13

STEREOSCOPY

STEREOSCOPY is essential to the photo-interpreter. It may be defined as the science and art of viewing two different perspectives of an object, recorded on photographs taken from nearby camera stations, so as to obtain the mental impression of a three-dimensional model of the object. Two photographs of ground objects taken in quick succession from an aircraft will provide a three-dimensional model of these objects when viewed through a stereoscope by the photo-interpreter. In the present study, it is principally the application of stereoscopy which is important. Of the facets of this subject, the ones requiring varying degrees of attention are the stereoscopic image, parallax, the stereoscope and testing for stereoscopic vision.

PROPERTIES OF THE EYE

The faculty of vision by a human eye involves the use of the eyeball, including the fovea, the retina and possibly the para-fovea, the optic nerve and the visual centres of the brain. Rays of light pass through the refracting media of the eye, similar to light passing through the camera-lens, and form an image at the back of the eyeball on the retina. The stimulus is then transmitted to the visual centres of the brain and co-ordinated with a similar perception from the other eye. The fovea is most sensitive to green light at $0{\cdot}555\,\mu$ whilst in subdued light the para-fovea responds most to light at $0{\cdot}515\,\mu$ (Middleton, 1952).

A single eye is able to determine accurately the direction of an object; but is only able to gauge distance qualitatively. This is illustrated by holding two pencils, with the points about a foot apart at nearly arm's length. and with one eye closed trying to bring the point of one in contact with the top or tip of the other. It will be found that the top of one pencil will pass usually behind or in front of the tip of the other pencil. If the exercise is repeated with both eyes open, it is easy to bring the pencils into contact. The reason is that a pair of human eyes possesses the faculty of *binocular vision* by which it is possible to obtain a conception of relief in space (i.e. a three-dimensional model). Wheatstone, in the mid-nineteenth century, showed that if two photographic images are placed in front of the eyes, so that a similar view of the image is seen by each eye, then an impression of the relief of the object is produced by the eyes in the form of a single model. To illustrate the relief effect of binocular vision hold a pencil end-on a few inches in front of one eye. Then view it with both eyes. It will be found that it is only when it is viewed with the two eyes that an impression of its relief in space is obtained. It is interesting to note that visual acuity in stereoscopic work depends on a number of factors including the physiological state of the individual, which may be affected by fatigue, noise, lighting and mental depression (Nowicki, 1952).

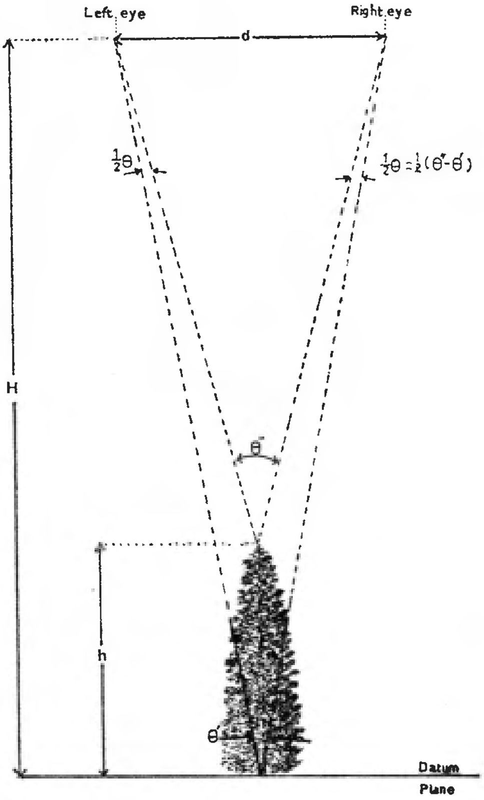


FIG. 13.1. Assume an observer in a helicopter at a flying height H above the ground sees a tree below him, then provided the disparity between the two angles of convergence (θ', θ'') is not excessive, he will experience an impression of depth for the tree (height: h) (see text). In reality the eye-base is extremely small compared with H.

