CHAPTER 6

Measuring Heights and Contouring

The simplest method of contouring from levels taken in the field is to arrange the field stations in the form of a rectangular grid and then to use a linear interpolation between pairs of such points to locate the points at which the required contours cut the grid lines. In Fig. 6.1 the heights of A_5 and A_6 are given as 99·61 and 100·62

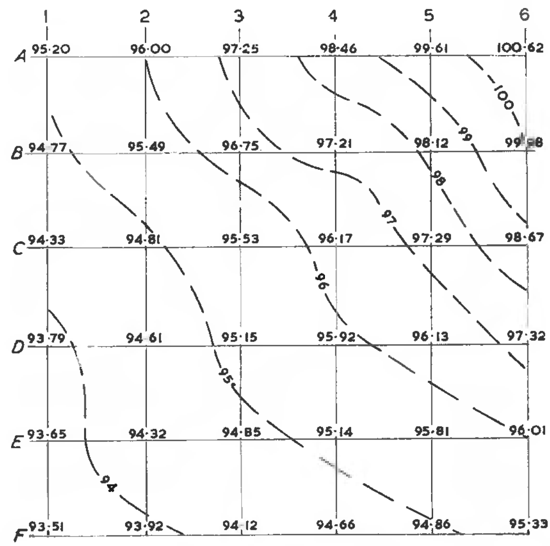

FIG. 6.1. CONTOURS INTERPOLATED FROM A GRID OF LEVELS

respectively, and it is obvious that the 100 m contour cuts the line joining A_5 and A_6. A_5 is only 0·39 m below 100 m, while A_6 is 0·62 above. The total difference in height between A_5 and A_6 is 1·01 m, and the distance A_5 to A_6 is 10 m. Therefore, the 100 m contour crosses the line A_5A_6 at a point $10 \times 0{\cdot}39/1{\cdot}01 = 3{\cdot}86$ m from A_5. Similarly for all other cutting points. The contours are then sketched in by joining all cutting points representing the same contour.

When contouring is done from air photographs, the method used is basically the same. One of the photographs is gridded and the height at each grid intersection point is determined. Contours are then interpolated between these points as for a ground survey. Sometimes an irregular pattern of points is used instead of a regular grid, heighting points being chosen at obvious changes of gradient.

PARALLAX EQUATIONS

In Chapter 3 and Fig. 3.24 we defined parallax in the sense understood by the photogrammetrist. We also stated that parallax was a

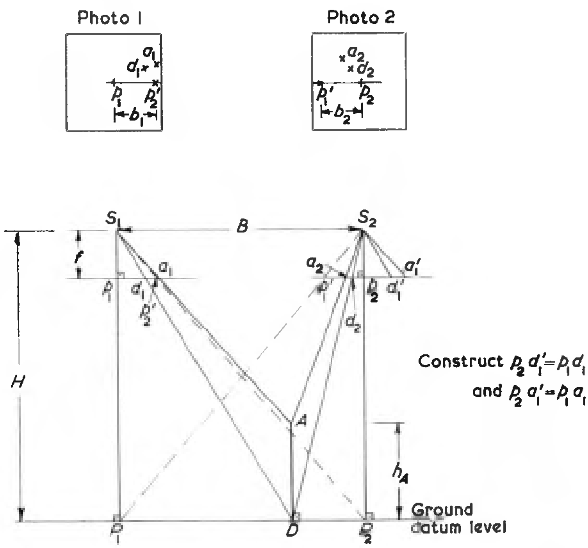

FIG. 6.2. PARALLAX OF A POINT—DIAGRAM FOR DETERMINATION OF PARALLAX EQUATION

B = air base
H = flying height
h_A = height of A above datum
b = base line

measure of the height of a ground point. Let us extend this argument by reference to Fig. 6.2, where D is a point at datum level and vertically below the point of ground detail A. The sketches of photo 1 and photo 2, above the main figure, show the positions on the two photographs of the images of A and D. It will be noticed that while there is a big difference between the x-coordinates a_1 and a_2, yet the y-coordinates of these two points are almost equal. The same may be said of the points d_1 and d_2. That is, there is relatively little y-parallax and the two photographs " correspond " fairly well.

In Fig. 6.2, $p_2d_1' = p_1d_1$ and $S_2d_1' \| S_1d_1$

Similarly $p_2a_1' = p_1a_1$ and $S_2a_1' \| S_1a_1$

But a_2a_1' represents the parallax of A, and d_2d_1' the parallax of D.

Triangles S_1DS_2 and $d_1'd_2S_2$ are similar.

