CHAPTER 5

Radial Line Methods of Increasing Control

We have seen in Chapter 2 how that if the ground is perfectly planar we can plot the position of points by intersections of radials from the isocentres of two or three consecutive photographs. We also saw how if all the photographs were untilted then plotting could be carried out by intersections of radials from the plumb point. In fact, of course, the ground is never truly planar and truly vertical photographs are extremely rare. However, in a very slightly tilted photograph, the plumb point and the isocentre will be very close together so that distortions due to height and tilt combined will be very nearly radial from either the plumb point or the isocentre. It is very difficult to identify either the plumb point or the isocentre on a photograph. Only the principal point may be readily found. In a nearly vertical photograph the principal point will be very close to the isocentre, and errors due to assuming that distortions are radial from the principal point will be small. Both the important graphical methods of plotting from air photographs involve the assumption that distortions are actually radial from the principal point. This principle is known as the *radial line assumption* or the *Arundel assumption*; it allows us to plot points of detail by intersecting radials from the principal points of consecutive photographs. The procedure is somewhat tedious and would not be suitable for the complete plotting of all detail, but only for increasing the number of plotted points on the map so that the bulk detail may be inserted by the methods suggested in Chapter 7.

The drill for plotting these additional control points, or *minor control points* as they are called, involves the construction of a series of *principal point traverses,* one for each strip of photographs. Each of these traverses is then replotted in relation to known ground (or major) control points. Every point must also be adjusted to its most probable position.

The first step is the plotting of what is known as the *base grid.* This is a series of grid lines, in relation to which the positions of previously surveyed ground control points are known. The grid is designed to satisfy the conditions of the particular projection chosen, and the ground control points are accurately plotted on it (see Fig. 5.1).

The base grid must be plotted on some form of *topographic base.* The most suitable materials are those which are subject to very

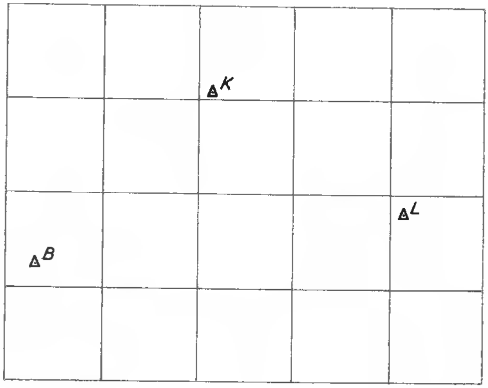

FIG. 5.1. BASE GRID WITH GROUND CONTROL POINTS PLOTTED AT *B*, *K* AND *L*

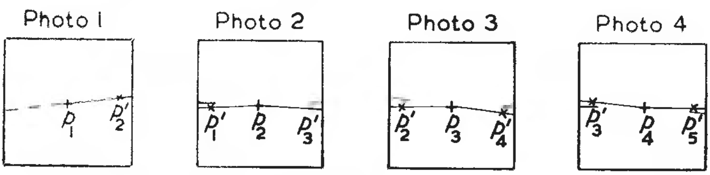

FIG. 5.2. BASE-LINING FIRST FOUR PRINTS OF A STRIP

little expansion and contraction, and have a clear white surface on which it is easy to draw. There are several well-known stable plastic sheets on the market. Some are obtainable as transparent, opaque, or translucent sheets; others are available in a range of thicknesses to suit different purposes. This type of sheet is sold under various trade names, such as Astrafoil, Cobex, Ethulon, Kodatrace and Permatrace. Other materials in use for the purpose include zinc or aluminium sheets surfaced with white enamel and butt-jointed sheets of plywood, coated with white enamel.

The next stage is to base-line all the photographs (see Fig. 5.2), and then to deal with each strip in turn as follows. Assemble the photographs so that the base $p_1'p_2$ on photo 2 falls over the base line p_1p_2' of photo 1, and so that the base line $p_2'p_3$ of photo 3 falls over the base line p_2p_3' of photo 2, and so on to the end of the strip (see Fig. 5.3). This layout is known as a *rough mosaic*, and enables a piece of transparency to be cut so that it is long enough to contain the full length of the strip, and wide enough to take the width of the mosaic about three times. At the same time the approximate

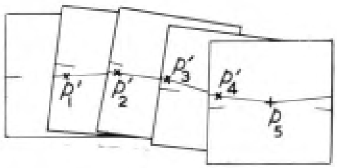

FIG. 5.3. A ROUGH MOSAIC

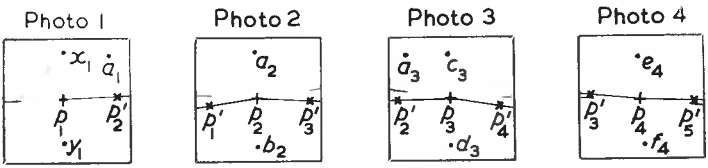

FIG. 5.4. CHOOSING THE PASS POINTS

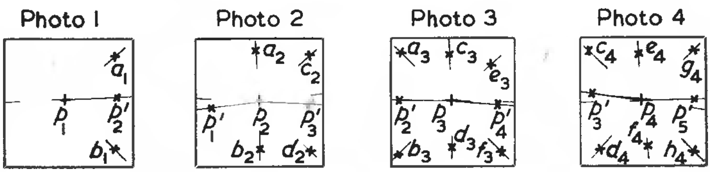

FIG. 5.5. RAYING-IN THE PASS POINTS

direction of the principal point traverse $p_1'p_2'p_3'p_4'$, etc. should be lightly indicated in a central position on the transparency. One of the less expensive transparent stable plastic sheets could be used for this purpose.

Points of detail are now chosen on each photograph roughly in the positions indicated in Fig. 5.4. These should be easily-recognized, but very small, points of detail, such as the image of ground point A, which appears at a_2 in photo 2, and can be easily recognized at a_1 on photo 1 and at a_3 on photo 3. Similarly for points *B*, *C*, etc. (see Fig. 5.5). These points are the pass points or minor control points.

