Chapter 5

Displacement Due to Topographic Relief and Tilt

31. Displacement. Conditions of Art. 28 were ideal, that is, the ground was level and the plate horizontal. This condition practically never exists. The errors which are bound to occur in a photograph are

(1) Displacement due to topographic relief.
(2) Displacement due to tilt.

32. Displacement Due to Topographic Relief. In Fig. 43 let $A - B$ be the ground relief with respect

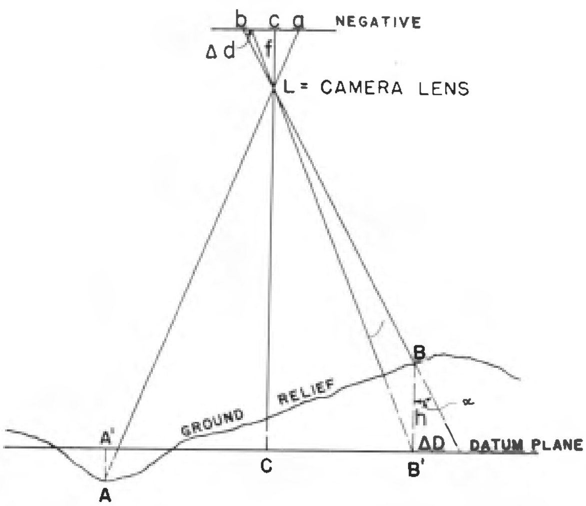


FIG. 43. Effect of Topographic Relief.

to the datum plane shown, point A being below the datum plane and point B above the datum plane. The camera lens is at L and the photographed positions of A and B are at a and b on the negative.

Since maps are orthographic projections, B would be projected on a plane at B' and A at A'. The positions of these points as photographed will be displaced. The point B is displaced away from the center C and point A toward the center C.

The amount of this displacement is given by the following equation:

$$\Delta D = h \tan \alpha \text{ or on the plate } \Delta d = \frac{h}{S} \tan \alpha$$

As α varies with the distance from the vertical axis, the displacement varies with this distance and the ground elevation h. From this it is seen that the farther away from the center of the photograph, the more the displacement. This would indicate that the central portion of a photograph is the least distorted, which is one reason for using overlapped negatives, because the edges can be discarded.

If we know h, the elevation of B from actual levels, and α, which can be figured from

$$\tan \alpha = \frac{bc}{f}$$

where bc is measured from the photograph.

The amount of displacement is given by

$$\Delta D = h \tan \alpha$$

The following example will show the amount of displacement when the area presents marked relief.

Assume: The flight is made at 10,000 ft. altitude; focal length of camera lens = 10 in., elevation of point A below datum = 520 ft.

Referring to Fig. 43 measure the distance a-c on the negative, which is found to be 2.5 in.

$$\text{The scale} = \frac{H}{f} = \frac{10{,}000}{10} = 1000 \text{ ft. per in.}$$

The horizontal distance on the ground to point A is

$$1000 \times 2.5 = 2500 \text{ ft.}$$

$$\tan \alpha = \frac{ac}{f} = \frac{2.5}{10} = 0.25$$

$$\Delta D = h \tan \alpha$$

$$\Delta D = 520 \times +0.25 = 130 \text{ ft.} \quad \text{(displacement)}$$

This shows that in a distance of 2500 feet the point is displaced 130 feet under the above assumed conditions and is displaced toward the center. If the point were above the datum plane 520 feet, it would have been displaced outward from the center by a like amount.

33. Difference in Parallax. Assume now that two overlapped photographs are used and placed

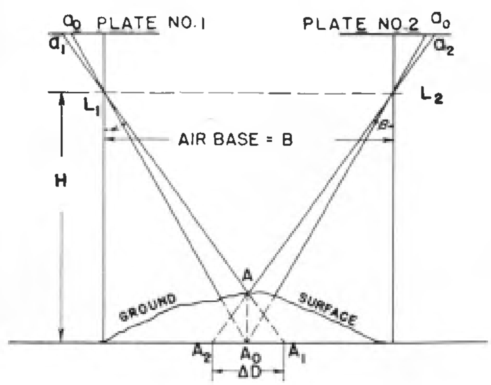


FIG. 44. Difference in Parallax.

in the positions shown in Fig. 44. Let a_1a_0 in plate 1 = Δd_1 (the displacement in the first photograph) due to elevation h and a_0a_2 in plate 2 = Δd_2 (the displacement in the second photograph) due to elevation h.

