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CALIBRATION OF PHOTOGRAMMETRIC CAMERAS

J.B. PATTERSON

National Measurement Laboratory, CSIRO, Sydney (Australia)

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ABSTRACT

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In order to obtain the maximum useful information from an aerial photograph it is necessary to have the full metrical information relating the camera lens to the picture format of the camera.

This includes the calibrated focal length and the distortion curves for each semi-diagonal of the picture format, as well as the mean curve, for this calibrated focal length.

Also required are the coordinates relative to the fiducial centre, of the fiducial marks, the point of autocollimation and the point of symmetry of radial distortion.

A description is given of the equipment and procedures used at the National Measurement Laboratory, Sydney, for the determination of these parameters.

INTRODUCTION

In order to obtain accurate information from aerial photographs it is necessary to have precise information on the metrical properties relating the camera lens to the picture format.

This information includes the calibrated focal length, the radial distortion corresponding to this focal length, the coordinates relative to the fiducial centre, of the fiducial marks, the autocollimation point, and the point of symmetry of radial distortion.

The methods of making these measurements at the National Measurement Laboratory were described by Bell and Mayer (1956) but the advent of cameras in which the register glass is superseded by the vacuum back for location of the film, and the introduction of relatively distortion-free lenses has necessitated a number of changes in both the observational and analytical procedures used in the calibration.

Cameras are calibrated in a goniometer specially designed for the purpose in which the lens cone of the camera is supported in its normal working position with the axis vertical. A glass plate carrying a pair of finely divided scales is located against the film-locating surface, and the angle in the object space corresponding to each scale line is measured with the goniometer.

The use of "distortion-free" lenses in which the residual radial distortion is less than 10 μm has made it necessary to modify the methods developed by Mayer (1957) for determining the coordinates of the point of symmetry.

EQUIPMENT

The goniometer

The general form of the goniometer is similar to a large theodolite and is shown in Fig. 1. The base *A* has three levelling screws and carries the main frame *B* through a vertical centre bearing *C*. Supported by this main frame on two trunnions *D* is a stirrup-shaped member *E* capable of rotation about a horizontal transit axis. A telescope *F* is contained within this stirrup so that its line of sight lies in the same vertical plane as the main vertical axis and passes upward from the objective, intersecting the horizontal transit axis at right angles.