The method of marking these points on the photograph varies, but perhaps the least objectionable is to make a minute hole with a very fine needle. If such a point is ringed round with a coloured chinagraph pencil it will be found again more readily.

Although some pass points will be readily recognizable on each of the photographs on which they occur, it will usually be necessary to use a magnifying glass or stereoscope. These points must be very accurately transferred, and it will often be best to set up a properly base-lined pair of photographs under the stereoscope as described in Chapter 3. With a needle, point accurately to the chosen point on say the left-hand print, then viewing stereoscopically move

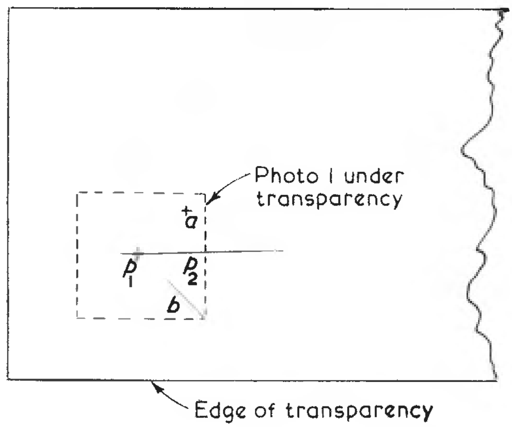

FIG. 5.6. THE TRANSPARENCY—TRACING FROM PHOTO 1

another needle over the right-hand print until the needle points appear to coincide: the second needle should now indicate the correct image on the right-hand print.

On photo 1 draw in from p_1 short radials through a_1 and b_1. On photo 2 draw in from p_2 short radials through a_2, b_2, c_2 and d_2; continue for each photo.

The next step is to take the transparent strip and place it over photo 1 so that the base line p_1p_2' lies approximately under the first leg of the lightly marked traverse. Trace in points p_1 and a_1, the base line, and a short length of radial through b_1. Figure 5.6 shows the detail traced and the way it should be marked up. The broken line indicates photo 1 in position under the transparency.

Fasten photo 2 face upwards on to the table, and place the transparency over it so that the line p_1p_2 on the transparency covers the line $p_1'p_2$ on photo 2. Keeping these two lines collinear, slide the transparency over photo 2 until the point a is exactly over the radial through a_2. Maintaining these positions by securing the transparency to the desk, trace the radials through p_3', b, c and d as in Fig. 5.7.

The point b is now fixed by an intersection of two rays.

Photo 3 is now fixed to the table top, and the transparency placed over it so that the points a and b lie immediately over the radials through a_3 and b_3, and p_2 lies immediately over the base line $p_2'p_3$.

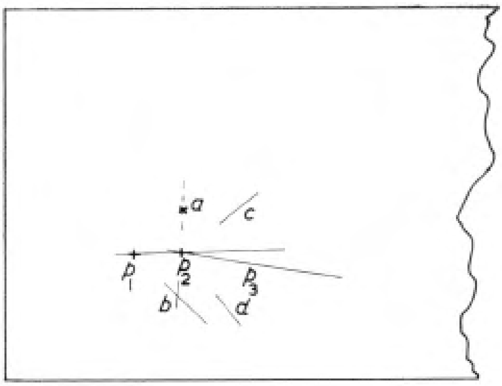

FIG. 5.7. THE TRANSPARENCY—TRACING FROM PHOTO 2

This will give a position for p_3 which might not lie exactly over the base line $p_2'p_3$. If the drawing has been carried out accurately, and the tilt distortion is not excessive, then this discrepancy will be very small, and can be neglected. Now trace in the radials from p_3 through c_3, d_3, e_3, f_3 and p_4'. Repeat this process as for photo 3, for each remaining photo of the strip in turn. We now have the complete principal point traverse plot on to the transparency as shown in Fig. 5.8.

The points B and L (see Fig. 5.1) have been deliberately chosen as pass points because they are ground control points. The traverse lines can now be transferred from the transparency to the base grid by plotting between these two points.

The scale of the strip is governed by the scale along the line p_1a_1 on photo 1, and a is therefore known as the *scale point*. The point A should be chosen, if possible, at about the average ground height of the area covered by the sortie; then the scale of the transparent plot will be about the same as the average scale of all the photographs, or *mean photo scale* as it is usually called.

The method of plotting the traverse on to the base grid is as follows. Choose any point *m* on the transparency (see Fig. 5.8), such that the points *b*, *l* and *m* form a well-conditioned triangle, i.e. a triangle in which no angle is less than 30°. *m* does not represent either a ground point or an image point but is chosen quite arbitrarily. If any one of the principal points or pass points itself forms a well-conditioned triangle with *b* and *l*, then there will be no need to introduce the point *m*.

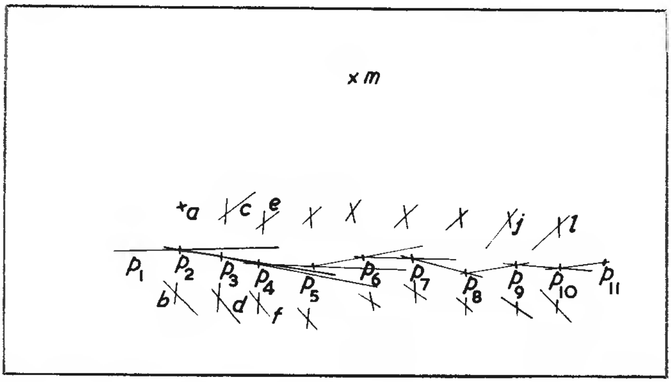

FIG. 5.8. THE TRANSPARENCY—PRINCIPAL POINT TRAVERSE COMPLETE

Join *bl* on the transparency, and *BL* on the base grid. Lay the transparency over the base grid so that *b* falls over *B* and *bl* lies directly over *BL*. Prick through *m* and call this point *M'*. Remove the transparency and draw a short radial from *B* through *M'*. Now set up the overlay so that *l* falls over *L*, and *bl* again lies directly over *BL*. Again prick through *m* and call the pricked point on the base grid *M''*. A radial from *L* through *M''* will meet the radial *BM'* in *M*, which will be the position of *m* on the base grid.