From Art. 32,

$$\Delta d_1 = \frac{h}{S} \tan \alpha \text{ and } \Delta d_2 = \frac{h}{S} \tan \beta$$

The total displacement Δd on the plate or negative is therefore the sum of Δd_1 and Δd_2.

$$\Delta d = \Delta d_1 + \Delta d_2 = \frac{h\,(\tan \alpha + \tan \beta)}{S}$$

Let B = the distance between exposure stations L_1 and L_2, called the air base. Then by similar triangles

$$\frac{B}{D} = \frac{H - h}{h}$$

Let ΔD_1 = the displacement on the ground $A_0 - A_1$, and ΔD_2 = the displacement on the ground $A_0 - A_2$. Then

$$\Delta D = \Delta D_1 + \Delta D_2$$

But $\Delta D_1 = (\text{scale}) \times \Delta d_1$, and $\Delta D_2 = (\text{scale}) \times \Delta d_2$.

Let scale $\frac{H}{f} = S$. Therefore

$$\Delta D = S\,(\Delta d_1 + \Delta d_2)$$

or, since $\Delta d = \Delta d_1 + \Delta d_2$,

$$\Delta D = S \Delta d$$

Substituting ΔD in the above equation we have

$$\frac{B}{S\Delta d} = \frac{H - h}{h}$$

or

$$\Delta d = \frac{Bh}{S(H - h)}$$

Substituting $S = \frac{H}{f}$, we have

$$\Delta d = \frac{f}{H} \times \frac{Bh}{H - h}$$

From this equation the difference in displacement can be computed for a given elevation or, conversely, the elevation can be determined if the displacement is accurately measured on the photograph.

This latter displacement can be measured in the stereoscopic treatment of a pair of overlapped photographs such as in the stereocomparator. (Principle described in Art. 20.) To illustrate, assume the following problem:

Let the air base B = 2500 ft., the altitude H = 10,000 ft., the focal length f = 10 in. Therefore

$$S = \frac{H}{f} = \frac{10{,}000}{10} = 1000 \text{ ft. to the in.}$$

The parallactic displacement measured from the two negatives is 0.147 in. and outward from the center. To determine h, the height above the datum plane (because the displacement is outward), take the equation just derived, $\Delta d = \frac{Bh}{S(H - h)}$, and solve for h.

$$\Delta dSH - S\Delta dh = Bh$$

$$h(B + dS) = dSH$$

Then

$$h = \frac{\Delta dSH}{B + \Delta dS}$$

Substituting the values of the problem, we have

$$h = \frac{0.147 \times 1000 \times 10{,}000}{2500 + 0.147 \times 1000} = 554 \text{ ft. approx.}$$

When a stereocomparator or stereocomparagraph is used to measure the difference in parallax, the above equation may be used to find the difference in elevation. A convenient method for this solution is to construct a diagram like Fig. 45. The ordinates are differences in elevation and the abscissas are differences in parallax. It will be noted that a curve must be plotted for each altitude and

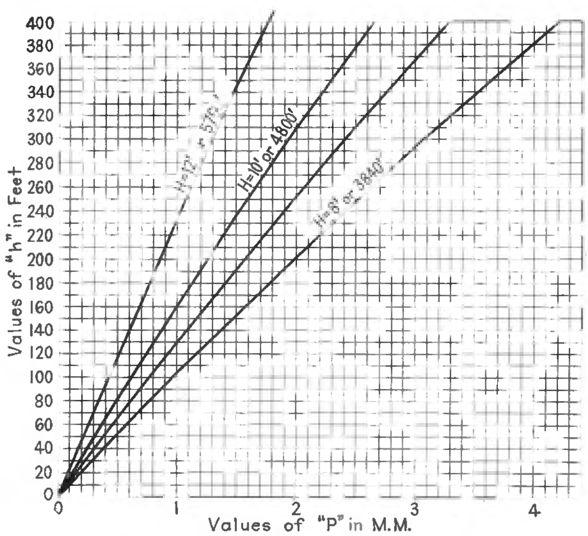


FIG. 45.

that the curve becomes steeper with an increase in altitude.

34. Tilt. In the foregoing discussion the negative was considered parallel to the datum plane and therefore the axis of the camera vertical. This condition is difficult to obtain. Air pockets, wind changes, banking, and tipping of the plane all tend to tilt the camera and therefore the focal plane in which the negative is located.

The displacement due to this tilt is illustrated by Fig. 46. Let this represent a section through the camera lens and at right angles to the line of sight.