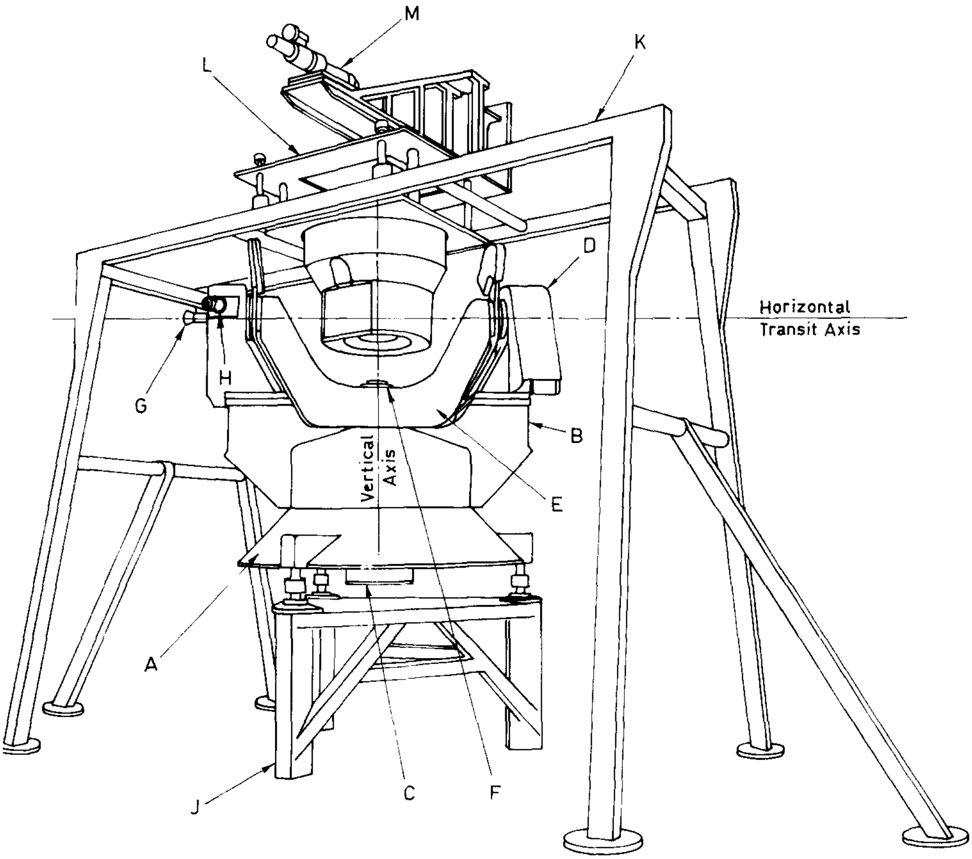

Fig. 1. Goniometer.

A cross line graticule, at the focal plane of the telescope objective, and the eyepiece *G* are carried outside one of the uprights, for easy viewing. This same upright contains a 152-mm diameter graduated glass circle with an optical reading system *H* permitting the rotation of the telescope to be read to the nearest arc second. This glass circle has been calibrated using a 72-sided polygon attached to the rotating axis, in conjunction with an autocollimator. The errors in angular reading for angles up to 60° either side of the central 90° reading have been determined.

The goniometer stands on a low metal tripod *J* at a convenient height for observation, and the camera is separately supported on a rigid tubular steel framework *K* fixed to the floor. The optical unit of the camera is bolted to an intermediate frame *L* with levelling screw adjustment, and after being lifted onto the steel framework by a hoist can be positioned so that the front nodal point of the lens lies approximately on the telescope transit axis.

Above the camera stand an autocollimator *M* is supported on a swinging platform pivoted on a plate attached to the wall. It is swung out of the way when the camera is being lifted into the calibrating position. When autocollimation measurements are required it is swung back and locked in a position above the camera with its viewing axis passing close to the camera axis.

Calibration plate

The calibration plate for cameras with flat picture surfaces consists of a glass plate 260 mm × 273 mm × 12 mm with one of the larger surfaces polished flat within 2 μm. Pairs of parallel lines of separation 1 mm are ruled mutually perpendicular, along the diagonals *EG* and *FH*. Scale lines with 10-

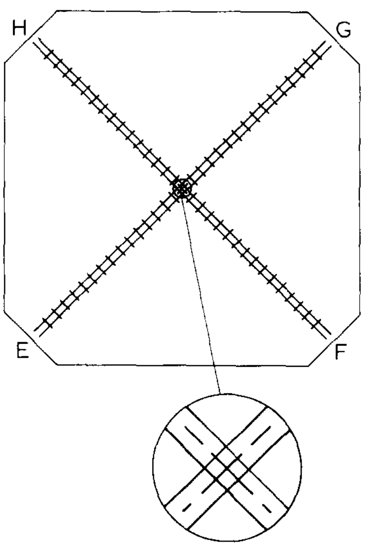

Fig. 2. Calibration plate.

mm interval are ruled perpendicular to each pair of parallel lines. The general arrangement is shown in Fig. 2.

The relative size of the scale lines has been exaggerated for the sake of clarity. The calibration of each scale is known with an uncertainty of $\pm 2\ \mu m$.

METHOD

Setting up

Vacuum-back photogrammetric cameras have fixed fiducial marks at each corner or midway along each side of the picture format. The origin of coordinates, or fiducial centre, is defined as the point of intersection of the straight lines joining these fiducials, and the centre cross of the precision scales used for calibration should preferably either coincide with this point or be at some known position very close to it.

The calibration plate is held flat against the picture frame in a suitable holder, which allows the plate to be positioned accurately by fine screw adjustments. One pair of parallel lines can be set to straddle two diagonal fiducial marks by viewing each mark in turn with a microscope. The other pair of lines can then be positioned relative to the other two fiducials, so that the centre cross of the plate coincides with the fiducial centre. The coordinates of the fiducial marks are then found using the first diagonal as a reference line and measuring the displacement of the fiducials in directions parallel and perpendicular to the nearest scale division with a travelling microscope.

