Chapter 3

Stereotopographic Mapping

16. The Stereoscope. Nearly everyone has looked through a stereoscope at one time or another; the old parlor stereoscope was common in every home at one time. It consists of two eyepieces with magnifying lenses and a carrier for two photographs. When these photographs are viewed the objects therein stand out in relief, that is, we not only see vertical and horizontal distances but are able to perceive depth.

In order to get stereoscopic vision the two photographs must be taken from two different viewpoints. In operating the stereoscope the right-hand view is seen with the right eye and the left-hand view with the left eye. These two views are merged together to form one picture in which length, height, and depth are clearly visible.

17. Stereoscopic Vision. The phenomenon of binocular or stereoscopic vision is exceedingly complex and investigators do not yet agree entirely as to its explanation. The following is an explanation of some of the simple principles, which are the only ones in which we are particularly interested.

In binocular vision there are two things which enable a person to judge distances.

1. The relative position and size of the objects.
2. The stereoscopic principle of parallax.

In Fig. 28 assume that E and E' represent the position of the two eyes of an observer, the curved arcs below E and E' the retina of the two eyes. In viewing a distant object O the retina of the left eye is impressed at o and the right at o', and therefore O is at infinite distance. Now assume that a nearer object is viewed, O_n being a near corner and O_f a far corner. The retina of the left eye is impressed at o and the right eye at o_f' for the far corner and o_n' for the near corner. The angles β_1 and β_2 are the angles of parallax for the two objects and α the differential parallax, expressed by the following equation:

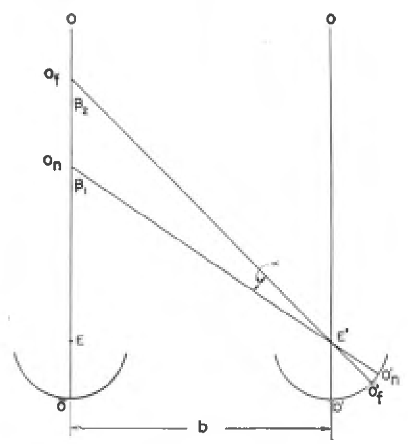

Fig. 28. Binocular Vision

$$\alpha = \beta_1 - \beta_2$$

Certain physical conditions limit the distance from which stereoscopic vision can be perceived.

1. When the angle β_1 becomes less than about 20 seconds the observer no longer perceives the spacial relation of points. This is the smallest angle which the eye can sense.
2. The normal distance between the human eyes is $2\frac{1}{2}$ inches.

From these limitations the distance E–O is about 2100 feet. This means that beyond 2100 feet we no longer get stereoscopic vision with the naked eye. It is true that we judge distance beyond this limit but we do so by the relative size and position of objects, light, and shadows.

It is possible to increase the distance of stereoscopic vision in two ways:

1. By increasing the base distance b.
2. By magnifying the field of view.

This principle is employed in the ordinary binoculars. By means of prisms the objective lenses may be extended so that they are 5 inches or more apart. Also the magnification of the image is accomplished by means of the lenses. If the base is increased to 5 inches the angle of parallax is increased by an amount equal to the ratio of 5 to 2.5 or 2 times. If the magnification is 4, the stereoscopic vision is increased $2 \times 4 = 8$ times.

18. Application of the Stereoscope to Mapping. In terrestrial photographic surveying, as discussed in Chapter 2, it was necessary to have base lines laid out and angles measured to numerous points. Surveyors using this method frequently experience difficulty in identifying the same feature in different photographs, and at times the intersection angle may be so small as to introduce errors in the graphical intersection of points.

These difficulties are eliminated by the use of a stereoscope. The stereocomparator (described later) was devised by Dr. Carl Pulfrick of Germany to this end.

Other instruments using the principle of the stereoscope were devised in subsequent years until today there is hardly a photographic survey made where the stereoscope is not used in some form.

The following is a list of some of the instruments and the dates of invention which use the stereoscopic principle.