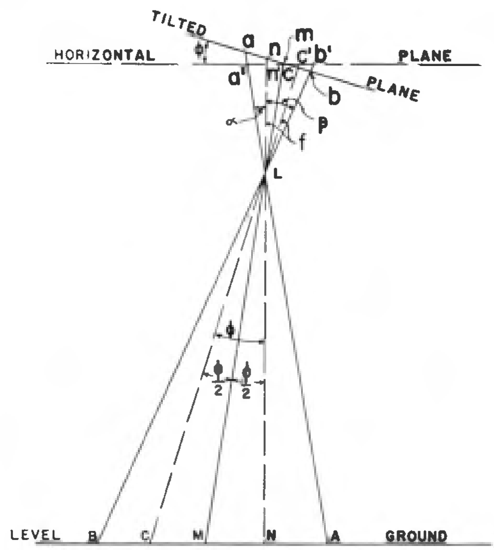


FIG. 46. Tilt.

The tilted plane represents the position of the negative at the instant of exposure and tilted through an angle which we will call φ

N = the nadir point, the point directly beneath the camera lens.

n = the image of this point on the tilted plane (nadir point).

n' = the image of this point on a horizontal plane.

c = the intersection of the optical axis of the lens with the tilted plane. (Principal Point.)

c' = the intersection of the optical axis of the lens with the horizontal plane.

A and B are any objects appearing on the photograph or tilted plane at a and b. Projected on a horizontal plane they are represented by a' and b'.

m = intersection of the two planes and is taken midway between n and c for practical purposes (isocenter).

$f = Ln' = Lc$.

$$cm = cL \tan \frac{\varphi}{2} = f \tan \frac{\varphi}{2}.$$

$$cn = f \tan \varphi.$$

For small tilt angles such as usually occur,

$$f \tan \varphi = 2f \tan \frac{\varphi}{2}$$

The tilt can usually be kept within 2 or 3 degrees.

The image of the point A (shown at a) will be displaced outward. Let this displacement equal d. Then

$$d = ma - ma'$$

$$d = (ca - cm) - (mn' + n'a')$$

Since m is taken as the midpoint,

$$mn' = mc$$

Then $d = ca - cm - cm - n'a'$

$$= ca - n'a' - 2cm$$

or $d = f \tan (\alpha + \varphi) - f \tan \alpha - 2 \tan \frac{\varphi}{2} f$

or $d = f(\tan [\alpha + \varphi] - \tan \alpha - \tan \varphi)$ displacement outward

This displacement is in terms of dimensions on the photograph and must be multiplied by the scale H/f to determine the displacement on the ground.

The displacement inward of the point B (Fig. 46) is shown on the tilted plane at b. Let this displacement be d'.

Then $d' = mb' - mb$

$$= (n'b' - n'm) - (mc + cb)$$

Since $n'm = mc$

$$d' = (n'b' - mc) - (mc + cb)$$

$$= n'b' - 2mc - cb$$

$$= f \tan \beta - 2f \tan \frac{\varphi}{2} - f \tan (\beta - \varphi)$$

or $d' = f(\tan \beta - \tan \varphi - \tan [\beta - \varphi])$ displacement inward

To illustrate the amount of displacement due to a tilt of 2 degrees, assume the following problem:

Height of exposure station = 10,000 ft.

Focal length = 10 in.

α = 20 degrees

The scale = H/f = 10,000/10 = 1000 ft. per in.

If the data are used as for point A, the displacement is outward. Substituting in the following equation, we have

$$d = f(\tan [\alpha + \varphi] - \tan \alpha - \tan \varphi)$$

$$d = 10(\tan 22° - \tan 20° - \tan 2°)$$

$$= 10 \times 0.00514 = 0.0514 \text{ in.}$$

D, the displacement on the ground, is:

$$D = d \times \frac{H}{f} = 0.0514 \times 1000 = 51.4 \text{ ft.}$$

In line manner the displacement inward can be found from the equation for d'.

An important fact to remember is that image displacements caused by tilt of the aerial camera are radial with respect to the isocenter (a point approximately half way between the principal point and the nadir point) and the displacements resulting from the relief of the terrain are radial with respect to the so-called nadir point of the photograph.