For cameras with fiducial marks midway along the sides of the picture format, a smaller subsidiary plate is used with only a centre cross and two pairs of parallel lines ruled on it. The position of the autocollimation point relative to the fiducial centre is determined using this plate, as described in a later section. The plate is then replaced by the calibration plate, which is positioned with the scales across the diagonals and the centre cross coincident with the autocollimation point.

A circular area, 60 mm in diameter, surrounding the centre cross on each plate is coated with a layer of aluminium to provide a good reflecting surface for the autocollimator when setting the picture surface horizontal.

For cameras with a curved film surface, suitable calibration plates are available.

Point of autocollimation

The method for determining the coordinates of the point of autocollimation is very similar to that already described in Bell and Mayer (1956). The calibration plate is positioned relative to the camera fiducials as described in the previous section, and the camera is levelled in the following manner. A dish containing some mercury beneath a layer of viscous silicone oil is placed on top of the calibration plate and the cross line of the autocollimator is set on the return image obtained from the horizontal mercury surface. The mercury pool is then removed and the camera level is adjusted so that the return image obtained from the reflective coating on the calibration plate coincides with the cross line of the autocollimator, and the picture surface is truly horizontal.

The autocollimator is then swung out of the way, and a low-power measuring microscope with vertical illumination is focused on the centre cross of the plate. A second image of the cross is obtained by reflection if the mercury pool is placed below the camera lens. The separation of the two images of the cross is measured with the microscope, or the plate can be adjusted until the cross and its return image are coincident.

The spaces left in the centre cross, as seen in Fig. 2, allow the position of the return image to be measured, even if partly obscured by the original cross.

In the original arrangement of the goniometer, the autocollimation measurements were carried out at a separate station from that of calibration measurements. This involved the movement of the camera between stations and the possible disturbance of the calibration plate. To avoid this, the autocollimator was incorporated into the goniometer station as described.

Measurement of focal length and radial distortion

When the measurement of the coordinates of the point of autocollimation has been completed, the measuring microscope is removed and replaced by a fluorescent tube lamp to illuminate the scales on the calibration plate.

The goniometer transit axis is adjusted to be accurately at right angles to the line of the diagonal along which the distortion measurements are to be carried out. The angle subtended in the object space by each scale line is measured with the goniometer. The circular scale in the goniometer is set to read 90° when the instrument is set on the centre cross of the plate, so that the circle corrections can be added directly to the readings taken.

If r_i is the calibrated distance of the ith graduation line from the centre cross, and α_i is the corrected object space angle corresponding to the ith division as measured with the goniometer, then:

$$v_i = r_i - f_c \tan \alpha_i \qquad (1)$$

where v_i is the radial distortion at r_i corresponding to the value f_c of the calibrated focal length. By choosing an appropriate value of f_c any prescribed criterion for radial distortion can be met.

Minimal values for the radial distortion are obtained by applying the principle of least squares, according to which a value of f_c is chosen such that Σv_i^2 is a minimum, or:

$$\frac{\partial}{\partial \alpha}\left(\Sigma v_i^2\right) = 0$$

The solution of this set of equations is:

$$f_c = \frac{\sum_{i=1}^{n} r_i \tan \alpha_i}{\sum_{i=1}^{n} \tan^2 \alpha_i} \tag{2}$$

The values of r_i and α_i are entered into a programmed desk computer and eq. 2 solved for f_c. The value of f_c is used in eq. 1 to obtain the radial distortion v_i at each point used. With values of r_i and α_i along semi-diagonals OH and OG designated as positive, and those along semi-diagonals OF and OE as negative, the values of v_i are plotted against values of r_i for each diagonal in a four-quadrant presentation as explained in Mayer (1957). This procedure enables a more accurate smooth curve to be drawn through the origin, and the values of distortion $v(r_i)$ read from these curves, for discrete values of r_i, are used to determine the point of symmetry.

Point of symmetry

Generally the distortion curves will not be symmetrical when referred to the fiducial centre as origin. A new origin may be chosen about which the radial distortions on each diagonal are symmetrical.

With reference to Fig. 3, let P be the fiducial centre of one diagonal, and C be the calibrated interior perspective centre as defined in Mayer (1957).

