

COMMONWEALTH OF AUSTRALIA  
DEPARTMENT OF NATIONAL DEVELOPMENT  
DIVISION OF NATIONAL MAPPING

TECHNICAL REPORT No.6

## VARYCORD

A FORTRAN PROGRAM FOR THE LEAST SQUARES  
ADJUSTMENT OF HORIZONTAL CONTROL SURVEYS

BY

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CANBERRA, A. C. T., AUSTRALIA.

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#### Abstract

This report gives a detailed description of Program Varycord, a computer program for the least squares adjustment of angles, azimuths and distances on the spheroid. The program is written in an advanced form of Fortran used on the Control Data Corporation 3600 computer. The program is designed to adjust traverses, triangulation, Hiran and Aerodist trilateration, separately or in combination, with lines up to 1000 miles long. It can also be used for adjusting photogrammetric radial-line triangulation. It does not adjust heights.

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AIMS OF THIS REPORT

1. This report has been written with three objects in view:
  - 1.1 To record the mathematical procedures used in the geodetic adjustment of Australia.
  - 1.2 To provide a manual for users of the Variation of Coordinates program on the CDC 3600 computer. Program VARYCORD has continually grown in complexity. There are now more options and diagnostics than can easily be held in the mind.
  - 1.3 To provide a general description of the method for students and for those intending to write similar programmes for their own computers.

A Fortran listing of the program is not included in the report, but copies can be supplied on request.

HISTORY

2. The adjustment of surveys by the method of variation of coordinates is not new. The adjustment on the Transverse Mercator projection has been described by Rainsford (1957). The more complex adjustment on the spheroid was fully described, with numerical examples, by Adams (1915). The method was extensively used by the Geodetic Survey of Canada (Ross, 1955) for the computation of the Canadian Shoran Survey. The method is particularly suitable for surveys containing numerous measured distances as well as angles. Since the introduction of tellurometers and electronic computers, it has become the standard method of adjusting horizontal control surveys.

GENERAL OUTLINE OF THE METHOD

3. Preliminary latitudes and longitudes for all points are first obtained by any method. From these coordinates, the mutual distance and azimuth is computed between every pair of points between which an observation has been made. If the observations and the preliminary coordinates were perfect, these computed azimuths and distances would accord with the observed angles, azimuths and distances. In practice, they will not accord exactly; and the problem is to find adjustments to the preliminary coordinates which will minimise the weighted sums of the squares of the differences between the observed and computed values of the angles, azimuths and distances. This particular program adjusts angles, not directions.

ADVANTAGES OF THE METHOD

4.1 No conditions have to be listed. The formation of conditions, and the confirmation that no redundant condition has been included in the least square solution, increases in difficulty with the size of the adjustment, and is a task difficult to program on a computer.

4.2 The number of unknowns is precisely twice the number of points to be adjusted. In a network containing many redundancies, this number of unknowns may be smaller than the number of conditions. Moreover, by careful allocation of serial numbers to the survey stations (see paragraph 11), the "band width" of the matrix of coefficients in the normal equations can be kept small, which reduces both the storage and computation time required for their solution.

4.3 There is no limit to the number of observations which may be included in an adjustment, as it is a simple matter to arrange for them to be added into the normal equation matrix one at a time. New observations can easily be added to existing data at any time.

4.4 The latitudes and longitudes of the adjusted points are produced immediately, without further computations. This is a very great advantage, though it is sometimes partially off-set by the need to list preliminary latitudes and longitudes for all points in the input. In triangulation and traverse, these are always easily found by a preliminary computation of unadjusted angles and distances. With Hiran or Aerodist adjustments, coordinates have sometimes to be read off a small scale map; it is then necessary to iterate the adjustment several times - see paragraph 77. It is particularly important that the preliminary coordinates of points at the ends of a short line are in sympathy with each other, else the azimuth of the line computed from the preliminary coordinates may be seriously in error.

BASIC FORMULAE

5.1 Symbols used in more than one paragraph are defined in Appendix X. The program works with latitudes positive north, and longitudes positive west; except that when computing Transverse Mercator coordinates, longitudes are converted to positive east. For use in Australia, east longitudes and south latitudes are tabulated in the data without sign, and are automatically made negative.

5.2 If the earth were a sphere, the basic formulae connecting changes in A and L with small changes  $\Delta\phi''$ ,  $\Delta\lambda''$  would be:

$$\Delta A_{12} = (R/L) \sin A_{12} \Delta\phi_1 + (R/L) \cos \phi_1 \cos A_{12} \Delta\lambda_1 \quad (1)$$

$$\Delta L_{12} = -(R \cos A_{12} \Delta\phi_1 + R \cos \phi_1 \sin A_{12} \Delta\lambda_1) \sin 1'' \quad (2)$$

It is often convenient to keep these simple approximations in mind when considering weights and other theoretical problems.