Date	Name of Instrument
1907	Modification of the stereocomparator by Lieut. F. V. Thompson.
1908–1910	Stereoautograph by Lieut. von Orell and Carl Zeiss.
1919	Autocartograph.
1922	Stereoplanigraph.
1922	Stereotopometer.
1924	Autograph.
1925	Stereotopograph by M. Poivilliers.
1925	Photocartograph of Italian origin.
1925	Photogrammetric plotter by Dr. Archibald Barr and Dr. William Strout, with modification by H. G. Fourcade and called stereogoniometer.

Improvements and additions have been made to some of these instruments. They are mentioned here simply to give some idea of the extent to which the stereoscope has been used. The latest development and the one which has been used to considerable extent in the United States is the *aerocartograph*—an automatic plotter using two photographs in stereoscopic fusion and the outgrowth of some of the earlier plotting instruments. Illustrations of this instrument are shown in Figs. 29 and 30.

The following explanation of the stereoscopic principle is reprinted * here by permission of Mr. E. R. Polley, vice president of Fairchild Aerial Surveys, Inc.

This principle can best be explained from the standpoint of the familiar operations of an ordinary ground survey.

The fundamental principle upon which measuring stereoscopes operate is simply triangulation. The principle remains the same regardless of whether terrestrial photographs or aerial photographs are used, but it is demonstrated most easily by reference to terrestrial photographs.

Imagine that a survey by triangulation is contemplated of a dam site, such as that shown in Fig. 31. Many points on the north side of the canyon are to be located by triangulation from a base line on the south side. For example, assume point X to be any point on the surface of the ground.

The ordinary procedure would be first to lay off a base line at a convenient location on the south bank, possibly 100 ft. in length. Then angles would be turned successively from the extremities of this base line to the various points (of which point X is an example), and the positions of each point would be calculated from the resultant data.

Next, assume that two transits are set up simultaneously at the extremities of this base line and that by a system of prisms the transitman stands in the middle of the base line, looking through the right transit with his right eye and the left transit with his left eye. Thus, the transits, when directed to the same object, give an effect similar to that observed through a pair of binoculars. The observer is not conscious of looking through two separate instruments. He enjoys a marvelous capacity for relief perception, because effectively his eyes are separated 100 ft. instead of 2½ in., as is the case when observing with the unaided eye. To understand the remarkable stereoscopic effect he secures by this means, consider similar effects from everyday experience.

When looking through a pair of binoculars, human vision and depth perception (stereoscopic vision) are greatly improved for two reasons: First, the binoculars give a high degree of magnification; and, second, they make it possible to judge differences in distance with much greater accuracy. This latter phenomenon exists because the binoculars increase the base line of the eye from about 2½ in. to possibly