If $v(r_i)$ is the distortion value corresponding to the radial distance r_i in the positive half of the diagonal, and $v(r_j)$ is the distortion value corresponding to the radial distance r_j in the negative half of the same diagonal, then:

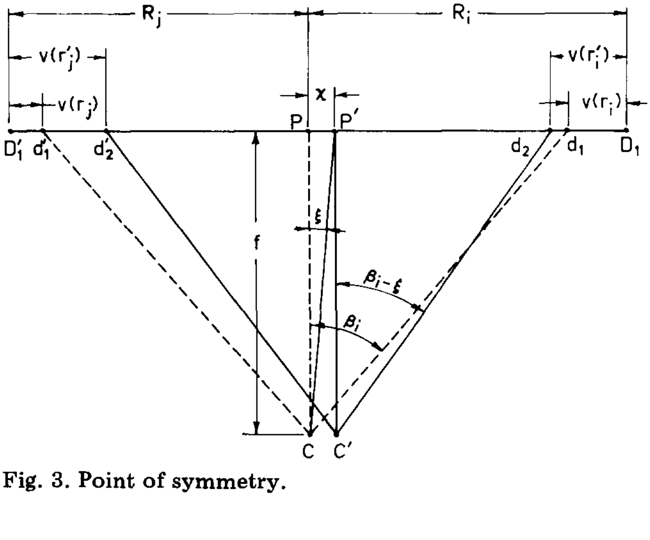

Fig. 3. Point of symmetry.

$$v(r_i) = R_i - f \tan \beta_i \tag{3}$$

and:

$$v(r_j) = R_j - f \tan \beta_j \tag{4}$$

where:

$$\tan \beta_i = r_i/f, \qquad \tan \beta_j = r_j/f$$

Note that r_j, R_j and β_j are of opposite sign to r_i, R_i and β_i.

If the position of P is changed by a distance χ along the diagonal corresponding to an angular change ξ in β then the new distortion value at a distance r_i' from the new origin is given by:

$$v(r_i') = R_i - \chi - f \tan(\beta_i - \xi) \tag{5}$$

Similarly:

$$v(r_j') = R_j - \chi - f \tan(\beta_j - \xi) \tag{6}$$

Also the distortion at distance χ is given by:

$$v(\chi) = \chi - f \tan \xi \tag{7}$$

For distortion to be symmetrical along a diagonal:

$$\sum_{i=1}^{n} v(r_i') = -\sum_{j=1}^{n} v(r_j')$$

$$\sum_{i=1}^{n} [R_i - \chi - f \tan(\beta_i - \xi)] = -\sum_{j=1}^{n} [R_j - \chi - f \tan(\beta_j - \xi)]$$

$$\sum_{i=1}^{n} R_i - n\chi - f \sum_{i=1}^{n} \tan(\beta_i - \xi) = -\sum_{j=1}^{n} R_j + n\chi + f \sum_{j=1}^{n} \tan(\beta_j - \xi)$$

i.e.

$$\sum_{i=1}^{n} R_i + \sum_{j=1}^{n} R_j = 2n\chi + f \left[\sum_{i=1}^{n} \tan(\beta_i - \xi) + \sum_{j=1}^{n} \tan(\beta_j - \xi) \right]$$

From (3) and (4):

$$\sum_{i=1}^{n} R_i + \sum_{j=1}^{n} R_j = \sum_{i=1}^{n} v(r_i) + \sum_{j=1}^{n} v(r_j) + f \left[\sum_{i=1}^{n} \tan \beta_i + \sum_{j=1}^{n} \tan \beta_j \right]$$

Now if r_i and r_j are chosen equidistant points from the origin, but opposite in sign, then:

$$\sum_{i=1}^{n} \tan \beta_i = -\sum_{j=1}^{n} \tan \beta_j$$

Thus:

$$\sum_{i=1}^{n} R_i + \sum_{j=1}^{n} R_j = \sum_{i=1}^{n} v(r_i) + \sum_{j=1}^{n} v(r_j)$$

$$= 2n\chi + f\left[\sum_{i=1}^{n} \tan(\beta_i - \xi) + \sum_{j=1}^{n} \tan(\beta_j - \xi)\right]$$

From (7):

$$\sum_{i=1}^{n} v(r_i) + \sum_{j=1}^{n} v(r_j) = 2n[v(\chi) + f \tan \xi] + f\left[\sum_{i=1}^{n} \tan(\beta_i - \xi) + \sum_{j=1}^{n} \tan(\beta_j - \xi)\right]$$