Therefore $\dfrac{d_2d_1'}{S_1S_2} = \dfrac{\text{perpendicular height of triangle } d_1'd_2S_2}{\text{perpendicular height of triangle } S_1DS_2}$

That is $\dfrac{d_2d_1'}{B} = \dfrac{S_2p_2}{H} = \dfrac{f}{H}$

Therefore $d_2d_1' = fB/H$

or parallax of D, $p_D = fB/H$ . . . . . (6i)

In this formula f will be expressed in millimetres, and the parallax will also be in millimetres. D is a particular point, being at datum level, but A is any general point at some height which we will call h_A above datum. If the parallax of A is written p_A, then from similar triangles $S_2a_2a_1'$ and AS_2S_1 we have—

$$\frac{a_2a_1'}{S_2S_1} = \frac{S_2p_2}{H - (AD)} = \frac{f}{H - h_A}$$

that is $\dfrac{p_A}{B} = \dfrac{f}{H - h_A}$ or $p_A = \dfrac{fB}{H - h_A}$. (6ii)

This is the general parallax equation, which expresses the parallax of a point in terms of the focal length of the lens, the length of the air base, the flying height, and the height of the point. The focal length will be recorded in the margin of the photograph; the parallax can actually be measured within limits as we shall see later; the air base and flying height can be estimated. The altimeter reading at the time of exposure is recorded in the margin of the photograph, but is not usually a sufficiently accurate value of H. The method used to determine H requires the existence of either four points of ground control on each photograph, or a minor control plot. For each photograph, four points are chosen so that, when opposite pairs of points are joined, the two lines intersect roughly at right angles and somewhere near the centre of the photograph. For instance, referring to Fig. 5.5, points a_3, f_3, e_3 and b_3 on photo 3 would be excellent. In this case the procedure would be to measure the distance a_3f_3, then to measure the distance AF on the slotted templet plot. If the scale of the slotted templet plot were 1 in 10,000, then the scale of photo 3 along the line a_3f_3 would be

$$\frac{a_3f_3}{10{,}000 \times AF}$$

Suppose a_3f_3 measured 284·1 mm and the distance AF measured 293·7 mm; then the scale along a_3f_3 would be

$$\frac{284{\cdot}1}{10{,}000 \times 293{\cdot}7}$$

or 1 in 10,338 approximately. We then require to find the scale along e_3b_3 by the same method, and the average of these two scales would then be accepted as the mean scale of the photograph. For good results, each of the four points A, F, E and B should be at, or near, the average height of the ground covered by the photograph. Since scale also equals $f/H - h$, then in the instance given

$$\frac{f}{H-h} = \frac{1}{10{,}338}$$

or $H-h = 0{\cdot}150 \times 10{,}338 = 1{,}551$ m, where the focal length is 150 mm. $H-h$ here represents the height of the aircraft above the mean level of the ground. h is determined as the mean height above datum of all the eight points used on the two photographs to determine the scales. However, an approximate value of h is often used.

The length of the air base, B, is represented by the base line on each photograph. If we measure the lengths of these base lines (p_1p_2' and $p_1'p_2$ in Fig. 6.2) and call the mean b (i.e. $b = \frac{1}{2}[p_1p_2' + p_1'p_2]$), then the average scale along the base lines $= b/B$. This is the same thing as writing $f/H - h = b/B$ where h is the average height above datum of the ground line P_1P_2.

$$\text{If } \frac{f}{H-h} = \frac{b}{B}, \text{ then } B = b \times \frac{H-h}{f} \qquad \text{(6iii)}$$

Thus we can determine the value of all the unknowns in the parallax equation, except h_A. So that we can write

$$p_A = \frac{fB}{H-h_A} \text{ as } H - h_A = \frac{fB}{p_A} \text{ or } h_A = H - \frac{fB}{p_A} \qquad \text{(6iv)}$$

and so find the value of h_A = height of A above datum (AD in Fig. 6.2). This presupposes that we can find a value for B, which depends upon the value of

$$\frac{b(H-h)}{f} \qquad \text{(as in (6iii))}$$

f is known accurately, b and H are estimated arbitrarily, but h depends upon our knowing the heights of p_1 and p_2 above datum—a most unlikely happening. h is therefore usually taken to be the average height of the ground above datum, and $(H - h)$ becomes the height of aircraft above mean ground level, which we are able to determine. Although this introduces an approximation, the error involved is negligible in practice.

Let us assume that we know the height of some ground point whose image falls in the overlap; let this point be E, and its height above datum be h_E. Then—

$$h_A = H - \frac{fB}{p_A} \qquad \text{(6iv)}$$

and

$$h_E = H - \frac{fB}{p_E} \qquad \text{(6v)}$$

Subtracting equation (6v) from equation (6iv)—

$$h_A - h_E = H - \frac{fB}{p_A} - H + \frac{fB}{p_E} = fB\left(\frac{1}{p_E} - \frac{1}{p_A}\right)$$

$$= fB\frac{p_A - p_E}{p_A \times p_E} \qquad \text{(6vi)}$$

It is usual to write $h_A - h_E$ as $\Delta\, h_{AE}$. This is purely conventional, Δ is a Greek letter, capital delta, often used to denote difference. By the same conventional notation $p_A - p_E$ would be written as Δp_{AE}. Thus—

$$\Delta h_{AE} = fB \times \frac{\Delta p_{AE}}{p_A \times p_E} = \frac{fB}{p_E} \times \frac{\Delta p_{AE}}{p_A}$$

But $\quad p_E = \dfrac{fB}{H - h_E} \quad$ i.e. $\quad \dfrac{fB}{p_E} = H - h_E$

Therefore

$$\Delta h_{AE} = (H - h_E)\frac{\Delta p_{AE}}{p_A} \qquad \text{(6vii)}$$

Since E is the point of known height, this will usually be written as

$$\Delta h_{EA} = (H - h_E)\frac{\Delta p_{EA}}{p_A} \qquad \text{(6viii)}$$

This formula will enable us to find the difference in height between A and E, simply by measuring the parallax of both A and E, and estimating the value of H as explained earlier.