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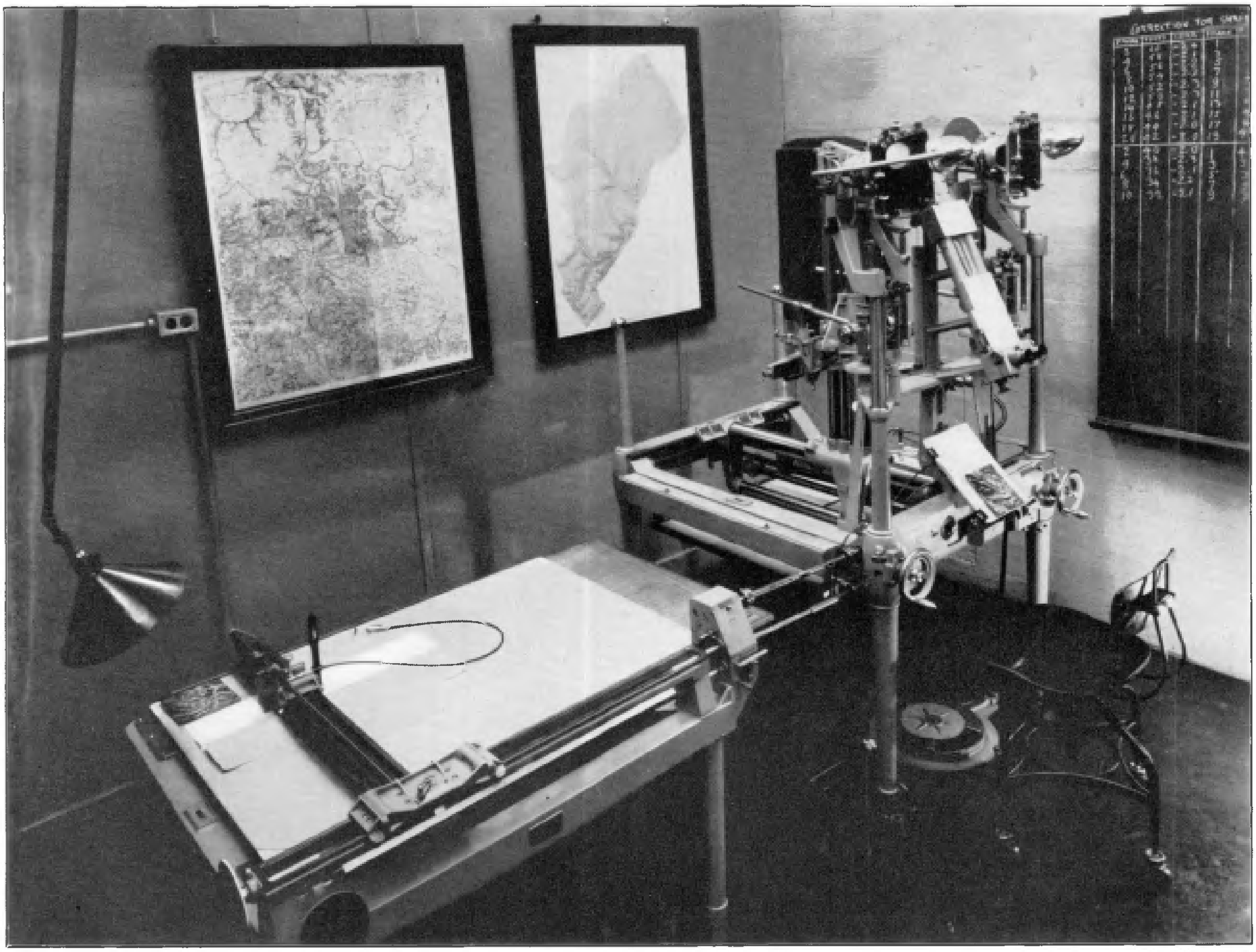

FIG. 29. The Aerocartograph.

5 in. This practically doubles the accuracy with which relative distances can be judged. Therefore, a superlative effect can be secured, if, instead of looking through binoculars with a 5-in. base, the observer looks through binoculars with a 100-ft. base, such as is secured with the two transits. The transitman perceives the most minute differences in distance. If one of two objects is only a few inches closer than the other, they appear to be separated definitely. This ability to judge easily slight differences in distance is effective up to distances of 1000 to 2000 ft., with transits separated 100 ft.

The two cross-hairs of the transits appear to merge into a single cross-hair, the center of which, lying in the lines of sight of the two instruments, appears to rest exactly at the point of intersection. In fact, the cross-hairs appear to stand in space as a vertical grid, the center intersection of which appears to rest exactly at the point of intersection. If the two transits are directed at point X, the center of the grid (the cross-hairs) will appear to rest exactly on that point. Below this, the grid appears to have receded under the ground. Now, if the convergence of the two transits is increased slightly, so that the intersection occurs perhaps 1 ft. nearer the base line, the transitman will still see point X stereoscopically. The only change which this increase in the convergence angle of the two transits causes is an apparent movement of the vertical grid (the cross-hairs) so that it appears to stand nearer the transitman than point X. If the angle of the two transits is diverged so that the point of their intersection occurs behind point X, the grid will appear to have receded into the ground. To the observer it will appear as if the side hill were composed of transparent jelly with a grid of wire appearing first in front of the surface of the jelly and subsequently buried in it.

In similar fashion the transits can be directed at any point or object and the apparent coincidence between this object and the cross-hair grids will permit precise setting of the transits on this object.

Next, assume that the observer replaces each of the transits temporarily with a camera and takes a picture with the center of the camera lens exactly at each transit station.