An observer out of doors also obtains the impression of relief of distant objects, provided the objects are not excessively far. Let us consider an observer in a helicopter. If, in fig. 13.1, the eye base of the observer in the helicopter is d, the angle subtended at the top of the object viewed, a tree, θ'', and the angle subtended at the base of the same object θ', the difference in the angles of convergence is θ ($\theta'' - \theta'$). The angle of convergence is also known as *angular parallax* and *parallactic angle*. It is the angle subtended by the eye base at the object viewed. The *eye base* (d) or *interpupillary* *distance* varies between about 2·0 and 2·9 in. for most Europeans. An average is 2·60 in. (65 mm) (Nowicki, 1952).

The minimum disparity in the *angles of convergence* (θ) for a person to distinguish between objects is in natural space about 20 seconds; but Pulfrich (1901) observed that for some people the angle is as small as 10 seconds. Later Fourcade concluded that the maximum is 2° for obtaining a stereoscopic image. Ogle in 1952 has shown that for quantitative estimates of depth the disparity at 6° from the fovea is up to 2°, but for qualitative estimates disparities up to $3\frac{1}{2}°$ can be tolerated. Above $3\frac{1}{2}°$ the eyes cannot form two photographic images into one relief model in space. Each eye behind a lens of the stereoscope observes the images recorded on the photograph from a different relative position, as it would when observing the object in natural space at a comparable relative distance. The maximum disparity varies according to the region of the retina stimulated.

The three-dimensional model using a stereo-pair of photographs is not formed where the convergent visual axes intersect, as was formerly thought; but, instead, the model is formed some distance below the table plane of a magnifying stereoscope. For example, a conventional lens stereoscope with a focal length of $4\frac{1}{2}$ in. and at $3\frac{1}{2}$ in. above the viewing table provides the image at about 16 in.

Some people, possibly 5%, do not have the faculty of stereoscopic vision and no amount of training in adulthood will give it to them. Unfortunately, there is no known physical aid to provide stereoscopic sight to a person who does not possess it naturally; but training can help those with weak stereoscopic vision. The interpreter, when beginning his day's work, will probably find that his stereoscopic vision improves for the first 20 to 25 minutes. A minimum practice period of 5 to 10 minutes is advisable before candidates are tested for stereoscopic acuity.

THE STEREOSCOPIC IMAGE

Let us now consider the same tree as recorded on a pair of stereo-photographs and examined under a stereoscope. The stereoscopic image of the tree will be formed by two photographs as shown in fig. 13.2. If dots g'_2, g''_2, g'_1, g''_1, are marked on the photographs at the top and bottom of the tree and the photographs are set up for stereoscopic viewing, then the fused dots (g'_2, g''_2) at the top of the tree will appear to float in space and above the fused dots (g'_1, g''_1) at the bottom of the tree. If the distance between the corresponding dots is measured, then it will be found that the distance between the dots at the top of the tree is less than the distance between the dots at the bottom of the tree.

The difference between these two measurements, i.e. (g'_1 to g''_1) – (g'_2 to g''_2) can be used for determining heights by parallax. The same effect of floating dots can be achieved by making two pencil spots ($\frac{1}{10}$ in. diameter) on a sheet of paper at the distance of the eye base apart or at the eye base less $\frac{1}{4}$ in. When viewed under a stereoscope these two spots will fuse into one; and if a further two spots are marked about $\frac{1}{5}$ to $\frac{1}{10}$ in. closer together and about $\frac{1}{2}$ in. below the first dots as shown in 13.2b, these (g'_2, g''_2) will fuse and float above the first pair of fused spots (g'_1, g''_1). Within limits, the closer the pairs of spots are together, the higher the fused spots appear to float, and the wider the spots are apart the deeper the fused spots will appear to sink into the paper. This phenomenon provides the principle used in the construction of parallax bars, dots being permanently marked on glass slides and the distance being recorded by micrometer screw-gauge. Nyyssonen (1955) commented that height of trees as measured by parallax can vary according to the colour and size of the floating marks. He found that red floating marks were difficult to use; and observed that the standard error of the estimate seems no greater when measuring by parallax wedge than by other methods.