Using the triangle bp_5m of the transparency (see Fig. 5.8), p_5 can be plotted on the base grid in the same way that *M* was plotted from the triangle *bml*. All the remaining principal points can be plotted by the same method.

If the strip we have just plotted is strip No. 2, and *l* lies in the overlap between this strip and strip No. 3, then in order to plot the principal point traverse for strip No. 3 we require another control point near the other end. Let us choose point *c* as falling in the overlap between the two strips; then we can plot *C'* from the transparency for strip No. 2, and plot strip No. 3 between *C'* and *L*.

To plot strip No. 4 we would first plot two points in the common overlap between strips 3 and 4 using the transparency of strip No. 3 for this purpose. Let these points be Q' and R' as in Fig. 5.9; then plot strip 4, including the two points S' and T' lying in the overlap with strip 5. S' and T' will be the control for strip 5.

Let us suppose that the third ground control point K lies in the 5th strip of photographs. Then we would plot on the base grid the

FIG. 5.9. THE BASE GRID—EXTERNAL ADJUSTMENT TO GROUND CONTROL

point K' as the position of k on the transparency, and using S' and T' as control. K' will not normally coincide with K; thus there will have been an accumulated error equal to $K'K$ in plotting the four strips. This means that we must make an adjustment to the positions of C', Q', R', S' and T'. This is done by joining $K'K$ and drawing short lines through each of the five points parallel to $K'K$ (see Fig. 5.9). The actual positions of the corrected points C, Q, R, S and T will be such that $S'S = K'K \times (BS'/BK')$ and $R'R = K'K \times (LR'/LK')$, etc. This is called the external adjustment. The principal point traverses would not be plotted on to the base grid until after the external adjustment had been made.

In making the external adjustment, the aim must be to find some strip having on it two recognizable points of ground control, and start plotting from this strip. The ground control points should normally be as far apart on the strip as possible (certainly not less than 70 per cent of the strip should lie between them), though the points concerned should lie on at least two and preferably on three photographs in the strip. These ground points will give the strongest control if they lie well away from the principal point traverse.

If there is no strip containing two ground control points then the task is not quite so simple. Suppose, for example, we are plotting the same sortie as before, except that the image of L did not fall on strip No. 2, but that it did fall in strip No. 3. We would complete the transparencies for strips No. 2 and 3 as before, and choose one pass point, say *j* on Fig. 5.8, such that *j* lies both in strip No. 2 and in strip No. 3. Choosing *c* as before as the other control point common to the two strips, we would now take a further transparency, and trace on to it the points *b*, *j* and *c* from strip No. 2. We now trace *l* on to the further transparency from the transparency of strip No. 3, using *j* and *c* to control this setting. From the further transparency we now plot the positions of C' and J' on to the base grid using *B* and *L* as the base. Then proceed as before for strips 4 and 5, and the external adjustment.

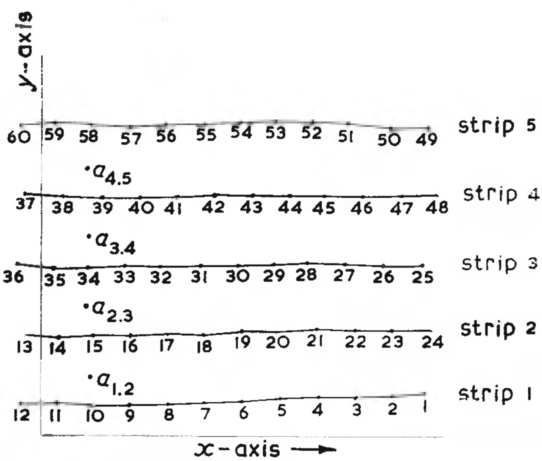

FIG. 5.10. THE BASE GRID—PRINCIPAL POINT TRAVERSES
The principal points are numbered 1 to 60.
The axes are used for internal adjustment.

Having got the principal point traverse for each strip plotted on to the base grid, we then plot the pass points from say strip 1. Next we plot the pass points from strip 2 on to the base grid. Unfortunately we shall probably find that points *d*, *f*, *h*, etc. (see Fig. 5.5) when plotted on the base grid as *D*, *F*, *H* do not coincide with the same points as plotted from strip 1. To overcome this difficulty the position of the principal point traverse must be adjusted. There are numerous methods of doing this, most of which are fairly involved and most of which are similar. Perhaps the simplest is as follows.

Let Fig. 5.10 represent the plotted positions on the base grid of the five principal point traverses. The principal points are numbered consecutively throughout all the strips from photo 1 to photo 60. $a_{1\cdot2}$, $a_{2\cdot3}$, $a_{3\cdot4}$ and $a_{4\cdot5}$ are pass points or other points in the overlaps, and when used for this purpose these points are often termed *tie* *points*. The suffixes denote the strips in which the point falls. Figure 5.11 is an enlargement of a portion of Fig. 5.10, showing that the point $a_{1\cdot 2}$ in Fig. 5.10 is really composed of two points, a_1 and a_2 in Fig. 5.11. The distance between a_1 and a_2 is exaggerated for clarity. a_1 is plotted by intersections from the principal point traverse for strip 1, and a_2 from the traverse for strip 2.