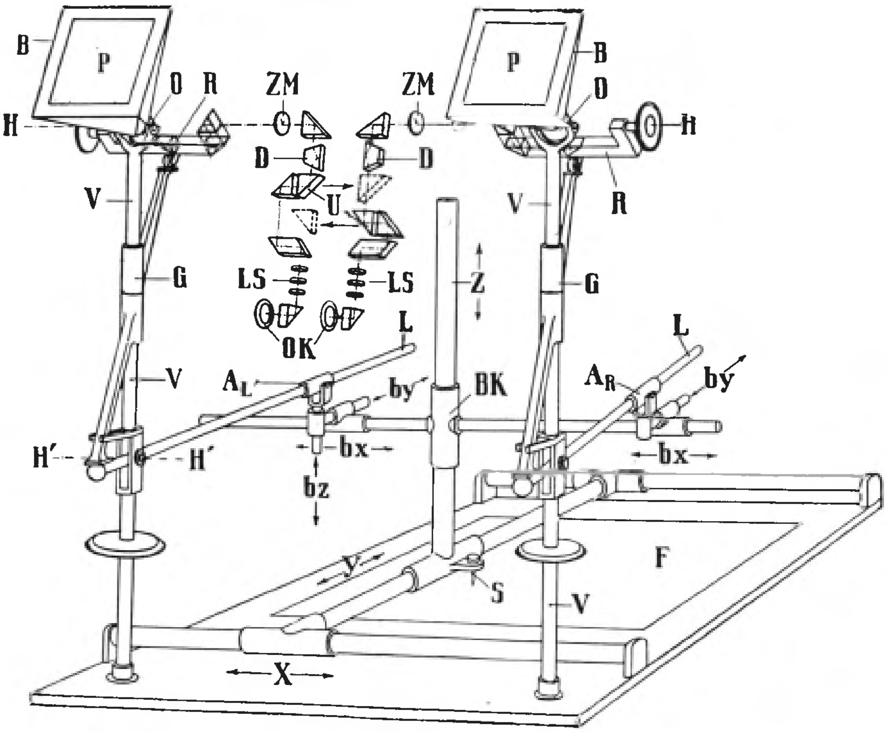

Fig. 30. The Aerocartograph (the Arrangement of Prisms and Lenses).