Dividing by $2nf$:

$$\frac{\sum_{i=1}^{n} v(r_i) + \sum_{j=1}^{n} v(r_j)}{2nf} = \tan \xi + \frac{\sum_{i=1}^{n} \tan(\beta_i - \xi) + \sum_{j=1}^{n} \tan(\beta_j - \xi)}{2n} + \frac{v(\chi)}{f}$$

Generally $\chi << 0.1$ mm and $v(\chi)/f$ is negligible. Thus:

$$\frac{\sum_{i=1}^{n} v(r_i) + \sum_{j=1}^{n} v(r_j)}{2nf} = \tan \xi + \frac{\sum_{i=1}^{n} \tan(\beta_i - \xi) + \sum_{j=1}^{n} \tan(\beta_j - \xi)}{2n} \tag{8}$$

Using selected values of r_i and r_j, the right-hand side of eq. 8 is calculated for $\xi = 1$ arc second, noting that:

$$\beta_i = \tan^{-1}(r_i/f), \qquad \beta_j = \tan^{-1}(r_j/f)$$

The left-hand side of the equation is calculated from the original distortion curves, for the same values of r_i and r_j, and by division, a value of ξ is obtained in arc seconds. Then:

$$\chi = f \tan \xi$$

The new distortion values are given by:

$$\begin{aligned} v(r_i') &= R_i - \chi - f \tan(\beta_i - \xi) \\ &= v(r_i) + f \tan \beta_i - \chi - f \tan(\beta_i - \xi) \\ &= v(r_i) - \chi + f[\tan \beta_i - \tan(\beta_i - \xi)] \end{aligned} \tag{9}$$

where $\beta_i = \tan^{-1}(r_i/f)$. Similarly:

$$v(r_j') = v(r_j) - \chi + f[\tan \beta_j - \tan(\beta_j - \xi)] \qquad (10)$$

This procedure is performed for each diagonal, and the two values of χ give the coordinates of the point of symmetry with respect to the fiducial centre, and the new symmetrical curves are given by eqs. 9 and 10.

Errors

The possible sources of systematic errors have been discussed by Bell and Mayer (1956) and the same considerations apply in this case. The error in setting the calibration plate, so that the scale centre is coincident with the fiducial centre of the camera, is estimated to be within 2 μm.

As stated earlier, the scales on the calibration plate have been calibrated, as well as the theodolite scale of the goniometer. The limit to which the goniometer can be set on any scale line with consistency seems to be 2 arc seconds.

Uncertainties in the calibration data corresponding to this 2 arc seconds uncertainty in the goniometer readings, are calculated as follows:

$$v(r_i) = r_i - f_c \tan \alpha_i$$

$$\frac{dv(r_i)}{d\alpha_i} = -f_c \sec^2 \alpha_i$$

$$\Delta v(r_i) = -\frac{f_c}{\cos^2 \alpha_i} \Delta\alpha_i$$

Table I gives the uncertainties in $\Delta v(r_i)$ for three values of f_c and for values of α_i from 0° to 45°.

In a camera with normal format, only values above the dotted lines are applicable.

TABLE I

Values of $\Delta v(r_i)$ in micrometres for $\Delta(\alpha_i)$ = 2 arc seconds

α_i (degrees)	f_c (mm)		
	88	150	300
0	−0.9	−1.5	−3.0
10	−0.9	−1.5	−3.1
20	−1.0	−1.7	−3.4
30	−1.2	−2.0	−4.0
40	−1.5	−2.6	−5.1
45	−1.8	−3.0	−6.0

Thus the random uncertainty at each calibration point can amount to 3 μm. However, the procedure for obtaining the calibrated focal length and residual radial distortion from the observed data gives the same weight to each observation and ensures that a continuous curve of radial distortion is obtained. A measure of the uncertainty in the data is given by the differences between each observed value and the corresponding smoothed value.

To summarize, it is probable that for the assigned value of the calibrated focal length, the calculated value of the residual radial distortion is in error by not more than ±5 μm.

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REFERENCES

Bell, G.A. and Mayer, I., 1956. The calibration of aerial survey cameras. Cartography, 1(4): 144—147.

Mayer, I., 1957. The presentation of the results of aerial camera calibration. Photogrammetria, 13(2): 49—66.