Measuring Parallax

We saw in Chapter 3 how, using a bar scale, we could actually measure the x-coordinates of the images on two overlapping photographs of the same ground point, and then by finding the algebraic difference between the coordinates we could determine the parallax of the ground point (see page 75).

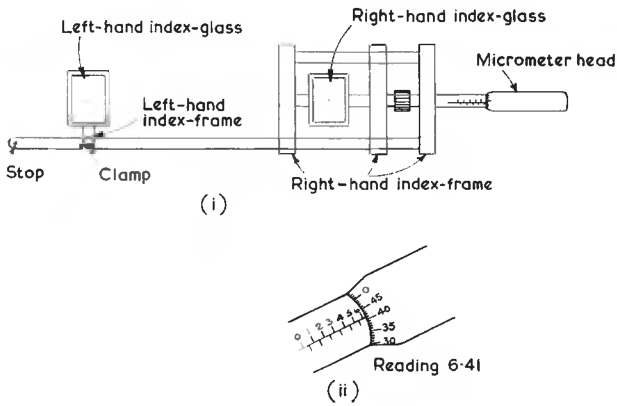

FIG. 6.3. (i) DIAGRAMMATIC SKETCH OF A PARALLAX BAR
(ii) MICROMETER HEAD (ENLARGED)

This type of measurement might enable us to read to the nearest 0·5 mm, but we need a much closer reading than this if we are to obtain ground heights with sufficient accuracy. A special hand instrument has been produced for this purpose. This is known as the *parallax bar* or *stereometer*, and is illustrated in Figs. 6.3 and 3.17. The instrument consists of a bar to one end of which is rigidly attached the right-hand index-frame. The left-hand index-frame is free to move along the bar but can be clamped in any desired position on the bar. Each index-frame contains a dot (or cross) engraved on the underside of a piece of glass or Perspex. The right-hand dot can be moved relative to the bar in a direction parallel with the bar, in such a way that the amount of its movement is recorded on the micrometer head. Thus once the left-hand index-frame is clamped into position we can measure the magnitude of all changes in the spacing between the two dots.

The main micrometer scale, which reads longitudinally to the bar, is graduated in millimetres and half millimetres. The bar bearing this scale is cylindrical in section and fits inside the barrel-shaped micrometer head (see Fig. 6.3 (ii)). It is by turning this head that the right-hand dot is moved in and out. The head itself also moves longitudinally in unison with the dot. One complete revolution of the head causes it to move one half-millimetre along the main scale. The leading edge of the head is equally divided into fifty parts so that the bar may be read directly to one-hundredth part of a millimetre.

If we place this instrument under a stereoscope so that the image of the left-hand dot is visible only to our left eye and the image of the right-hand dot only to our right eye, then by slightly adjusting the spacing of the dots we shall be able to fuse the images stereoscopically. We shall, in fact, see the fused dot floating in space; this is referred to as the *floating mark*. At the same time we must set up a pair of photographs for viewing stereoscopically under the parallax bar. The position now is that we have a view in the stereoscope of the floating mark superimposed on a three-dimensional image of the ground covered by the photographic overlap. Put in another way, we shall see two entirely separate stereoscopic models: one of the photographs, and the other of the dots.

To obtain these two images in satisfactory relationship with each other, base-line the pair of photographs and set them up under the stereoscope as in Chapter 3. Tape the photographs securely to the table, remove the stereoscope and, after bringing the micrometer of the parallax bar to the centre of its run, set the right-hand dot over a definite but very small point of detail on the right-hand photograph. (Some instruments read from zero to 13 mm; in such a case the reading should be set at 6·50 mm approx.) Let this be the image of the ground point *Q*. Slacken the clamp-screw on the left-hand index-frame and move the left-hand dot so that it falls over the left-hand photographic image of the same point of detail, *Q*. Tighten the clamp, making sure that the right-hand dot remains over the appropriate point of detail. This is known as *zeroing the bar*, and the clamp should subsequently remain tight throughout the heighting procedure on this pair of photographs.

Set the left-hand dot over a fairly clear area of ground, and keeping the bar parallel with the eye base, move the micrometer head in and out until fusion of the dots is obtained. Let the actual point chosen be the image of *A*. Now move the micrometer so that the right-hand dot moves inwards, that is to the left. Continue until fusion is lost, then move the micro-head back just enough to regain fusion. The dot should now appear to be floating above the model of the ground. Move the right-hand dot gradually outwards, and watch the floating mark gradually sink down to ground level. If you continue moving the right-hand dot outwards, the floating mark may appear to penetrate the ground surface of the model, though such a phenomenon might be rejected by your subconscious as unnatural. In the latter case fusion of the dots will be lost. At the time at which the floating mark appears to be exactly at ground level, the spacing of the dots is the same as the spacing of the pair of image points a_1 and a_2 (see Fig. 6.4 (ii)). That is to say the parallax of the dots is the same as that of the point of detail A.