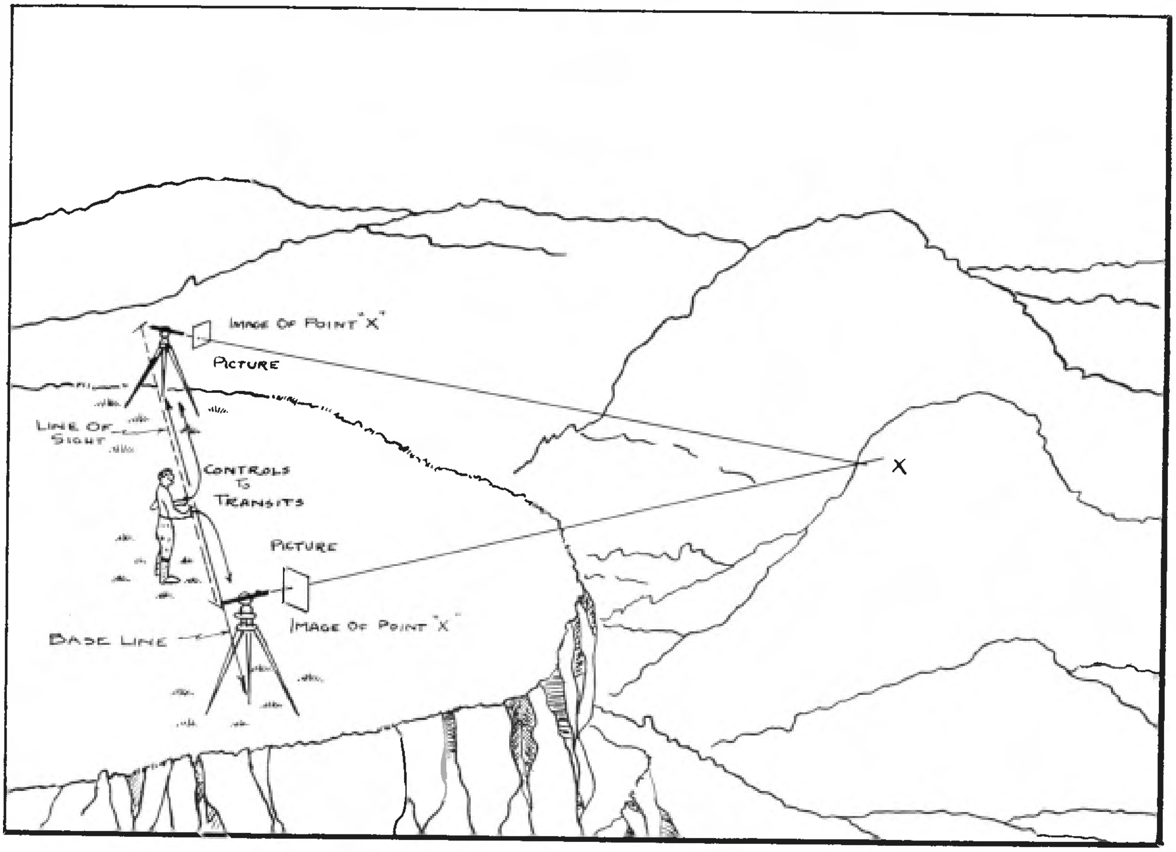

Fig. 31. Principle of Aerocartograph as Applied to Ground Survey.

Then, assume that he replaces the transits and supports the pictures directly in front of them, taking extreme care to make the distance between the pivot point of the transits and the principal point of the photograph precisely the same as the focal length of the camera that took the pictures.

Now, if the observer directs the transit at the tiny dot on the picture, that is the photographic reproduction of the point X, and then removes the photograph from in front of the transit, he will find the transit pointing directly at point X on the hillside. Therefore, the observer can direct his transits to point X, or to any other point he wishes, by observing the corresponding image point on one photograph through one transit and the same point as shown on the other photograph through the other transit.

The application of the stereoscopic principle in the measuring stereoscope might be described very broadly by stating that the pictures are brought to the office and set up before the two observing systems of the aerocartograph instead of in front of two transits.

The foregoing description refers to phototheodolite or ground pictures for which length of base and angles at which pictures are taken are known. Therefore, it is relatively simple to reproduce the relative positions and separation (all to scale) in the aerocartograph. In taking aerial pictures, the camera is supported by a rapidly moving airplane; the base line is the distance traveled by the airplane in the interval between exposing two photographic plates. This distance is not measured; camera angles are unknown. Therefore, it is necessary to know the elevation and horizontal positions of several points on the ground photographed in order that the base-line distance and camera angles can be reproduced in the aerocartograph. There is no other essential difference; the fundamental principle is simply photographic triangulation regardless of whether aerial photographs, ground photographs, or combinations of the two are used.

19. Fundamental Principles (Stereoscopic Parallax).

Horizontal Distance

Assume that two photographs are taken of a range of hills and are represented in Fig. 32 as photograph 1 and photograph 2. The base distance B is known and f, the focal length of the camera lens.

To find the distance to the point A. By similar triangles

$$\frac{D}{B} = \frac{f}{p} \quad \text{or} \quad D = \frac{Bf}{p}$$

From this equation we can find D because B and f are known and the value of p can be measured from the photographs.

$$p = a_1c_1 + c_2a_2$$

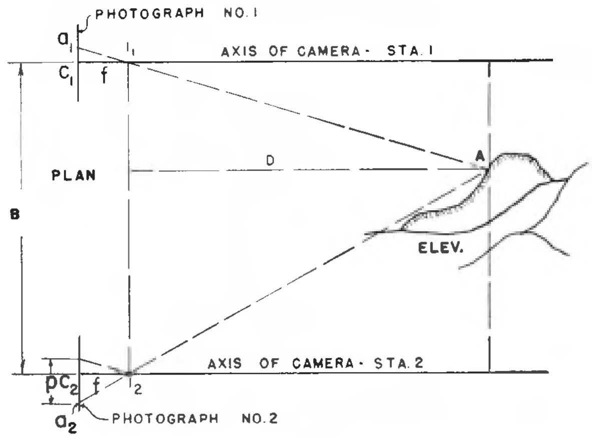

Fig. 32. Horizontal Distance by Stereoscopic Parallax.

where p is called absolute parallax (not to be confused with the parallax of Art. 17).