The problem is to adjust the positions of each principal point traverse in the sortie so that all points common to two strips will plot as a single point. If we adopted the mean position for $a_{1\cdot 2}$, i.e. if we treated the mid-point of a_1a_2 as the correct position, and replotted p.p.s (principal points) 14, 15 and 16 accordingly, we might find that we had made the error at $a_{2\cdot 3}$ greater than before. So that to find the most likely positions for $a_{1\cdot 2}$, $a_{2\cdot 3}$, etc., we must find the correction for each which will give the least total of corrections. This is achieved by measuring the errors a_1a_2 in Fig. 5.11 in two directions at right angles to one another and applying two corresponding corrections to each point. Choose the two directions such that one is roughly parallel to the direction of the principal point traverses (x-axis in Fig. 5.10), and the other is perpendicular to this (y-axis in Fig. 5.10). Draw lines through a_2 and a_1 parallel to the x-axis and y-axis respectively. Measure the distance marked x in Fig. 5.11 and call this the x error. Similarly measure the distance y as the y-error. Measurements made on the face of a photograph are invariably in millimetres.

A set comprises one pass point from each lateral overlap, chosen so that the whole set lies as nearly as possible on the same perpendicular to the general direction of the principal point traverses. A schedule as in Table 1 is made out for each set of tie points. The first column relates to the strip from which the tie point has been plotted. The next three columns refer wholly to x-errors and corrections, and the next three columns to y-errors and corrections. The second column gives the measured x-error. Consider that we are measuring the error of the plotting of a_2 relative to a_1 in Fig. 5.11; then if we find a_2 is to the right of a_1 the error will be positive, and if to the left it will be negative (similarly for the point $a_{2\cdot 3}$, should the position as plotted from strip 3 be to the right of the position as plotted from strip 2, the error is positive); that is, we take the lower strip first. In the same way if the position of say $a_{2\cdot 3}$ is too high when plotted from strip 3 and compared with strip 2 the y-error is positive. Thus for a positive y-error the p.p. traverses are too far apart, and for a negative y-error they are too close together. Other conventions can be adopted if thought desirable.

In the schedule of Table 1 the horizontal lines represent the individual strips, but the x- and y-errors refer to the overlaps between any two strips, and are therefore entered in the spaces between the two appropriate lines.

In the third column of Table 1 it has been assumed that strip 1 is correct, and that a_1 in Fig. 5.11 is therefore given zero correction. This being so, a_2 must be given a correction to the right, that is in a positive x-direction, so that this correction must represent the

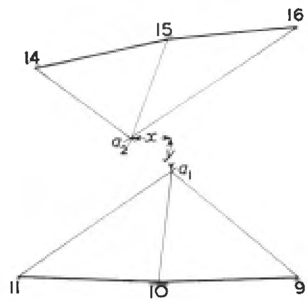

FIG. 5.11. THE BASE GRID—PASS POINT PLOTTED FROM DIFFERENT TRAVERSES
This is an enlargement of part of Fig. 5.10.

Table 1
EXTRACT FROM SCHEDULE FOR INTERNAL ADJUSTMENT

	x-error			y-error			
	Error	Adjusted to strip 1	Correction	Error	Adjusted to strip 1	Correction	
		−0·1	−1·1		+1·0	+0·9	strip 5
Point $a_{4\cdot5}$	+1·3			−1·2			
		+1 2	+0 2		−0·2	−0·3	strip 4
Point $a_{3\cdot4}$	+0·9			+0·7			
		+2·1	+1·1		+0·5	+0·4	strip 3
Point $a_{2\cdot3}$	−0·4			−1·3			
		+1·7	+0·7		−0·8	−0·9	strip 2
Point $a_{1\cdot2}$	−1·7			+0·8			
		0	−1·0		0	−0·1	strip 1
		sum +4·9			+0·5		
		average +1·0			+0·1		

whole x-error for the point $a_{1\cdot2}$ in Fig. 5.10. If such a correction were given to strip 2, then the position of $a_{2\cdot3}$ as plotted from strip 2 would be moved 1·7 mm to the right. Thus the negative x-error applying to $a_{2\cdot3}$ would be increased from − 0·4 to − 2·1, and the position as plotted from strip 3 would need moving 2·1 mm to the right, i.e. the whole strip, at this cross-section would be moved 2·1 mm to the right. These figures then apply to the individual strips and must be entered on the appropriate lines.

The assumption that no correction should be made to strip 1 is obviously fallacious, and we must find the correction to be applied to this strip to give the least overall total correction to all the strips combined. This is done by taking the algebraic sum of the adjustments in the third column, and finding the mean of these corrections, in this case + 1·0 mm. This means that the whole sortie has at this cross-section been moved bodily 1·0 mm to the right. This can be offset by moving each strip 1·0 mm to the left; thus we obtain the final adjustments in the fourth column.

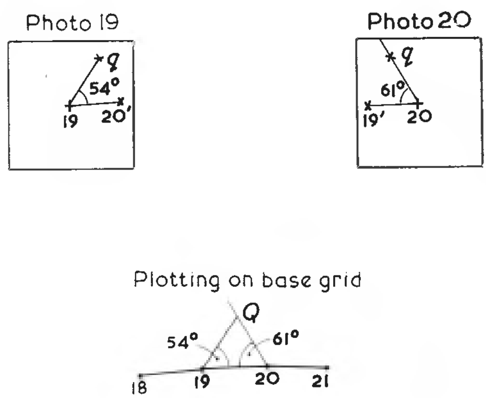

FIG. 5.12. PLOTTING FURTHER CONTROL POINTS BY INTERSECTIONS

When the same procedure has been followed for the *y*-errors, then corrected single positions of $a_{1\cdot 2}$, $a_{2\cdot 3}$, $a_{3\cdot 4}$ and $a_{4\cdot 5}$ can be plotted.

The plotting of strip 5 may again show an error at *K*, and the external adjustment may need to be repeated.

Using these corrected pass points, together with the ground control points, the p.p. traverses are replotted from their respective transparencies by the method of intersections as before.