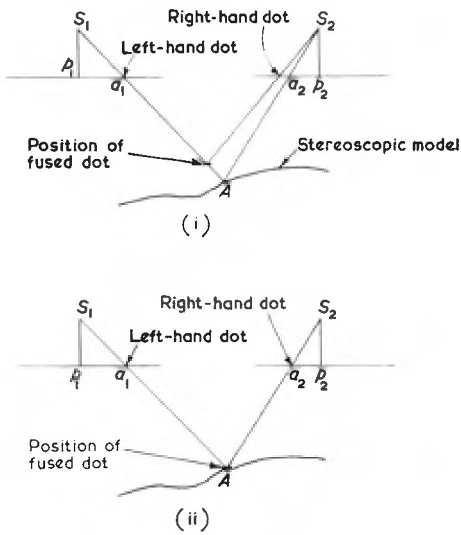

FIG. 6.4. VIEWING WITH A PARALLAX BAR UNDER THE STEREOSCOPE
(i) FUSED DOT APPEARS TO BE FLOATING ABOVE GROUND
(ii) FUSED DOT APPEARS TO BE AT GROUND LEVEL

The parallax bar reading is a measure of the parallax of the point A. In some instruments the readings increase with the parallax, but in others the readings increase when the parallax decreases. One of the first requirements is to find out which type of bar you are using; this can be done before zeroing, by turning the micrometer head so that the readings are increasing slowly. During this movement, watch the right-hand dot—if it is moving to the right the space between the dots is increasing and the parallax, and therefore the height of the floating mark, is decreasing. This instrument is then of the inverse reading type. In a direct reading instrument the right-hand dot would move to the left (decreasing separation) as the readings increased.

Although the bar readings are a measure of the parallax of a point, they do not give the actual parallax.

In Fig. 6.5 the parallax of A, $p_A = x_{1A} + x_{2A} = K - X_A$.

But $$X_A = L - m_A$$

Therefore $$p_A = K - L + m_A \quad . \quad . \quad (6ix)$$

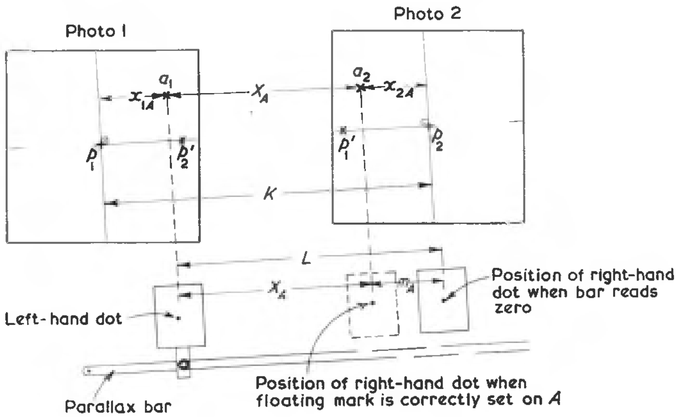

FIG. 6.5. DIAGRAM TO ILLUSTRATE THE RELATIONSHIP BETWEEN BAR READING DIFFERENCE AND PARALLAX DIFFERENCES

K = constant spacing of photographs during heighting operations
X_A = spacing of photo images A
m_A = parallax bar reading when mark is set on A
L = length of bar between dots after zeroing and when reading is reduced to zero.

Similarly it can be shown that for the ground point E,

$$p_E = K - L + m_E \quad . \quad . \quad . \quad (6x)$$

Therefore subtracting equation (6x) from equation (6ix),

$$p_A - p_E = m_A - m_E \quad . \quad . \quad . \quad (6xi)$$

That is to say that the *difference* in parallax between A and E is equal to the *difference* in the bar readings for the two points.

Example

We have a pair of vertical air photographs, both of which contain the image of the ground points A and E. We know the height of E above datum to be 38 m. The flying height has been estimated as 1,562 m, and the mean height of the ground above datum is 34 m. The base lines are 87·2 and 89·2 mm respectively. The focal length of the camera is 152·4 mm and the parallax bar readings are 5·31 for A and 6·12 for E. Find the height of A above datum. The bar is such that an increased reading indicates an increase in parallax.