In terrain where differences of elevation are small, the displacements caused by relief are small and in many cases negligible. In such areas, the most accurate determinations of the positions of natural features can be obtained by using the isocenter. In mountainous regions the displacements caused by relief are so large that they inevitably detract from the accuracy of the map. The method used to reduce the effect of these errors to a minimum is to determine approximately by graphical methods the amount and direction of tilt, so as to be able to plot the nadir point and the isocenter, on the photograph. Then the point of origin for radial lines can be selected for each photograph somewhere on the line joining the nadir point and isocenter. The exact position is usually determined by the draftsman's judgment as to the best location in each particular case. Experience has shown that results obtained by shifting the point of origin for radial lines in this manner are definitely better than can be obtained if the tilt is ignored.

35. Graphical Determination of Tilt. In Fig. 47 are illustrated a tilted photograph represented as a positive and also the horizontal plane which is the position that a horizontal photograph would take if the camera axis were vertical. The direction of flight is along the tilt axis.

Three ground control points in the same horizontal plane are shown at A, B, and C, and the images of these ground control points are shown at a, b, and c on the tilted photograph.

The principal point *o* on the photograph may be located and *oa*, *ob*, and *oc* may be measured from the photograph where *o* is determined from the fiducial marks on the photograph.

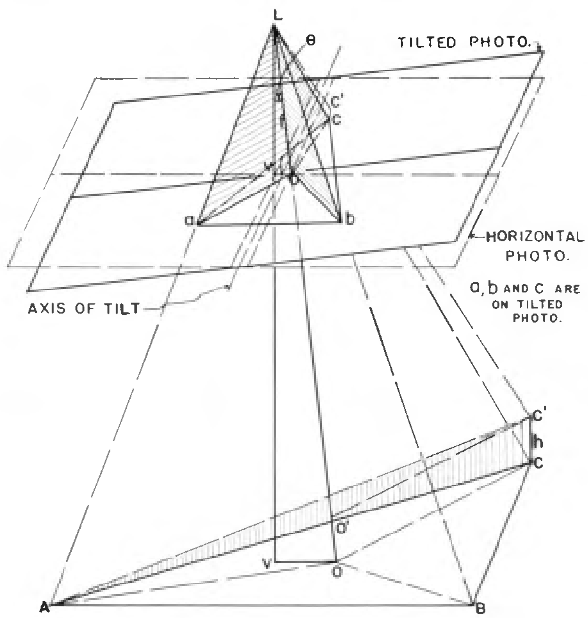


FIG. 47.

Find an approximate value of LO by similar triangles. (Approximate because LO is not vertical for a tilted photograph.)

$$\frac{LO}{f} = \frac{OA}{oa} \qquad \frac{LO}{f} = \frac{OC}{oc} \qquad \frac{LO}{f} = \frac{OB}{ob}$$

$$LO = \frac{f \times OA}{oa} \qquad LO = \frac{f \times OC}{oc} \qquad LO = \frac{f \times OB}{ob}$$

To determine OA, OB, and OC it is necessary to spot O on the control sheet. This may be done by the usual tracing-cloth method for the three-point resection.

The radials on the tracing cloth or photographic matte sheeting are determined by placing the tracing cloth on the photograph and drawing radials *oa*, *ob*, and *oc*. The tracing cloth is then laid over the ground control and the radials made to pass through their corresponding ground control points A, B, and C. The point O is then pricked through to the control sheet. With O plotted on the control sheet it is possible to scale the distances OA, OB, and OC for use in the equations for LO.

Next average the three values of LO which may be used for the average altitude of the camera station.

If the control points A, B, and C are of different elevations it becomes necessary to reduce the elevations to one plane.

The following demonstrations will illustrate a method of reduction to the plane ABC when C' is the ground control point instead of C which was used in the first part of the article.

Let h = the elevation of C' above the plane ABC.
C' = the photographed position.

In the triangle $AC'C$, AC is known from the horizontal ground control and CC' is known from the vertical ground control.

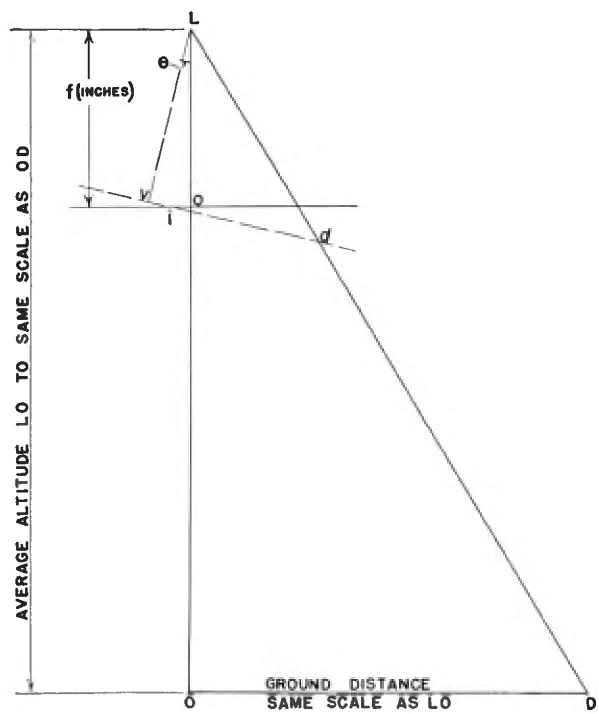


FIG. 48.