Further points of detail can now be added as required by using the adjusted principal point traverses as base lines, and transferring radials from the principal points direct from the photographs (see Fig. 5.12). This is not a practical method for plotting the full topographic detail, but it is quite suitable for obtaining sufficient points of minor control to enable the use of some of the simple methods of plotting the final detail, such as those described in Chapter 7.

The purely graphical drill described above is suitable when no instruments, other than ordinary drawing office equipment, are available. It is obviously tedious, but with only comparatively minor equipment a simple and efficient method of applying the radial line assumption is available. This is known as the graphical mechanical, or *slotted templet* method.

SLOTTED TEMPLET METHOD

Every photograph of the sortie is base-lined as before, and pass points are chosen in similar positions to those in Figs. 5.4 and 5.5. These points are pricked through and ringed as before. Transparencies are now cut to represent each photograph. For each photograph a piece of relatively thick (about 0·25 mm) stable plastic transparency is cut to a size about one inch greater than the photograph in each direction, and is carefully numbered with the serial number of the photograph which it is to represent. It is then placed over the appropriate photograph, the principal point is marked, and base lines and pass point radials are very carefully traced in with fine lines. A purpose-made hand-punch (see Fig. 5.13) is now used to make a 5 mm diameter circular hole at the principal point. This is made very accurately since the punch is centred by means of a needle point at the centre of the circular cutting edge. The templet is then removed and placed on the table of the slotted templet cutting machine (see Fig. 5.14) so that the p.p. hole lies over the spindle. This spindle is free to move only in a straight line because it is confined within a slot. The cutting edge of the machine is so placed that it will always contact the table top centrally over this slot. When the handle is depressed the cutting edge will contact the templet and cut a slot in it precisely 5 mm wide, and 40 to 80 mm long. Such a slot is cut for each of the pass points and base lines, i.e. eight slots per templet. Accuracy is maintained by sighting the pass point mark on the templet through a small telescope. Each templet slot is radial from the p.p. because both p.p. and pass point lie centrally along the machine slot. The exact distance of the slot from the p.p. can be adjusted by moving the p.p. pivot along the machine slot. Fig. 5.15 shows a marked-up and partly cut templet.

It must be remembered that the slotted templet cutting machine is a precision instrument costing between say £300 and £400, and the slots are precisely cut. There are variations of the above drill to suit the particular equipment available, and one instrument obviates the need for tracing the pass points from photo to templet. In this instrument the photo and templet are mounted simultaneously on one vertical spindle and each is securely taped to a separate

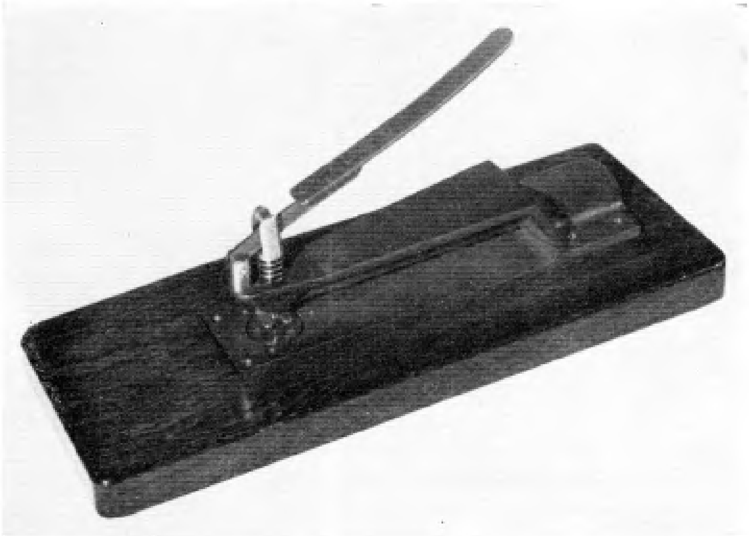

FIG. 5.13. ONE TYPE OF CENTRE PUNCH
(*C. F. Casella & Co. Ltd.*)

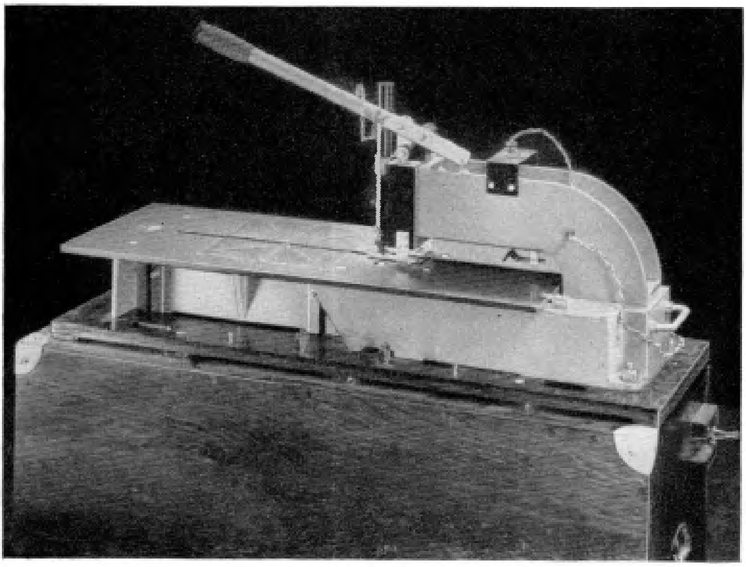

FIG. 5.14. SIMPLE SLOTTED TEMPLET CUTTING MACHINE
(*C. F. Casella & Co. Ltd.*)

turn-table. The spindle and turn-tables are rotated until one of the pass points lies centrally under the telescope. At this stage the turn-tables are clamped, and the cutting edge depressed to cut the appropriate slot in the templet.