Difference of Elevation

In Fig. 33 choose photograph 2 and use the plate positive instead of the negative. This is similar to

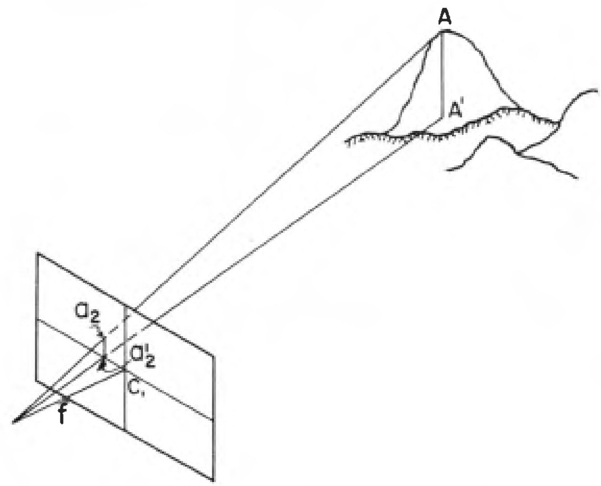

Fig. 33. Graphical Relation for Difference of Elevation.

the condition used in Art. 9 and Fig. 25. It is possible to measure the x and y coordinates of point A from the photograph.

Figure 34 shows the construction necessary to determine the required information.

Referring to Fig. 34.

1. Draw a line Ic_2 equal in length to f, the focal length of the camera lens (actual length). If the camera lens has a focal length of 12 inches lay out 12 inches.

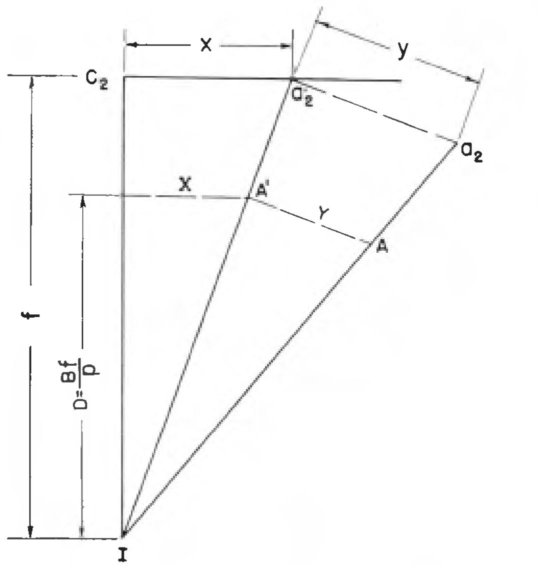

Fig. 34. Graphical Solution for Difference of Elevation.

2. Measure x on the photograph and lay off as c_2a_2' and perpendicular to c_2I.

3. Draw Ia_2'. This fixes the direction of the ray of light to A'.

4. Erect a perpendicular to Ia_2' at a_2 and on this lay off the distance y (measured from the photograph).

5. Draw Ia_2. This will fix the direction of the ray of light to A.

6. From the point I lay off $D = \dfrac{Bf}{p}$ and erect a perpendicular intersecting Ia_2' at A'. This locates the horizontal position of A on the map and gives the value X.

7. Erect a perpendicular at A' to Ia_2' intersecting Ia_2 at A. The distance $A'A$ is the difference of elevation between I and the point A and is designated by Y.

This process can be repeated for any number of points to be plotted.

20. The Stereocomparator. From the preceding article it is seen that the location of a point depends on B (the base line), f (the focal length), and p (the absolute parallax), and x and y.

The measurements x, y, and p can conveniently be measured in an instrument called a stereocomparator. Then referring to Fig. 34, we see that the following equations will locate any point desired:

$$D = \frac{Bf}{p}$$

$$X = \frac{Bx}{p}$$

Note:

$$\frac{X}{D} = \frac{x}{f} \text{ or } X = \frac{Dx}{f}, \text{ but } \frac{D}{f} = \frac{B}{p}, \text{ therefore } X = \frac{Bx}{p}.$$

$$Y = \frac{By}{p}$$

$$\textit{Note: } \frac{Y}{y} = \frac{IA'}{Ia_2'} = \frac{D}{f} = \frac{B}{p} \text{ or } Y = \frac{By}{p}.$$

The stereocomparator as used in practice is a precision instrument, made to take very fine measurements of x, y, and p usually by means of micrometer attachments. The principle, however, can be illustrated by a line diagram such as Figs. 35*a* and 35*b*.