5.3 On the spheroid the standard formulae (Ross, 1955) are:

$$\Delta A_{12} = (p_1/L) \sin A_{12} \Delta \phi_1 + (p_2/L) \sin A_{21} \Delta \phi_2 + (v_2/L) \cos \phi_2 \cos A_{21} (\Delta \lambda_2 - \Delta \lambda_1)$$

or

$$\Delta A_{12} = P_{12} \Delta \phi_1 + Q_{12} \Delta \phi_2 + R_{12} (\Delta \lambda_2 - \Delta \lambda_1)$$

defining P, Q and R.

$$-\Delta L_{12} = (p_1 \cos A_{12} \Delta \phi_1 + p_2 \cos A_{21} \Delta \phi_2 - v_2 \cos \phi_2 \sin A_{21} (\Delta \lambda_2 - \Delta \lambda_1)) \sin 1''$$

or

$$\Delta L_{12} = S_{12} \Delta \phi_1 + T_{12} \Delta \phi_2 + U_{12} (\Delta \lambda_2 - \Delta \lambda_1)$$

defining S, T and U. Along a geodesic  $U_{12} = U_{21} = \text{constant}$ .

5.4 These equations require a prior knowledge of L and A, and are only accurate if  $\Delta \phi$  and  $\Delta \lambda$  are small. With an electronic computer it is easy and wise to iterate every adjustment, to ensure that no error follows from this cause - see paragraph 77.

5.5 Equations (1), (3) and (4) are dimensionless; equations (2), (5) and (6) have the dimensions of length. They cannot therefore be used as they stand to form observations equations in a mixed least square adjustment. See paragraph 6.3.

5.6 For deriving azimuths and distances from latitudes and longitudes, the formulae of Robbins (1962) are used. Robbins distances are correct to 2 cms at distances of 1000 miles, after which they fairly rapidly deteriorate. Normal section azimuths are always correct. Geodesic azimuths; (G. Bomford, 1962, p. 95; formula (3.8)), if preferred to normal sections azimuths, are accurate to 0".06 at 1000 miles. The program is therefore suitable for adjusting all Aerodist and Hiran lines, but not for satellite triangulation.

5.7 For computing Transverse Mercator coordinates from latitudes and longitudes, the formulae derived by Redfearn (1948) have been used in full.

5.8 For meridian distance, formula (8.39) in Geodesy (G. Bomford, 1962) is used, with the first term corrected to  $a A_0 \phi$ :

$$m = a (A_0 \phi - A_2 \sin 2\phi + A_4 \sin 4\phi - A_6 \sin 6\phi)$$

### OBSERVATIONS EQUATIONS

6.1 In the following equations, the term "O-C" denotes the difference between an observed quantity and the corresponding value computed from the preliminary coordinates.

6.2 Subscripts A, Z and L refer to angles, azimuths and distances respectively.

6.3 Every term in an observation equation must contain a divisor  $k\sigma$  which is either the standard error  $\sigma$  of the observed quantity, or some quantity bearing a constant proportion  $k$  to it, the value of  $k$  being the same for all observations in any one adjustment. Thus we may divide through by standard errors, probable errors ( $=0.6745 \sigma$ ), or in general by  $k\sigma$ . Only then will the method of least square give us the most probable solution to our adjustment. This point will be readily grasped if we consider running the same adjustment twice, once with distances expressed in millimetres, and once with distances expressed in kilometres. If we omit the division by  $k\sigma$  ( $\sigma$  of course being expressed in the same units as the distances) the coefficients in the observation equations for distances will differ by a factor of one million, those for angles and azimuths will be unchanged, and quite different solutions will be obtained. After dividing through by  $k\sigma$  all observation equations are dimensionless, and observations of angles, azimuths and distances may be combined in a single adjustment.