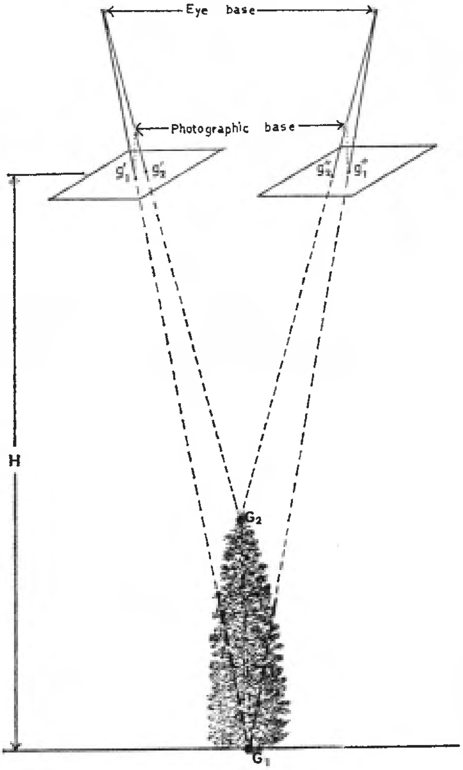


FIG. 13.2a. If instead of observing the tree from a helicopter (fig. 13.1) a pair of stereo-photographs are taken and set up for stereoscopic examination (of the image of the tree recorded on each photograph under a stereoscope), the interpreter will obtain the impression of depth for the tree (in the form of a three-dimensional model).

As mentioned above, two dots viewed under a stereoscope can be made either to fuse and float above the stereo-model by moving them a little closer or to sink into the ground by moving them apart. If instead of moving the dots laterally two long rows of dots are marked on a transparent base in such a way that each two opposite dots are closer together, then by placing this V- or 'dot-wedge' on the stereo-pair of photographs a similar effect will be observed as the parallax wedge is moved at right-angles to the air base. Each pair of dots is usually 0·002 in. further apart.

To make a reading, the transparent wedge is placed over the base of the image to be measured so that the left dot is at the base of the left-hand image and the right dot is over the base of the right-hand image. When the photographs are examined stereoscopically the dots fuse and float at the base of the image. The parallax wedge is then adjusted so that the fused dot floats at the top of the image. The parallax difference between the two readings is then determined from the side-scale on the parallax wedge and substituted in the parallax formula to determine the height in feet or metres. In some wedges, lines are substituted for dots. When viewed stereoscopically, these form a wedge the apex of which provides the parallax reading.

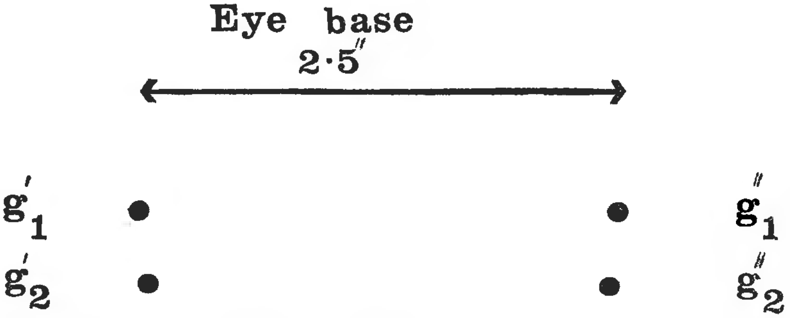


FIG. 13.2b. If the base and the tip of the tree imaged on the photographs in fig. 13.2a are replaced by spots (see text), then under the stereoscope spots g'_1 and g''_1 and g'_2 and g''_2 will be observed to fuse separately and float one above the other.