When all the templets of the sortie have been cut, they can be mounted over the ground control points plotted as before on to the base grid. For this operation small studs are required (see Fig. 5.16)

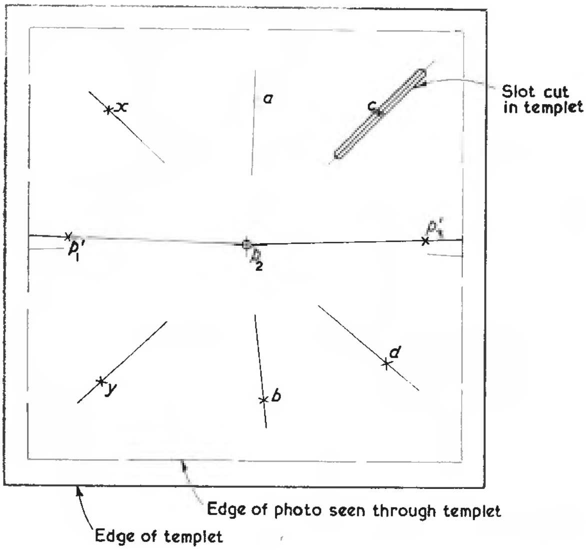

FIG. 5.15. PARTLY-CUT TEMPLET SUPERIMPOSED ON ITS MARKED-UP PHOTOGRAPH ($\frac{1}{4}$ × full size)
Natural dia. of hole = 5 mm
Natural width of slot = 5 mm

with a centrally-pierced hole through which the pin must be just capable of passing. The external diameter of the shank is 5 mm and is only just able to enter the slot or centre hole of the templets. These studs may be of either metal or plastic but the metal ones are usually considered the more satisfactory.

A pin is inserted vertically in every ground control point on the base grid, and, as before, a fairly central strip containing two well-spaced ground control points should be chosen first. Let us assume this to be strip 2 in Fig. 5.10, i.e. the strip shown in Fig. 5.8, where the two ground control points are *B* and *L*. Place a stud over the pin at B and then mount templet No. 2 by placing the slot representing the radial through b_2 over this stud. Place further studs through the hole and remaining slots in this templet; these will represent the points P_2, A, C, D, P_1 and P_3. Now mount the appropriate slots of templet No. 3 over the studs A, B, C, D and P_2, and fill in all empty slots and the p.p. hole with a stud. These studs will represent points E, F and P_4. Continue in the same way with

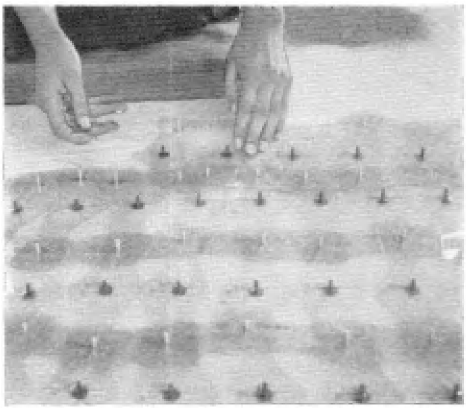

FIG. 5.16. SLOTTED TEMPLET ASSEMBLY UNDER CONSTRUCTION

templets 1, 4, 5, 6, etc. in turn, until the templets containing radials through L are reached. When the stud representing L is inserted in the templet assembly it should also be placed over the pin at L in the base grid. It will be found to be perfectly easy to adjust the scale of the assembly between B and L by pulling out or pushing in; the studs will act like the pivots in adjustable trellis work. The remaining templets of the strip can then be added, followed by the laying of the adjacent strips. Control for the latter will be provided by the studs representing pass points in the lateral overlaps concerned. Each of the other strips will then be laid in a similar manner until the assembly is complete. Each time a stud representing a ground control point is reached it is placed over the appropriate pin, and the assembly is pushed into shape to accommodate it. Figure 5.16 shows a slotted templet assembly under construction. On completion, the assembly is allowed to rest for at least twelve hours to give all the templets time to settle. Then all the studs are pricked through with pins which are left sticking in the base grid. The templets are then removed carefully one by one; the pass points and p.p.s are ringed and numbered as the pins are removed. It is usual to mark p.p.s in a different colour from that used for pass points. Principal point traverses can now be drawn in, and used for the plotting of further control as before, if this is required.

Using the slotted templets we have achieved at least as good a result as before, without the need for complicated internal and external adjustments. These adjustments are made mechanically by the templets sliding over one another. If the assembly shows signs of buckling, then this is an indication of lack of precision in making the templets, or of poor flying, or excessive variations in height of the terrain. Generally speaking, for slotted templet work, the variation in ground heights should not exceed 7 or 8 per cent of the flying height. Occasionally one templet, or a group of templets, will not fit into the assembly at all. To remedy this it might be necessary to make new templets from rectified prints (see Chapter 7).

To facilitate the assembly and subsequent adjustment to scale of parts of the assembly, it is usual to pare off the unnecessary portions of the cut templet, particular attention being paid to the removal of angular parts of the edges (see Fig. 5.16). If the assembly is particularly big, it may be necessary to wax each of the templets to enable them the better to slide one over the other, and so take up their relative positions of equilibrium. Sponge-rubber mats are used as stepping stones when working on large assemblies, which are generally made on the floor because the area covered may be the size of a large hall.

The only disadvantage of the slotted templet relative to the purely graphical radial line plot is the additional cost both for instruments and material.

In recent years an empirical formula relating the necessary amount of ground control with the number of templets in the assembly has become fairly widely accepted. This formula is usually stated as if it were a guide to accuracy in the form

$$e = 0{\cdot}16\sqrt{(t/c)},$$

where t = number of templets,
c = number of ground control points,
e = average point displacements, in millimetres.

Such an equation has more application if expressed as a method of determining the amount of control required, that is as $c = t(0{\cdot}16/e)^2$. The accuracy required is determined by the ability of the draughtsman, who should be capable of drawing a line not greater than 0·25 mm thick. It would therefore be wasteful to plan for an accuracy greater than this on the compilation sheet. Since e is an average value, it will not normally be less than say 0·4 of a millimetre. It is generally agreed that at publication scale the accuracy should be such that all well-defined points of detail should be within 0·5 mm of their true position. If publication scale is the same as compilation scale then the accuracy at compilation scale can be reduced to give e a value of 0·5 mm.