Solution

$$H = 1{,}562 \text{ m} \qquad m_A = 5{\cdot}31 \text{ mm} \qquad h_E = 38 \text{ m}$$

$$b = \frac{87{\cdot}2 + 89{\cdot}2}{2} = 88{\cdot}2 \text{ mm} \qquad m_E = 6{\cdot}12 \text{ mm}$$

$$f = 152{\cdot}4 \text{ mm} \qquad h = 34 \text{ m}$$

From the parallax equation (6ii)

$$p_E = \frac{fB}{H - h_E} \simeq \frac{b(H-h)}{H-h_E} = \frac{88{\cdot}2 \times (1{,}562 - 34)}{1{,}562 - 38}$$

$$= 88{\cdot}43 \text{ mm}$$

We have shown that

$$\Delta h_{EA} = (H - h_E)\frac{\Delta p_{EA}}{p_A} \quad . \quad . \quad \text{(see eq. 6viii)}$$

But $\Delta p_{EA} = p_E - p_A = m_E - m_A$. (see eq. 6xi)

$$= 6{\cdot}12 - 5{\cdot}31 = 0{\cdot}81 \text{ mm}$$

therefore $p_A = p_E - 0{\cdot}81 = 88{\cdot}43 - 0{\cdot}81 = 87{\cdot}62$ mm

therefore $\Delta h_{EA} = (1{,}562 - 38)\ \dfrac{0{\cdot}81}{87{\cdot}62}$ m

i.e. $h_E - h_A = 1{,}524 \times \dfrac{81}{8{,}762} = 14$ m

i.e. $38 - h_A = 14$ m

i.e. $h_A = 24$ m

Answer: A is 24 m above datum.

This can be summarized in schedule form as follows—

Ground point	Bar readings	Difference in parallax in mm	Parallax in mm	Difference in height in m	Height above datum in m
E	6·12		88·43		38
A	5·31	− 0·81	87·62	14	24

Inaccuracies of Parallax Heighting

In this way the height of any point whose image appears on the overlap could theoretically be found, provided that we know the

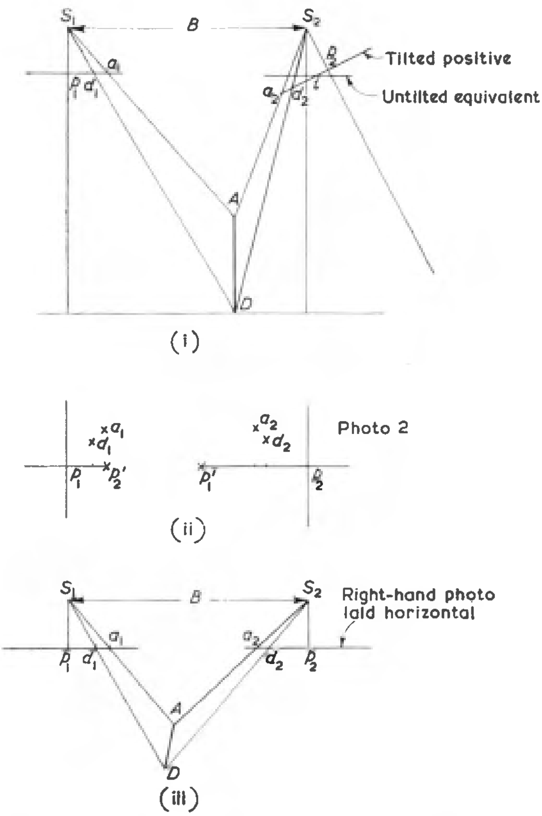

FIG. 6.6. EFFECT OF TILT OF MODEL AS RECONSTRUCTED UNDER A SIMPLE STEREOSCOPE

(i) Light rays at time of exposure. Left-hand photo untilted. *AD* is a vertical chimney.
(ii) Plan view of photos.
(iii) Model of *AD* as reconstructed under a simple stereoscope. Both photos are horizontal; thus the image is distorted because rays through the right-hand photograph are incorrectly reconstructed.

height of any other point on the overlap. However, there are many factors giving rise to errors in the parallax equation, and heights calculated as above are known as crude heights, because they need adjusting in some way before they can be said to give even reasonable results. We have already seen that only an approximate value can be found for the flying height, and an even less accurate value for the length of the air base. In addition, the setting of the floating mark on a particular point of detail is a very skilled operation, and even the expert will require to take the mean of at least five separate readings for each point. Further, there will be errors in the position of the image on each photograph due to tilt, and also to displacements caused by the lens (see Chapter 9). It will be found in practice that the procedure just given will give grossly inaccurate results except for pairs of points which are very close together. Since tilt is the principal cause of these errors, let us examine what happens to the parallax equation when tilt is present in only one photograph of the pair, though we must remember that in practice both photographs are almost certain to be subject to some degree of tilt.

In Fig. 6.6, the diagram (i) is a correct reconstruction of conditions at the time of exposure, with the optical axis S_2p_2 tilted in a fore and aft direction. The diagram (ii) shows the location of the images of the points A and D, and (iii) shows the reconstruction of the stereoscopic image which would actually take place under a mirror stereoscope. Note that under the stereoscope the photographs are necessarily mounted in a horizontal plane. Compare Fig. 6.6 (iii) with Fig. 6.2: a_2 and d_2 in Fig. 6.6 (iii) are displaced relative to their correct position as shown in Fig. 6.2, but a_2p_2 in Fig. 6.6 (iii) is equal to the x-coordinate of a_2 in Fig. 6.6 (ii), and similarly for d_2p_2. The result is that the three-dimensional image of AD is distorted—not only are the absolute depths below eye base (S_1S_2) of the points A and D reduced, but the distance AD is reduced and the line AD is no longer vertical. Using exactly the same arguments as applied to Fig. 6.2, we can show that parallax of A in Fig. 6.6 (iii) is still $p_A = fB/H - h_A$, and that parallax of D is still $p_D = fB/H - h_D$. Thus, the equation

$$\Delta h_{EA} = \frac{(H - h_E)\Delta p_{EA}}{p_A}$$

still holds good, but the difference in height now relates to the erroneous model of Fig. 6.6 (iii), and not to the correct difference in height, nor even to the actual length AD.