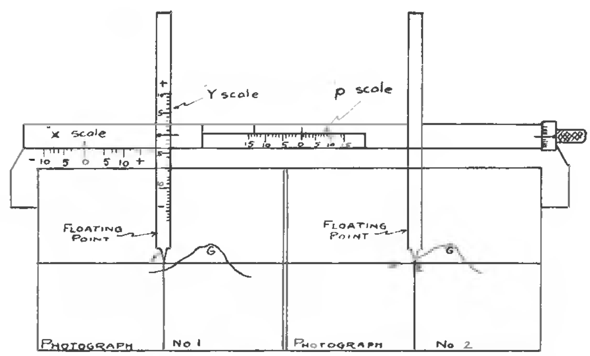

Fig. 35*a*. Stereocomparator.

In Fig. 35*a* is shown a frame holding two overlapped photographs taken from two different viewpoints. These are viewed through a pair of magnifying lenses and are so adjusted that they appear as a stereoscopic pair. On the top of the frame holding the photographs is a movable x scale which can be moved in a horizontal direction. The y scale which is held in a slot in the x scale can be moved in a vertical direction. This y scale is attached to the left-hand "floating point." The right-hand floating point is also held in the x scale frame and is moved horizontally when the x scale is moved; but in addition it can be moved independently in both horizontal direction and vertically.

Referring now to Fig. 35*a* and remembering that the observer is looking at the two photographs stereoscopically, we find that he sees only one picture.

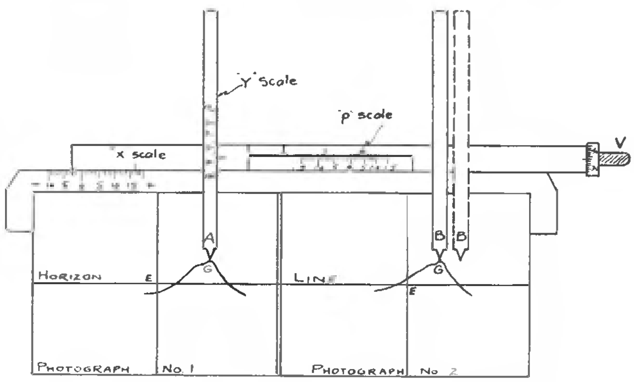

Fig. 35*b*. Stereocomparator.

Furthermore, he sees only one floating point because the floating points are fused together owing to the stereoscopic vision.

Assume that when the floating points are at the origins of coordinates and the floating mark rests on a distant point E in the stereoscopic model (an unusual condition in practice but used here to simplify the discussion), as shown in Fig. 35*a*, the scales all read 0.

Now let us determine the location and elevation of the top of the hill shown. We need to measure x, y, and p to use in the set of equations for D, X, Y determined in the first part of this article.

While sighting through the stereoscope move the frame carrying the scales horizontally and to the right until the floating points (only one as the observer sees, but really two) are in line with the top of the hill. The points will travel along the horizon line. The value of x is read from the x scale.

Next move the floating points vertically until the points are in line with the top of the hill. The value of y is read from the y scale. The floating points will be in the positions A and B.

As the observer sees the merged floating point it will appear to be in line with the top of the hill but back of it. The floating point can be brought forward and in contact with the top of the hill by moving the right-hand floating point toward the left. This is accomplished by turning the vernier V. In this operation the floating point B is moved to B'. The amount that it is moved is recorded on the p scale. This is called absolute parallax. Thus we have measured x, y, and p. The value D determined will be the distance in space (depth) from E to G. (See Fig. 35*b*.)

Parallax differences are usually employed as the difference in position of two points is desired. The symbol used for differences is Δd. (See Fig. 58, page 50.) The value Δd is determined by taking a reading on the p scale for one photographed point and then a reading on the same p scale for a second photographed point. The difference in the two readings is Δd.

If terrestrial photographs are being used and the camera axis is horizontal, Δd corresponds to the difference in horizontal distance. If aerial photographs are being used and the camera axis is vertical, Δd corresponds to the difference in elevation of the two photographed points.