PARALLAX

It is convenient now to introduce the parallax formula used in the determination of heights. In the United Kingdom, the term 'parallax' is often preferred to 'absolute parallax' or 'absolute stereoscopic parallax'. The latter are commonly used in the United States. The term *x*-parallax is also sometimes used.

Parallax is the apparent displacement of the position of an object (e.g. top of tree) in relation to a reference point. In fig. 13.3 it is assumed that the reference point is the centre of each photograph (i.e. principal point), the photograph having been taken with the camera axis in a truly vertical position as occurs when the film is parallel to the ground. On a pair of stereoscopic photographs, the position of the recorded object in relation to each principal point (p_1p_2) will vary due to a shift in the point of observation or camera station in the aircraft. The distance between the principal points ($p_1p'_2$ or $p_2p'_1$) as measured on the photographs is termed the air base; and the parallax (P) of a point common to the pair of stereoscopic photographs (i.e. *the absolute stereoscopic parallax*) is the algebraic difference, measured parallel to the air base, of the distance of the two images of the same object from their respective principal points. The object being measured may be a tree, as shown in fig. 13.3. If the tip of the tree is considered as point G_2. and the base of the tree point G_1 then from what has been said previously it will be expected that, in stereoscopic vision, point g_2 will float above point g_1, g'_2 to g''_2 being less than g'_1 to g''_1. Examination of two photographs stereoscopically will confirm this as being correct.

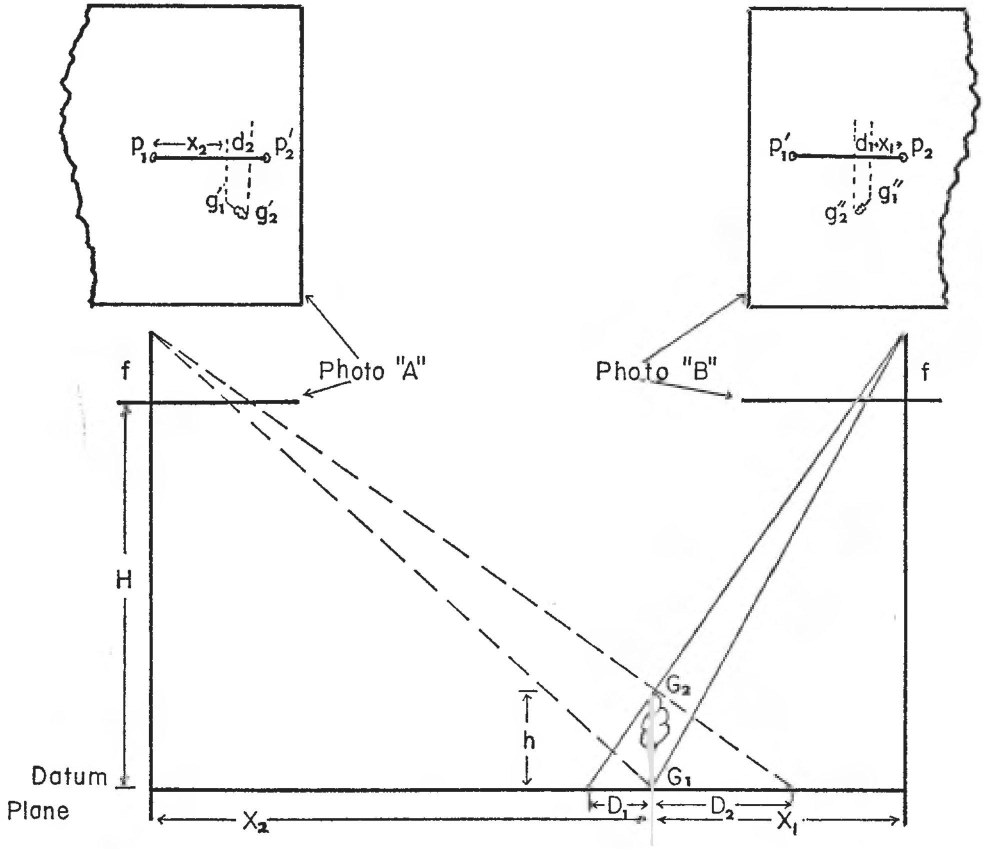


FIG. 13.3. Assume a plane flies from left to right at an altitude of H ft above the datum plane on which there is a tree G_1G_2 of height h. On the pair of contiguous photographs of the area, the tree will be recorded as $g'_1g'_2$ and $g''_1g''_2$. A formula, the parallax formula, can be derived for determining the height of the tree recorded on the stereo-pairs of vertical photographs (see text).