In triangle $LO'C'$, compute LO'.

$$\frac{LO'}{f} = \frac{O'C'}{oc'}$$

$$LO' = \frac{f \times O'C'}{oc'}$$

where $O'C' = OC$. Therefore

$$LO = LO' + h$$

This same procedure may be followed for additional control points and the average of the values of LO used for the altitude.

To Determine ov and Tilt Graphically

1. Select a piece of drawing paper and on it lay off a straight line LO (Fig. 48).
2. From L measure a distance $Lo = f$ (actual length). If the focal length is $8\frac{1}{2}$ in., lay off $8\frac{1}{2}$ in. (Fig. 48).

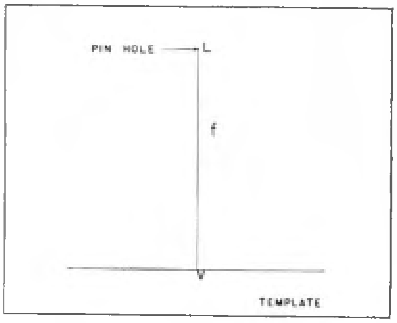


FIG. 49.

3. Draw a line through O at right angles to Lo extended (Fig. 48).
4. Select a piece of tracing cloth or, better, a piece of celluloid or photographic matte sheeting. Lay off two lines at right angles to each other and make a pinhole at L (see Fig. 49).
5. Make $Lv = f$ (actual measurement like $8\frac{1}{2}$ in.)
6. On Fig. 48 make LO equal LO, the average altitude to some convenient scale.
7. Since O is on the ground plane, lay off OD, the computed ground distance to the same scale as used for LO (Fig. 48).

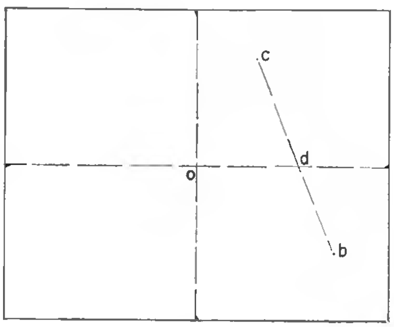


FIG. 50.

8. Superimpose the template (Fig. 49) on the graphical layout (Fig. 48) and insert pin at L so that the template swings about this point.
9. Measure the distance od on the photograph (Fig. 50).
10. Swing the template about point L until od on the template equals the distance od from the photograph (Fig. 50).
11. The angle vLo is the tilt angle which may be scaled from the diagram or computed by the formula $\tan\theta = \frac{ov}{f}$.

Approximations Used in the Above Solution

1. o on the photograph was taken as the origin of radials, whereas it is really i. o and i are close together, and no appreciable error is introduced in making this assumption.
2. LO is assumed to be a vertical line in determining the altitude. LV is the vertical line but V is not known. A 2-degree tilt with altitude of 20,000 ft. would cause a difference of only 12 ft. in the length of LV and LO and a 2-degree tilt with altitude of 10,000 ft. would cause a difference of 6 ft.

These small differences would cause no difference in the determination of the scale fraction or in determining the angle θ.

A more precise method of determining the tilt is found in Chapter 10.

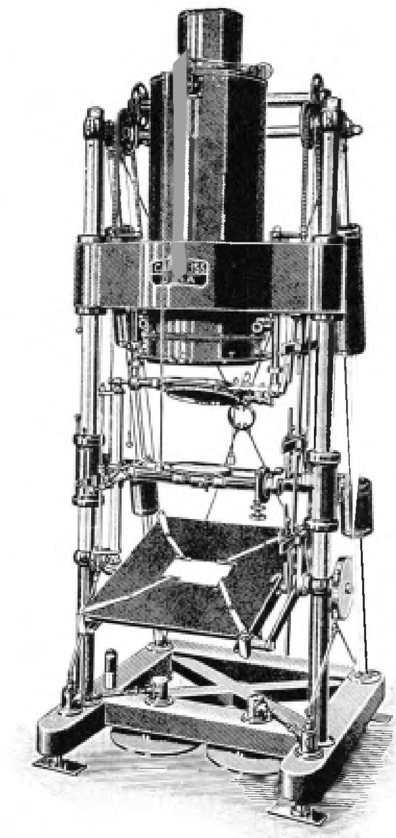


FIG. 51*a*. Automatic Rectifier for Plates or Film.