6.4 For each observed angle, by subtracting equation (4) for the line 1-2 from equation (4) or the line 1-3, we obtain:

$$\frac{1}{k\sigma_A} [(P_{13} - P_{12}) \Delta\phi_1 - (R_{13} - R_{12}) \Delta\lambda_1 - Q_{12} \Delta\phi_2 - R_{12} \Delta\lambda_2 + Q_{13} \Delta\phi_3 + R_{13} \Delta\lambda_3] = (O-C)_A / k\sigma_A$$

6.5 For a Laplace azimuth, we use equation (4) for the azimuth line, but the matter is complicated by the need to satisfy the Laplace equation. If the longitude of the observation station is changed by  $\Delta\lambda$ , the Laplace azimuth is changed by  $-\Delta\lambda \sin\phi$ . Accordingly, the observation equation becomes:

$$\frac{1}{k\sigma_z} [P_{12} \Delta\phi_1 - (R_{12} - \sin\phi_1) \Delta\lambda_1 + Q_{12} \Delta\phi_2 + R_{12} \Delta\lambda_2] = (O-C)_z / k\sigma_z \quad (8)$$

6.6 For an observed distance, from equation (6):

$$\frac{1}{k\sigma_L} [S_{12} \Delta\phi_1 - U_{12} \Delta\lambda_1 + T_{12} \Delta\phi_2 + U_{12} \Delta\lambda_2] = (O-C)_L / k\sigma_L \quad (9)$$

6.7 No other types of observation equations are used in the program.

WEIGHTS - see paragraph 21.

7.1 It is often convenient to allocate general values for standard errors  $\sigma_A, \sigma_z, \sigma_L$ , which are used for all angles, azimuths and distances in an adjustment, rather than to tabulate individual weights for each observation. We may call  $(1/\sigma^2)$  general weights.

7.2 An individual weight may be allocated to any azimuth or distance. Individual weights cannot be allocated to the angles, but may be allocated to any of the tabulated directions, from which the angles are derived. If no individual weight is tabulated on the data sheets, the individual weight is taken as unity.

7.3 Systematic weights can also be allocated - for example, distances can be weighted inversely as their length, or as the square of their length. Azimuths can also be weighted inversely as the lengths of the ray.

7.4 Each observation equation is then multiplied by the square root of the product of the general weight, the individual weight and the systematic weight.

#### FIXED POINTS

8.1 Points are held fixed by setting the relevant  $\Delta\phi$  and  $\Delta\lambda$  to zero in any observation equation in which a fixed point occurs. Thus terms relating

to a fixed point are simply omitted.

8.2 In this program, the coordinates of fixed points are listed before the adjusted points in both the input and output.

8.3 The maximum number of fixed points is currently limited to 40. This number has never yet been approached in practice. The restriction is however artificial, and it should not be difficult to increase the maximum number of fixed points if a need arises. See Appendix B, note O.2.

8.4 The minimum number of fixed points is in general one; but if no azimuth or no distance has been observed, then the minimum number of fixed points is two. When these minima are not attained, it is easily seen that the network is free to move, rotate, expand or contract. The solution is indeterminate, and the matrix of the coefficients in the normal equations is singular. A diagnostic is printed - see paragraphs 12 and 67.1

#### FIXED OBSERVATIONS

9.1 No specific method is provided for holding an angle, azimuth or distance fixed, unless the stations at the end of the line are held fixed.

9.2 If it is desired to hold an observation fixed, while the terminal stations are free, it usually suffices to give the observation a large individual weight, say one million, which is achieved by tabulating "1E6" in the weight box. If the fixed quantity is seriously out of sympathy with the free quantities, this method fails. This is a useful indication that the fixed quantity is seriously inaccurate.

#### EXTERNAL ANGLES

10.1 If we consider, as an example, a station in a chain of braced quadrilaterals, the question arises whether to include the exterior angle of about  $180^\circ$  as an observation, or only the four interior angles of about  $45^\circ$ .

10.2 The advantage of omitting external angles are, firstly, that there is a significant reduction in the band-width of the matrix of the coefficients of the normal equations, which speeds up the adjustment and cuts down the storage required; secondly, if one believes that angles of  $180^\circ$  are less accurately measured than angles of  $45^\circ$ , omission of the external angles ensures that they receive slightly bigger adjustments. In the adjustment of single figures, external angles have traditionally been omitted.

10.3 The disadvantages are, firstly, that directions have to be listed in a special order on the data sheets, so that the external angle occurs after the last and before the first direction. If an external angle has occasionally to be included, as with the centre point of a polygon, then the first direction

must be tabulated again after the last direction. Secondly, if angles are observed in rounds, and the selection of the opening RO is arbitrary, it is hard to see why the external angle should be treated differently from any other angle.

10.4 The matter may be important. For example on a traverse, omitting the "external" angles gives the angular work only half the weight it would have if external angles were included.

10.5 In the geodetic adjustment of Australia, external angles were normally included as observations, and no precautions were taken in this respect when listing the data. External angles, can, however, be excluded if desired - see paragraph 23. Care must then be taken in the order with which directions are tabulated.