Suppose then that $e = 0{\cdot}5$ mm and there are 130 templets in the assembly; then

$$c = 130\left(\frac{0{\cdot}16}{0{\cdot}5}\right)^2 = 13$$

The disposition of these 13 control points is also important though no formula can be applied. It has been found experimentally that control in the four corners of an assembly is of paramount importance, and that the bulk of the remaining ground control should be near the ends of each strip. Further control points should be located in the two extreme lateral overlaps, and only a few points would be needed over the central part of the assembly. This pattern appears to be the best for even very large assemblies.

ANALYSIS OF ERROR

In Fig. 5.17 (i) a represents the image of any ground point A. a' is the corrected position of a, and aa' the tilt distortion. a' must lie on ai, and since a is on the same side of the isometric parallel as n, the scale at a must be too great. Therefore the correct position, a' of a must be such that $ia' < ia$. Had a fallen at e the corrected position would have been further from i, e.g. at e'. d lies on the isometric parallel and therefore has no tilt distortion.

Figure 5.17 (ii) illustrates the error due to the radial line assumption, assuming that the ground is a perfectly horizontal plane. The assumption is that a' falls on ap, and since its position along pa is influenced by radial lines from other photographs, the tendency is for g (where g is the plotted position of a on the minor control plot) to take up the least erroneous position consistent with its lying along pa. That is, $a'g$ will tend to be perpendicular to pa, since $a'g$ is the error in position due to the radial line assumption. In this figure the tilt distortion $aa' = cc' \sec \phi$.

Figure 5.17 (iii) shows a section through the perspective centre of the perspective diagram and containing the line *ia*. As relationships in the principal plane are more easily established, Fig. 5.17 (iv) would be preferred in any analysis.

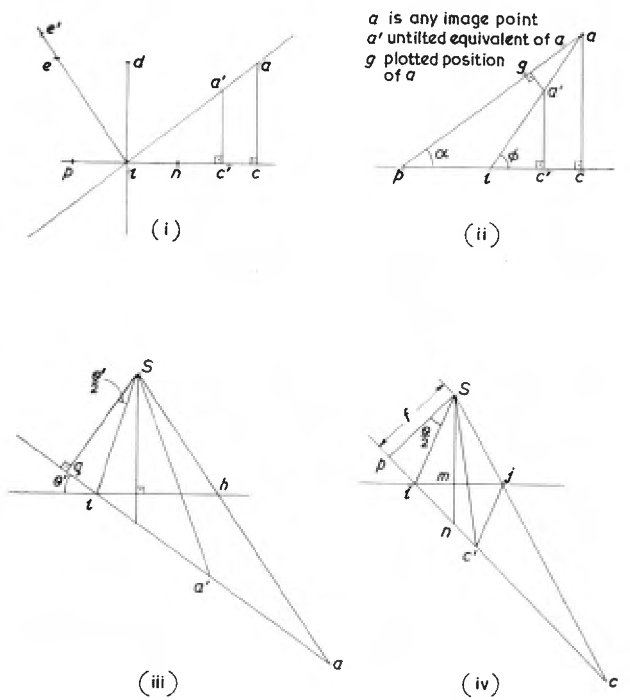

FIG. 5.17. ANALYSIS OF ERROR IN RADIAL LINE PLOTTING
(i) PLAN VIEW OF PRINT
(ii) PLAN VIEW OF PRINT
(iii) SECTION IN EPIPOLAR PLANE *iSaa′*
(iv) SECTION IN PRINCIPAL PLANE

Provided that tilts do not exceed 3° for any photograph, and that variations in ground height do not exceed 8 per cent of the flying height, then errors due to the radial line assumption should not exceed 0·4 mm. Thus it is unusual to make a mathematical analysis in practice.

In the following example the error due to the combined tilt and height distortions is shown to be 0·009 in., a barely plottable amount, but it can be seen that, if the variations in ground heights were to approach nearer to the allowable 8 per cent instead of $2\frac{1}{2}$ per cent of the flying height, the error due to making the radial line assumption would become appreciable. It must be remembered,

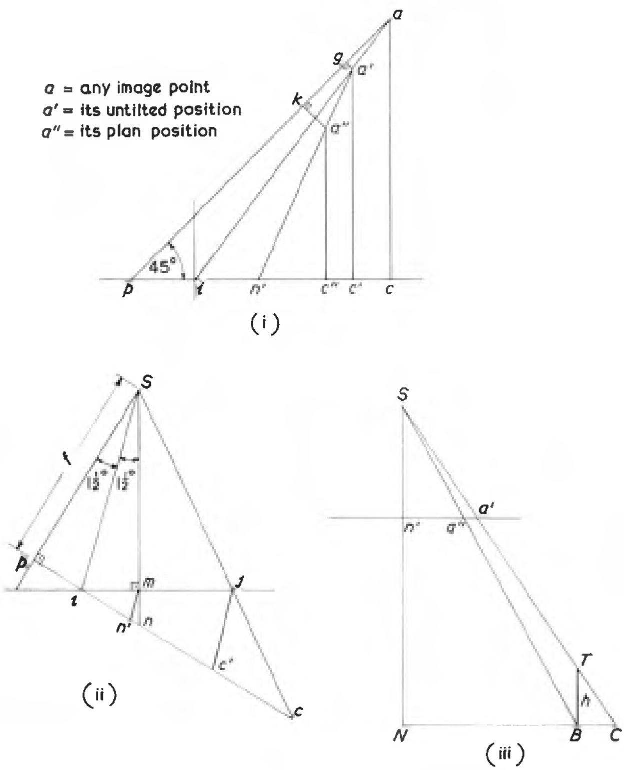

FIG. 5.18. ERRORS DUE TO COMBINED TILT AND HEIGHT DISTORTION
(i) PLAN VIEW OF PRINT
(ii) SECTION IN PRINCIPAL PLANE
(iii) VERTICAL SECTION THROUGH $a''a'$

however, that not only have we assumed the worst conditions for the tilt distortion, but we have also assumed a height displacement in the same general direction as the tilt distortion, i.e. both have been assumed to have displaced *a* away from *p*.