Instead of measuring these distances direct, the distances when using the absolute parallax formula are measured in respect of two known controls for all measurements, namely the principal points and the air base. The differences measured in this manner for the top of the tree and the bottom of the tree is termed parallax difference or differential parallax. The *parallax difference* (Δp) of an object on a pair of stereoscopic photographs is defined as the sum of the radial displacements of the top of the object from its base, measured parallel to the air base on each photograph.

In fig. 13.3 the location of a tree on the ground is shown in relation to the photographic station of the aircraft when the pair of photographs were taken. If it is assumed that the tilt of the camera is negligible and that the photographic scale is constant, then it may be shown that $h = H\Delta p/P + \Delta p$, where '$h$' is the height of the tree.

1. The parallax (P) by definition is: $P = x_1 + x_2 =$ Air base $(X_1 + X_2)$ in level country, or if algebraic signs are considered $P = x_1 - (-x_2)$.
2. The parallax difference (Δp) by definition is:

$$\Delta p = d_1 + d_2.$$

3. From similar triangles:

$$h/H = \frac{D_1}{X_1 + D_1} \qquad [H + f \doteqdot H]$$

or:

$$h/H = \frac{d_1}{x_1 + d_1}$$

$$d_1 = \frac{h(x_1 + d_1)}{H} = h/H\,(x_1 + d_1).$$

4. Similarly

$$d_2 = \frac{h(x_2 + d_2)}{H} = h/H\,(x_2 + d_2).$$

5. Substituting in (2)

$$\begin{aligned}\Delta p &= h/H\,(x_1 + d_1) + h/H(x_2 + d_2)\\ &= h/H\,\{x_1 + d_1 + x_2 + d_2\}\\ &= h/H\,\{(x_1 + x_2) + (d_1 + d_2)\}\end{aligned}$$

or

$$h = H(\Delta p)/(x_1 + x_2) + (d_1 + d_2) = H(\Delta p)/P + \Delta p.$$

For many purposes the formula may be used as $h = \dfrac{H\,(\Delta p)}{P}$ if $h/H < 2\%$, since there are other factors affecting the accuracy which are greater than any error caused by using the adjusted formula. The parallax difference (Δp) will then be: $\dfrac{P\,(h).}{H}$

Sometimes the formula is written as $h = H \dfrac{(dP)}{b + dP}$ where the photographic air base (b) or mean of the two air bases is substituted for the absolute parallax (P). It will be observed that, ideal conditions excepted, $P \neq b$; and also that $P + \Delta p$ is the parallax of the top of the object. Often h and H are expressed in feet and P and Δp in millimetres. Although these are different units of measurement they can be applied directly in the formula. The absolute parallax is increased by longer air bases and parallax difference by shorter focal lengths. By increasing the ratio: $\dfrac{\text{flying height}}{\text{air base}}$ the intersection of the rays on a pair of stereo-photographs is at a greater angle, thus increasing the accuracy in preparation of the map base. The model is then termed a 'harder model'.

It is interesting to note that to provide a scale model for use in advanced instruments the $\dfrac{\text{eye base}}{\text{space image}}$ ratio should be the same as the $\dfrac{\text{air base}}{\text{flying height}}$ ratio (see Miller, 1958).

