stereoscope with the shadows falling towards the observer, and with the common image portions adjacent to one another. Choose a particularly conspicuous group of images which can be easily recognized on both photographs, e.g. two intersecting hedges, one roughly at right angles to the other. Now look through the eye-pieces, cover up the right-hand photograph, and pick out the chosen detail on the left-hand photograph; do the same with the detail on the right-hand photograph. Move the photographs laterally (in both the x- and the y-directions) until the two images of the hedge intersection coincide; then rotate the prints until the two images of both hedges coincide. Study the stereogram until the stereoscopic model becomes apparent, adjusting the lateral position and rotating the photographs until the most comfortable fusion is obtained. The prints are now secured to the table with masking tape to form a stereogram.

To transfer the principal point p_1 to p_1' on photo 2, we require two fine needles, which we hold one in each hand. With all the magnification we can get, and looking only at photo 1, we make the left-hand needle-point touch exactly the left-hand principal point p_1. Now again view the stereoscopic model and (keeping the left-hand needle point in contact with p_1) move the right-hand needle-point gradually over the face of the right-hand photograph until the two needle-points coincide exactly. The right-hand needle-point should now indicate the correct position of p_1'.

Transfer the principal point of photo 2 to photo 1 in the same way. It should now be possible to lay a straight-edge so as just to touch each of the points p_1, p_2', p_1' and p_2; if this cannot be done, the stereogram must be adjusted and the procedure repeated. Join p_1p_2' and $p_1'p_2$. Check by turning the two photographs through a right angle and viewing the pseudoscopic model.

This technique is not so easy as it sounds, mainly because under most conditions of lighting the needle-points tend to disappear from view at frequent and unpredictable intervals. Perseverance is required, but it is still a lot quicker than the graphical method.

FURTHER READING

GROUND PHOTOGRAMMETRY

(i) Schwidefsky, pages 91–106.
(ii) Moffitt, Chapter 14.
(iii) Hallert, pages 73–102.
(iv) Zeller, pages 34–103.
(v) *Manual of Photogrammetry*, Vol. II, Chapter 19.

Stereoscopy

(i) Schwidefsky, pages 59–74.
(ii) *Manual of Photogrammetry*, Vol. I, Chapter 11.
(iii) Hart, pages 156–81.
(iv) Moffitt, Chapter 7.
(v) Hallert, pages 52–62.
(vi) Lyon, Chapter 2.

Plane Table Surveying, Resection and Intersections

H.M.S.O., *Manual of Map Reading, Air Photo Reading and Field Sketching*, Part III.

(See Bibliography (page 346) for the full titles.)