HEIGHTING DRILL

Figure 6.6 obviously grossly exaggerates the errors, since a photo having the tilt of photo 2 in 6.6. (i) would certainly be rejected. However, the distortions of the model produced by an acceptable photograph would be sufficient to make the parallax relationships completely unreliable, but there are several methods of correcting the heights derived from differences in parallax. The simplest of the drills involved, assumes that the errors are strictly linear in any direction. That this assumption is untrue will become apparent later, but some reasonable results have been achieved, and we will consider the way in which it has been used.

Suppose we know the heights of the two points *A* and *E*, whose images lie in the overlap between two vertical air photographs, then we can find the heights of points *B*, *C* and *D* lying on the straight line *AE* as follows: First find the crude height of *B* by parallax measurements and calculations as before, and using the known height of *A*. Repeat this for *C*, *D* and *E*. The crude height of *E* will differ from the true height, so that we may readily determine the correction required at *E*; the corrections required at *B*, *C* and *D* can then be calculated as proportional to their distances from *A*. For example, suppose the true heights of *A* and *E* are 30 and 37 m respectively, and that the crude heights of *B*, *C*, *D* and *E* are respectively 34, 29, 23 and 27 m. Let the measured distances on the map be $AB = 10$ mm, $AC = 20$ mm, $AD = 30$ mm and $AE = 40$ mm. Correction required at *E* is +10 m; therefore correction at *B* must be $10 \times 10/40 = +2{\cdot}5$ m, at *C* it is $10 \times 20/40 = 5$ m and at *D* it is $10 \times 30/40 = 7{\cdot}5$ m. Thus the corrected heights of the five points *A*, *B*, *C*, *D* and *E* would be 30, 36·5, 34, 30·5 and 37 m respectively.

L. G. Trorey in his book *Handbook of Aerial Mapping and Photogrammetry* described how he uses an adaptation of the above method for contouring. This method uses heighting drill only along lines perpendicular to the base line; i.e. points *A*, *B*, *C*, *D* and *E* of the above example must not only be collinear, but also lie on the same perpendicular to the base line. Considerable accuracy is claimed for this method, but the requirement of several pairs of points of known height, whose positions are so rigidly governed, must lead to considerable difficulty and inconvenience in the field. Although the heights of height control points will normally be obtained from barometric heighting, each control point has to be accurately recognized on ground and photograph, and has to be visited in the field.

More recently a theoretically sound method of heighting using a parallax bar has been developed. The corrections to be applied to the crude heights are a combination of a parabolic and a hyperbolic correction. A full investigation of the method is reported by Professor Thompson in the *Photogrammetric Record*, Vol. 1, No. 4

Expressed mathematically the method depends on a correction to any crude height of $a_0 + a_1x + a_2y + a_3xy + a_4x^2$, where x and y are the coordinates on the left-hand photograph of the image of the point being heighted, and a_0, a_1, etc. are constants which must be determined by experiment for each overlap. Since there are five unknown constants we must have five height control points per overlap. The solution simultaneously of the five resulting equations is the most tedious part of the drill, and the operation should be carried out according to a strict form of computation: one method is illustrated in Chapter 10, as part of a practical example of contouring.

Once the values of the constant coefficients are known, the appropriate correction at any other point may be ascertained by substituting the coordinates of the point for x and y in the equation

$$h' - h = a_0 + a_1x + a_2y + a_3xy + a_4x^2 \quad . \quad . \quad (6xii)$$

in which h' is the correct height and h the crude height.

The procedure is as follows. First base-line the photographs, choose the five control points and plot them on the left-hand photograph. Ideally these points should be located one in each corner of the overlap and one in the middle, but the rules to remember are that no four of these points should be in a straight line and no three of them should lie on the same perpendicular to the base line. The heights of these points must now be obtained by ground methods, usually by barometric heighting.

Now fix a transparency to the top edge of the left-hand photograph, and with this transparency folded down over the photograph draw in the X and Y axes as shown in Fig. 6.7. The centre of the base line is chosen as the origin, the X axis is drawn to overlie the base line, and the positive direction of the Y axis is downwards. A grid is now drawn on the overlay with lines at 20 mm intervals, and based on the new axes.

Coordinates of any photo-point can be read off the grid to the nearest 2 mm.

Carry out the heighting drill for each point in turn using say point No. 2 as datum; find the crude heights of the other four points. Now in equation (6xii) make substitutions for each of the control points in turn. $h' - h$ will be the difference between the crude height and the known correct height: in the case of point No. 2, this will be zero. In the right-hand side of the equation we must substitute the X-coordinate of the point for x, and the Y-coordinate for y. The solution of the five equations so formed gives us numerical values for a_0, a_1, a_2, a_3 and a_4.