Johnson (1957) provided a mathematical proof that parallax difference smaller than 0·001 ft cannot be detected by average photo-interpreters. He suggested logically that instruments graduated in excess of 0·001 ft do not necessarily increase the accuracy of the measurement of height. This provides weight to the argument that simple instruments measuring to this degree of accuracy may be just as satisfactory as more complex instruments having a greater precision of accurate measurement. Thus a parallax wedge reading to 0·001 ft should be as satisfactory in practice as a parallax bar reading to 0·1 mm (i.e. 0·004 in.).

Focal lengths of the stereoscope and camera, and magnification produced by the lens of the stereoscope, influence the relief exaggeration when stereoscopically viewed. The lower the magnification and the longer the focal length of the stereoscope the greater is the stereoscopic exaggeration, but details are less conspicuous and eye-strain is increased. A camera with a short focal length will provide greater stereoscopic exaggeration than a longer focal length, e.g. 12 in. A wide-angle lens with a short focal length (e.g. 3½ in.) and large scale photography will give for tall trees a difference in the angles of the convergence in excess of 3½° over much of the photograph; and thereby make it impossible for the eyes and brain to fuse the two complete images into a single three-dimensional model. For a given focal length, the greater the flying height the less will be the stereoscopic relief for the same scale photographs. The stereoscopic exaggeration will also increase towards the edge of the photographs. When examining photographs of eucalypts over 150 ft tall taken with both an 8¼ in. lens and a 6 in. lens, it was found impossible to obtain a single model of many of the tree images from photographs of the shorter focal length at a scale of 1/7,920.

Pseudoscopic views. Two items of interest involving the application of stereoscopy also require a brief description. These are pseudoscopic views and anaglyphs. In viewing terrain on aerial photographs a reversal of the relief is sometimes obtained by the eyes. Such a phenomenon is known as pseudoscopic illusion.

Are these lines water courses? The answer is no. They are actually the crests of sand-dunes in the Simpson desert, Australia. Pseudoscopic illusions can also be obtained if two photographs, having been flown right to left, are placed in the opposite order, left to right, for stereoscopic viewing. A volcanic crater may give the impression of being a volcanic cone. Viewing so that the shadows fall away from the observer can also result in depressions seeming to be elevations and vice-versa. Photographs are normally viewed under the stereoscope with the shadows towards the interpreter.

Anaglyphs. An anaglyph is a picture resulting from the printing or photographic projection of two nearly identical images of a stereoscopic pair in colour, e.g. red and blue. When the images are viewed in these colours, using spectacles with one lens of each colour, a stereoscopic image is formed. The observer sees the red image through the red lens and a blue image through the blue lens. The images recorded by the eyes are then fused by the brain into a relief model in space. The principle of anaglyphs has been used in the Multiplex by projecting the black-and-white diapositives through blue and red filters on to the mapping table.

STEREOSCOPES

These may conveniently be grouped into three basic types, namely lens, mirror and prism. By far the most popular is the *lens stereoscope*, which is also called a hand stereoscope or a pocket stereoscope. It comprises two semi-convex simple lenses mounted in a frame a few inches above the photographs (fig. *13.5*). Magnification is usually 2, 4 or 6 times. In some models, the eye base is adjustable between 2·2 in. and 3·0 in. (55 mm to 75 mm). The price varies from £2 to £5. The principal disadvantages are that only a small section of the photograph can be viewed at a time and, except for a narrow strip, viewing requires flipping up the side of one photograph.

Abrams and Zeiss have developed the pocket stereoscope into a *bridging stereoscope* so that flipping is not necessary. Further modifications include the addition of a parallax bar fixed to the stereoscope as in the Abrams' model; and the replacement of the simple lenses by binoculars. Rhody (1962) in conjunction with Wild S.A. developed a *'zoom' lens stereoscope*, or as he termed it 'un stéréoscope variable'. Several manufacturers are now producing zoom stereoscopes at prices between £1,500 and £2,000. The interpreter will find that maximum magnification is limited by the photographs, being possibly ten times for prints and twenty times for diapositives.