21. Demonstration of Stereoscopic Fusion. The following simple demonstration will illustrate the stereoscopic fusion of points. Take a card and on it place two black dots as shown in Fig. 36.

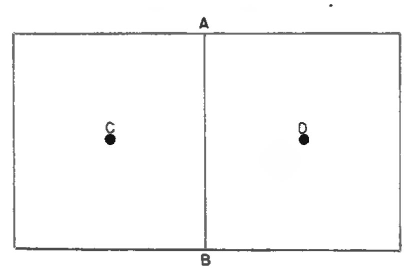

Fig. 36. Experiment with Two Dots.

Next, place a second card perpendicular to the first as at A–B. Hold this combination directly in front of the observer's two eyes, so that the right dot is seen with the right eye and the left dot with the left eye. Now imagine that you are looking through card 1. The two dots will slide in toward each other until they are merged into one single dot.

Figure 37 illustrates the geometric condition of this experiment. The two dots have been brought together and appear to the observer to be at the point *O* back of the card on which they were drawn. This is stereoscopic fusion. The depth or distance to *O* back of card 1 will vary with the spacing of the dots.

Now let us take two pairs of dots as in Fig. 38.

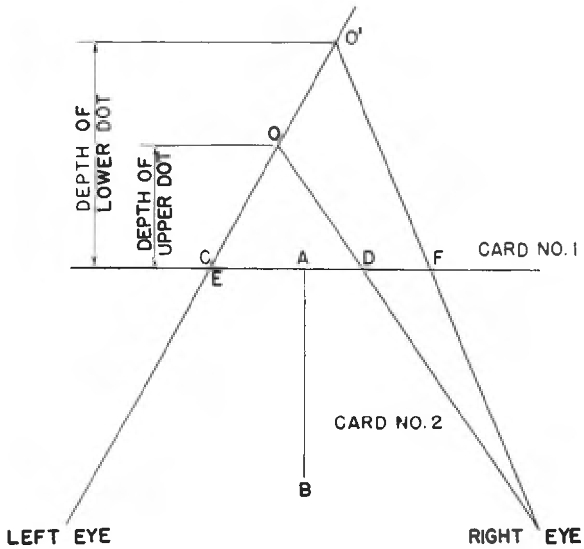

Fig. 37. Graphical Condition with Experiments.

The dots *C* and *D* are closer together than the dots *E* and *F*. As before, a second card is placed perpendicular to the first and the observer views the dots. The dots *C* and *D* will merge together and the dots *E* and *F* will merge together but the top dot will appear closer to the observer than the lower dot.

Figure 39 illustrates geometrically the fusion which takes place and why the dots *C* and *D* appear at *O* and the dots *E* and *F* at *O'*. Thus the spacing of the dots is a measure of the depth. This explains why the floating point in the stereocomparator can apparently be moved toward and away from the observer by shifting the right-hand point-

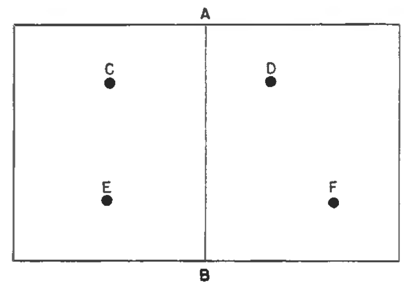

Fig. 38. Experiment with Four Dots.

er. In bringing the floating point in contact with the top of the hill (as in Art. 20) we might consider this pointer moved from *F* to *D* and in doing so the floating point has been brought toward the ob-

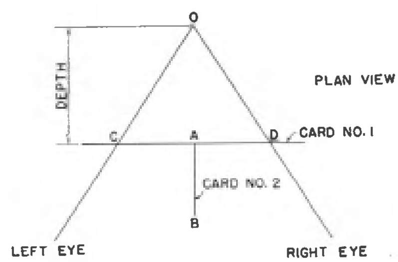

Fig. 39. Fusion of Two Dots.

server from *O'* to *O*. The distance *D–F* is the parallax difference and is measured on the *p* scale by first taking a reading on the *p* scale for the point *O* and then a reading for *O'*. The difference between the two readings is Δd, the parallax difference.