To find the height of any other point q on the overlap, we must first carry out the heighting drill with the parallax bar, again using point No. 2 as datum; this gives us h in the correction equation. We then measure the coordinates of q and substitute these in the right-hand side of the equation, making a full numerical determination of this side of the equation. Thus we can find h', the corrected height of the point q.

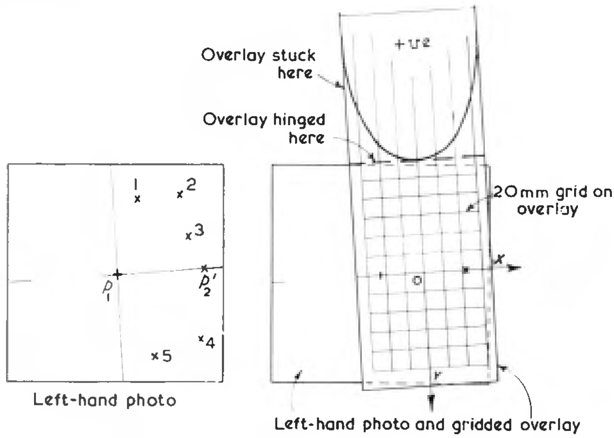

FIG. 6.7. HEIGHTING DRILL—THE GRIDDED OVERLAY

This drill could be followed to find the height of any point whose image falls on both the photographs under investigation; but the determination of contours calls for the heighting of so many points that it is usually advisable to draw two sets of graphs on to the overlay (see Fig. 6.7).

The parabolic graph at the top of the overlay is plotted by substituting a series of arbitary values for x in the equation $Y = a_4x^2$, where the value of a_4 has already been calculated. Y is the parabolic correction to apply. and is represented by the ordinate of the graph.

The series of graphs across the face of the photograph (see Fig. 10.5) represent the hyperbolic correction and are obtained from the equation

$$Hyp = a_0 + a_1x + a_2y + a_3xy$$

where a_0, a_1, a_2 and a_3 have already been determined. Hyp is the hyperbolic correction, and each separate curve is a rectangular hyperbola drawn through points having the same hyperbolic error.

If x remains constant, the right-hand side of the equation is linear in y, and if y remains constant that side of the equation is linear in x. Therefore we may adopt linear interpolations between the curves in directions parallel with either the X-axis or the Y-axis. The curves themselves can be very simply drawn in by finding mathematically the hyperbolic correction for the four corner points, and making successive linear interpolations between these four points.

The total correction to the crude height for each point will then be

$$Hyp + Y = a_0 + a_1x + a_2y + a_3xy + a_4x^2$$

as before. That is the sum of the parabolic and the hyperbolic corrections.

The network of points whose heights are required for interpolation of the contours is now plotted on the photograph. The crude height of each point is determined by parallax bar, and corrected heights are found by reading off the corrections from the graphs. The contours are finally drawn in on the photograph itself, using the stereoscopic model to help maintain correct contour heights between successive interpolated points.

Some Instrument Refinements

The main practical difficulties involved in using the parallax bar concern the maintenance of the bar parallel with the eye base and photo base, and also the bringing of the floating mark exactly to the " ground level " of the model.

A parallel guidance mechanism (see Fig. 6.8) solves the problem of parallelism. This mechanism consists of a base on which is mounted an x-carriage, free to move in only the x-direction. A y-carriage is mounted on this x-carriage, and is free to move only in the y-direction. The y-carriage carries both the stereoscope and the parallax bar, mounted parallel with each other and with the direction of movement of the x-carriage. The two prints are mounted on the base of the mechanism, and oriented so that, with the y-carriage clamped, the right-hand dot can be made to pass accurately along the base-lines of both photographs. The separation of the prints is set to give comfortable fusion.

The difficulty of fusing the dots when viewing points having appreciable wants of correspondence is reduced by making the left-hand stereometer dot adjustable in the y-direction.

Further help in reducing the floating mark to the model surface is obtained by viewing through a pair of magnifiers, which resemble field glasses, placed in front of the eye-pieces (see Fig. 3.17). One effect of this attachment is to restrict the field of vision, which is often a severe disadvantage; but by making each image point and both dots appear bigger, more exact placing of the floating mark becomes possible. As experience is gained, magnification will allow a smaller dot (or other shape of reference mark) to be used, which will cover a smaller area of ground and so increase the accuracy of heighting.

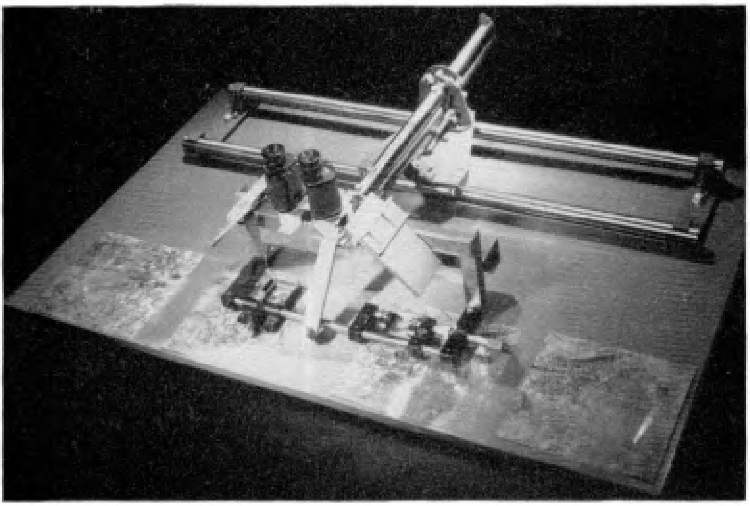

FIG. 6.8. PARALLAX BAR AND STEREOSCOPE WITH PARALLEL MECHANISM
(*C. F. Casella & Co. Ltd.*)