Example to illustrate the error in plotting a particular point.

Figure 5.18 (i) shows a point *a*, such that *ap* makes an angle of

45° from the principal line, and we are going to assume that *ac* is 100 mm long and is perpendicular to the principal line. The tilt of the photograph is assumed to be 3°. These conditions should make as great an error as may normally be expected when dealing with the points used in a slotted templet plot.

The following analysis uses elementary coordinate geometry, without introducing any of the less obvious approximations or other mathematical short cuts.

Our work is simplified by using the principal line as the x-axis, and i as the origin; but we must remember that it is the base line which is normally the x-axis.

We shall assume the flying height to be 3,000 m, and the principal distance to be 150 mm.

In Fig. 5.18 (ii) $pi = f.\tan 1\frac{1}{2}° = 3{\cdot}9285$,

$\therefore$ in Fig. 5.18 (i) coordinates of p are $(-3{\cdot}9285, 0)$

But $\angle api = 45°$ $\quad \therefore \quad pc = ac = 100$ mm

$\therefore$ coordinates of a are $(100 - pi, 100)$ i.e. $(96{\cdot}0715, 100)$

The equation of the line pa is $y = x + 3{\cdot}9285$, since the slope is 1 and the intercept on the y-axis equals pi,

i.e. ap is the line $x - y + 3{\cdot}9285 = 0$

To find the coordinates of a':

$$x\text{-coordinate is } ic' = ij = im + mj = f\left(\tan\frac{\theta}{2} + \tan \angle mSj\right)$$

$$\text{But } \tan \angle mSj = \tan(\angle pSc - \angle pSn)$$

$$= \frac{\tan \angle pSc - \tan \angle pSn}{1 + \tan \angle pSc \times \tan \angle pSn}$$

$$= \frac{\dfrac{pc}{ps} - \tan\theta}{1 - \dfrac{pc}{pS}\tan\theta}$$

$$= \frac{\dfrac{100}{150} - 0{\cdot}05241}{1 - \dfrac{100}{150} \times 0{\cdot}05241}$$

$$= 0{\cdot}59352$$

$\therefore$ x-coordinate of a' is $150(\tan 1\frac{1}{2}° + 0{\cdot}59352) = 92{\cdot}9565$

and y-coordinate of a' is $a'c'$; but $\dfrac{a'c'}{ac} = \dfrac{ic'}{ic}$

$$\therefore \quad a'c' = \frac{ic' \times ac}{ic} = \frac{92{\cdot}9565 \times 100}{96{\cdot}0715} = 96{\cdot}7576$$

$\therefore$ coordinates of a' are (92·9565, 96·7576)

Using the formula $p = \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$

for the perpendicular distance from the point (x_1, y_1) on to the line $ax + by + c = 0$, we have

$$a'g = \frac{1 \times 92{\cdot}9565 - 1 \times 96{\cdot}7576 + 3{\cdot}9285}{\sqrt{1^2 + 1^2}} = 0{\cdot}1274\,\text{mm}$$

Thus the error due to the radial line assumption is 0·1274 mm.

Height distortion is radial from the plumb point; but as we have corrected the position of a, we must also correct the position of n. If n' is the position of the image of N when the photograph is corrected for tilt, then $in' = ip = 3{\cdot}9285$, i.e. coordinates of n' are (3·9285, 0).

Let the homologue of a have a ground height of 75 m above datum; then if a'' is the height corrected position of a', a'' must lie on $n'a'$ in such a position that $n'a'' < n'a'$.

In Fig. 5.18 (iii),

$$\frac{a''a'}{BC} = \frac{f}{H}; \quad \text{therefore} \quad a''a' = \frac{f}{H} \times BC = \frac{f}{H} \times h \tan \angle BTC$$

$$\text{but} \quad \tan \angle BTC = \tan \angle n'Sa' = \frac{n'a'}{f}$$

$$\therefore \quad a''a' = \frac{75}{3{,}000} \times n'a' = \frac{n'a'}{40}$$

$$\therefore \quad \frac{a''c''}{a'c'} = \frac{n'a''}{n'a'} = \frac{39}{40} \quad \text{(Fig. 5.18 (i))}$$

$$\text{i.e.} \quad a''c'' = \frac{39\,a'c'}{40} = \frac{39}{40} \times 96{\cdot}7576$$

$$= 94{\cdot}3387$$

$$\text{Similarly} \quad c''c' = \frac{1}{40} \times n'c' = \frac{92{\cdot}9565 - 3{\cdot}9285}{40} = 2{\cdot}2318$$

$$\therefore \quad ic'' = ic' - c''c' = 92{\cdot}9565 - 2{\cdot}2318 = 90{\cdot}7247$$

$\therefore$ coordinates of a'' are (90·7247, 94·3387)

$$\therefore \quad a''k = \frac{1 \times 90{\cdot}7247 - 1 \times 94{\cdot}3387 + 3{\cdot}9285}{\sqrt{1^2 + 1^2}} = 0{\cdot}3145 \text{ mm}$$

∴ total position error of *a* due to the radial line assumption can be expected to be about 0·3 mm., which is barely plottable.

FURTHER READING

(i) Hart, pages 182–202.
(ii) Moffitt, Chapter 6.
(iii) *Admiralty Manual*, pages 267–80.
(iv) Trorey, pages 93–108.

(See Bibliography (page 346) for full titles.)