36. Change of Scale (Rectification). It is seen from the foregoing discussion that any change in the altitude of flight will introduce a change in the scale of the photograph and, since it is difficult to maintain a true altitude, the scale will frequently be changed. This change of scale can be corrected for in a rectifier, types of which are shown in Figs. 51*a*, 51*b*, and 51*c*.

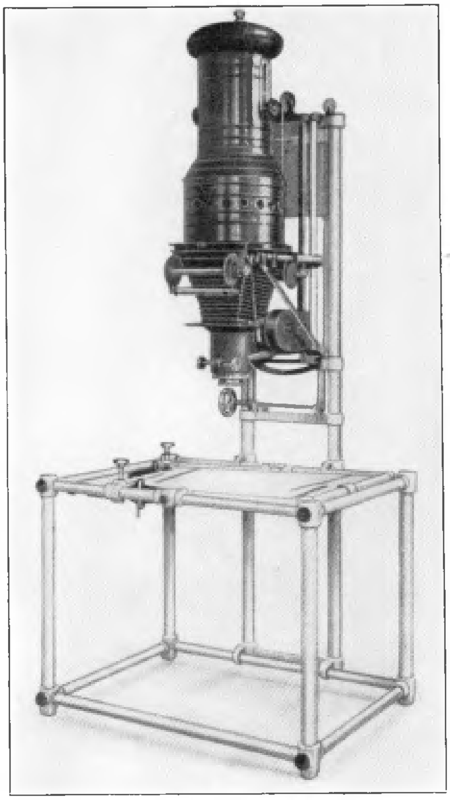


FIG. 51*b*. Small Rectifier for Plates or Film.

Rectifiers are usually constructed so that the projector is in a vertical position. Provision is made for inserting a negative between a source of light and a table or screen beneath. This table can be tilted in either direction, and upon this is placed a piece of sensitized paper or a sensitized plate.

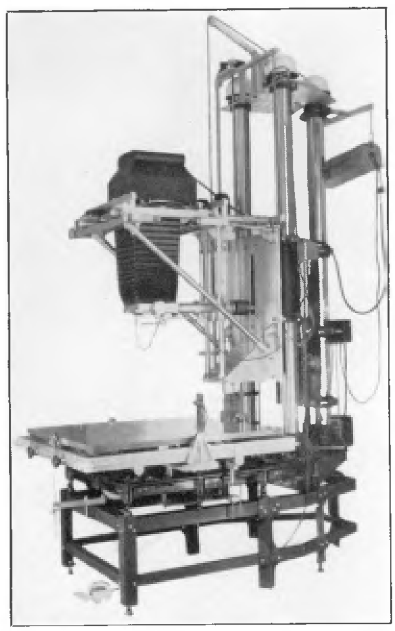
Change of scale is affected by varying the distances in the projecting camera, between the screen and the lens and between the original plate or film and the lens. If F is the principal focal length of the lens in the projector, f_1 the distance from the lens to the original plate or film, f_2 the distance from the lens to the table or screen, and m the desired ratio of linear values on the new photograph to corresponding ones on the original,

$\frac{f_2}{f_1} = m$ to produce the desired enlargement or reduction

Also $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$ to produce sharp definition

These two equations together give the proper distances f_1 and f_2 for settings on the projecting camera to produce any desired enlargement or reduction. The value of F has no effect upon the projected photograph, when the above conditions are fulfilled.

Beside being a projector, the rectifier is provided with adjustments so that the table or screen can be tilted in any direction. The angle of tilt can thus be used to eliminate the effect of tilt in the original photograph. The projector is also provided with an adjustment for tilting the lens at the same time the table is tilted so as to preserve sharp definition.



Courtesy J. G. Saltzman, Inc.

FIG. 51*c*. Large Rectifier.

In theory there are some slight errors in the above method but for practical purposes the scheme is satisfactory.

The resulting print is a rectified photograph which is the same as if the plate had been horizontal in the camera when the original exposure was made. A series of flight pictures can be horizontalized and brought to the same scale in this manner.