Since the magnifiers restrict vision to paraxial rays, the horizontal scale of the model will increase proportionally with the magnification. The effect on the vertical scale will depend upon the interpretation that the brain puts on the apparent enlargement of two flat pictures, both of which apparently remain at a great distance from the eyes—this will be complex, but will certainly not result in so great an increase in the vertical direction as in the horizontal. It is generally accepted that magnification decreases the impression of excessive gradients, though this is difficult to prove—it is probably almost entirely due to the greater apparent increase in horizontal relative to vertical scale.

In the Hilger and Watts folding mirror stereoscope, the reference marks of the parallax bar are replaced by two spots of light which are introduced into the viewing system, and can be fused to form a floating mark in relation to the stereoscopic model formed from the pair of photographs. Want of correspondence can be removed by a y-adjustment of the left-hand spot. The separation of the spots, and therefore the apparent height of the floating mark, can be adjusted by an x-movement of the right-hand spot. This x-movement is

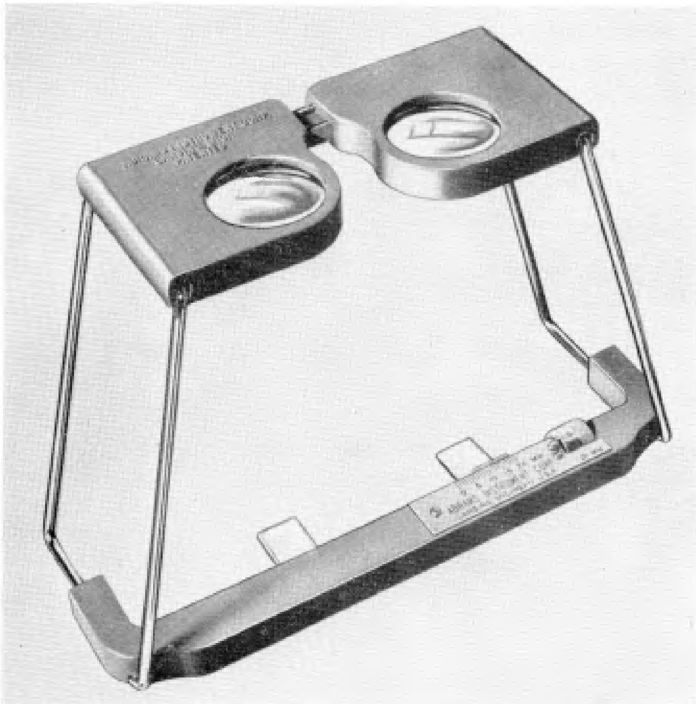

FIG. 6.9. ABRAMS HEIGHT FINDER ATTACHED TO POCKET STEREOSCOPE
(*Abrams Instr. Corp.*)

operated by a micrometer head, similar to the one illustrated in Fig. 6.3, which enables readings direct to 0·01 mm. The photographs are mounted on a carriage which is attached to the stereoscope by means of a pantograph type of parallel-guidance mechanism. Scanning is then carried out by a controlled movement of the photographs, thus avoiding the necessity for the operator to move his head. The brightness of the spots of light can be adjusted by a rheostat control, and the floating mark seems a little easier to bring to "ground" level than does that of the parallax bar. However, the main advantage is that there are no bar frames to interrupt the view of the photographs or to damage the emulsion by physical contact.

The binocular unit gives a magnification of 4×.

Figure 6.9 illustrates a pocket-size parallax bar which clips on to a stereoscope. It is especially useful for work in the field, including interpretation, and is shown attached to a pocket stereoscope.

In Fig. 6.2 we have seen that $p_D = fB/H$ (eq. 6i), so that if f remains constant, p_D varies directly as B/H. Fig. 3.13 is a similar figure, and we have shown that as H increases so the appreciation of variations in ground height decreases, whereas an increase in B has the effect of increasing depth perception. The B/H ratio is then a measure of the appreciation of stereoscopic depth.

FURTHER READING

(i) *Admiralty Manual*, pages 285–98.
(ii) Hart, pages 203–35.
(iii) Moffitt, Chapter 8.
(iv) Trorey, pages 56–92, and, using obliques, pages 37–55.

(See Bibliography (page 346) for full titles.)

The following articles in the *Photogrammetric Record* suggest that simple stereometer heighting accuracies can be increased by further refinements—

1. "Corrections to *x*-parallaxes", by E. H. Thompson (October 1968).
2. "Heights from parallax bar and computer", by B. D. F. Methey (April 1970).