For viewing a pair of stereoscopic photographs with a hand stereoscope, the best position as determined by the distance between the images is about ¼ in. to ⅜ in. less than the eye base, or can be determined more precisely by the optical formula for a lens stereoscope. By increasing slightly the distance between the centres of the photographs, the apparent depth of the stereoscopic image can be increased. The optical formula is: $\eta = SUD$, where

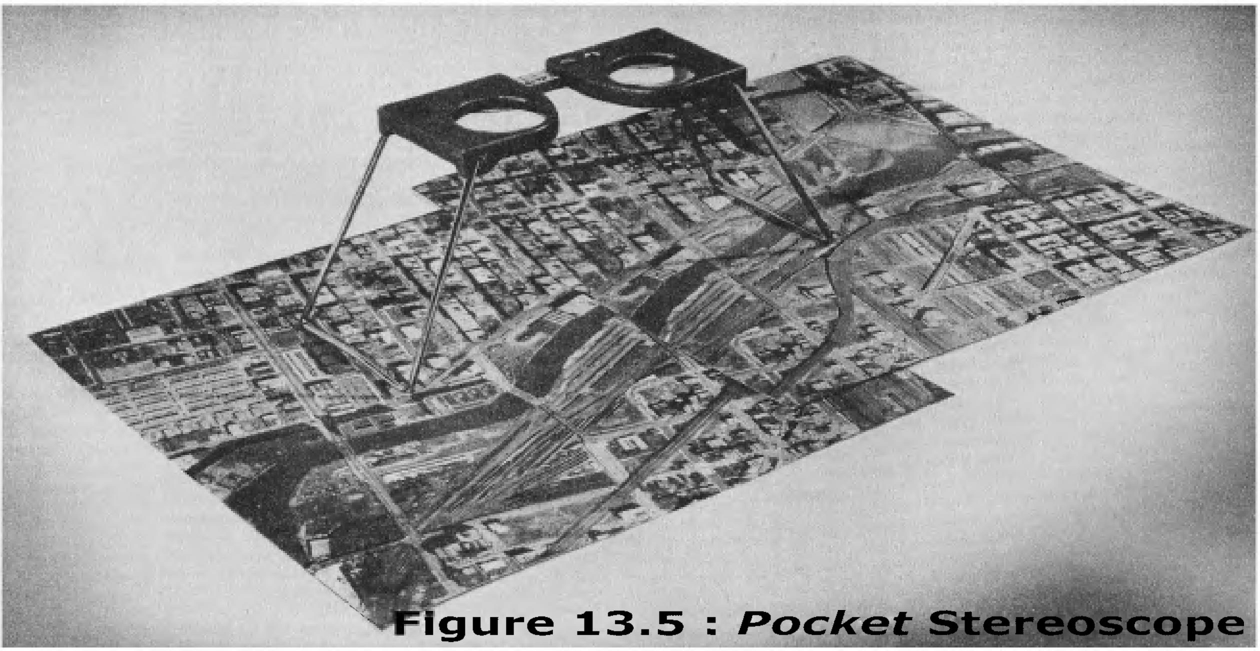


Figure 13.5 : ***Pocket*** **Stereoscope**

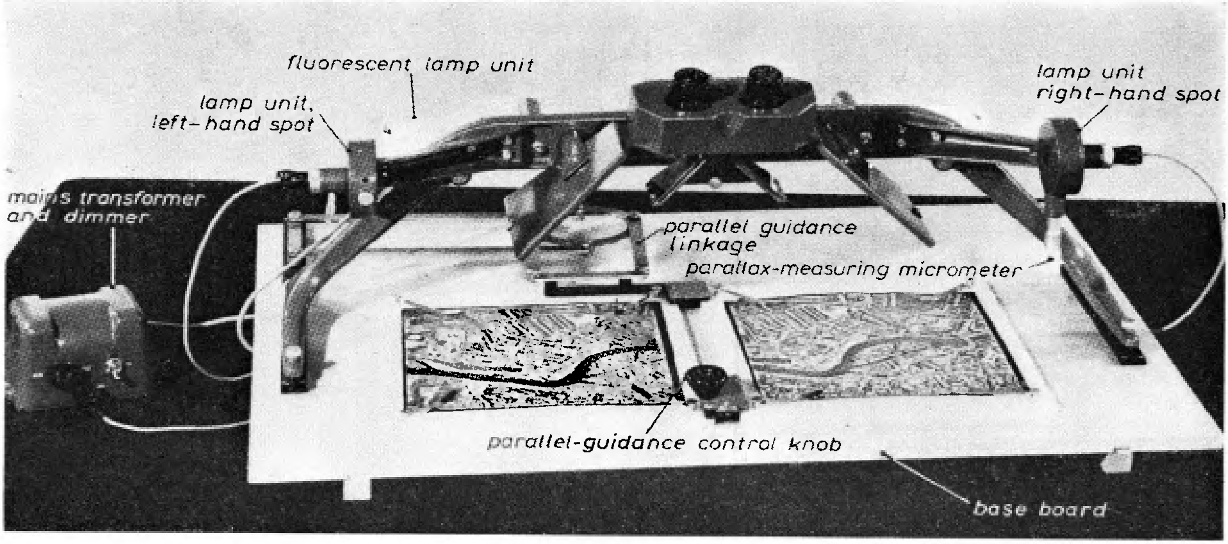
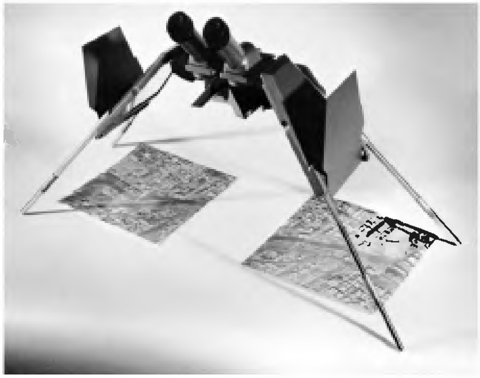


FIG. 13. 5b. The Hilger and Watts mirror stereoscope. Instead of a parallax bar having graticules in contact with each photograph two small spots of light are injected into the optical system of the stereoscope, when measuring parallax differences.

η: the separation distance between a point common to the two photographs,
S: distance between the optical centres of the lenses; and is independent of the eye base of the interpreter,
U: distance between lens and the photograph,
D: power of the lenses in diopters.

The second type of stereoscope uses mirrors to supplement the simple lens, so that a relatively large area of each pair of photographs is viewed. This avoids frequent moving of the photographs and flipping up the side of the photographs. As in the Zeiss and Wild mirror stereoscopes, viewing is usually through 'binoculars', although viewing can be carried out without the 'binoculars'. As the silver of the mirrors is on the outer surface of the glass to avoid refraction, it can easily be damaged by careless fingering.



A separate parallax bar often forms an accessory to the mirror stereoscope. A mirror stereoscope with 'binoculars' forms part of several other instruments, e.g. the Zeiss Stereopret. In the Old Delft Scanning Stereoscope viewing is achieved via mirrors and 45° rhomboidal prisms. External levers control the positions of the 45° prisms and internal mirrors for scanning. The photographs are not moved during the course of scanning. In the Hilger-Watts mirror stereoscope, the parallax bar is replaced by two pin-head light sources located behind the semi-transparent silver surfaces of the mirrors. When the photographs are viewed, a single point of light is seen in the stereo-model after correcting for parallax by adjusting the micrometer-screw of one light source (fig. *13.5*).

For viewing colour transparencies stereoscopically through the stereoscopes described above, it is necessary to provide illumination below the transparencies in the form of a light table. The brightness of the fluorescent lighting and the illumination table on which the transparencies are placed should be similar or the same as that used initially in processing. Often viewing is improved if a black paper mask is placed over the photographs except for the small area under close scrutiny as this reduces glare and eye-strain. Of course, colour prints are examined as black-and-white prints; but as mentioned in Part II, they are more expensive.