### STATION SERIAL NUMBERS AND THE BAND WIDTH

11.1 All stations in an adjustment, fixed and free, must be allocated a serial number. No two stations may have the same number, but there is no objection to gaps in the sequence. The highest permissible serial number is 140 (see Appendix F, 22.1).

11.2 In the matrix of the coefficients in the normal equations, the first two columns are for  $\Delta\phi, \Delta\lambda$  for the variable station with the lowest serial number, and next two for the variable station with the next lowest serial number, and so on.

11.3 In most survey adjustments, for example, in a traverse or triangulation chain, observations occur between stations which are relatively close to one another. It is possible that a Hiran line may extend from one end of the chain to the other, but this is exceptional. Usually, if we start numbering stations from one end of the chain and proceed steadily to the other end, observations will occur between stations whose numbers are not widely dissimilar.

11.4 When the normal equation matrix is formed, the coefficients will also lie in columns fairly close to one another, and all the non-zero coefficients will tend to lie in a band parallel to the principal diagonal. The maximum width of this band, outwards from and excluding the principal diagonal, is called the band width. If the maximum difference in the serial numbers of stations joined by any one observations is  $n$ , then the band width is  $2n + 1$ .

11.5 In this program, the zero coefficients outside the band are not stored, and play no part in the solution of the normal equations.

11.6 In a chain of braced quadrilaterals, properly numbered from end to end, the band width is 9 if external angles are included, and only 5 if they are omitted. In a more or less square triangulation network, or in one which also contains Hiran lines, the band width may rise to 50 or more; but even so there is usually a useful economy.

11.7 In a matrix of order  $N$  and band width  $M$  (on one side of, and excluding, the principal diagonal), the number of elements in the band matrix is

$$(M + 1) (2N - M) / 2$$

11.8 In a typical case, where  $N = 150$  and  $M = 20$ , the number of coefficients to be stored is 2940, compared with 22,500 in the full matrix, or 11,325 on and above the principal diagonal.

11.9 To number a rectangular block, it is best to start from a narrow edge, and number across the block, always in the same direction, as when reading a printed page. See Figure 1.

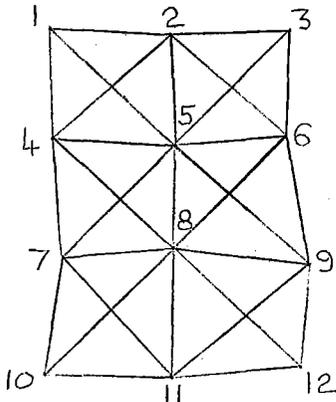


Figure 1: Station serial numbers in a rectangular block.

In an irregular block, one should follow this advice as far as possible. The aim is to minimise the largest range of serial numbers which occurs in any one observation. The average range is immaterial. It is the maximum range which decides the band width.

11.10 Figure 1 also makes it clear why omitting external angles reduces the band width. If external angles are included, there is an angle at 6 connecting station 3 with station 9, giving a band width of 13. If the external angles are omitted, the maximum band width is only 7. This can be an important consideration with large adjustments, which approach the limit of available computer storage.

11.11 If a new station is added to an old network which has no gaps in its sequence of serial numbers, and the new station is given the next serial

number available, the effect on the band width, and thus on the expense of computation, can easily exceed a factor of ten. For this reason, it has been necessary to write a simple program called RENUMBER, which renumbers all the cards in a deck of data.

### SINGULAR AND UNSTABLE MATRICES

12.1 The matrix of the coefficients in the normal equations will be singular, and the normal equations insoluble, in the following cases:

- a) No fixed points. The network is floating loose, and values for  $\Delta\phi, \Delta\lambda$  are indeterminate.
- b) No azimuth observation and less than two fixed points. The network is free to rotate.
- c) No distance observation and less than two fixed points. There is no fixed scale, and the network can expand or contract.
- d) Insufficient observations to fix one or more points - for example, no observations into or out of a point, or only one azimuth, one distance or one direction.

12.2 When the matrix is singular, a diagnostic is printed, which gives a clue to the faulty station - see paragraph 67.1

12.3 It would not be difficult to design a network, in which the matrix, though not singular, approached singularity and was unstable, - for example, if a station were fixed only by two nearly parallel directions, or by two almost parallel distances. However, this is a state of affairs which surveyors take care to avoid. In the adjustment of over 200 networks, no hint of instability has yet been detected.

### SOLUTION OF THE NORMAL EQUATIONS

13.1 The normal equations are solved by the Choleski or Square Root method, following the routine given by Faadeeva, 1959. This method is convenient for any symmetric matrix, such as occurs with normal equations, and it is particularly satisfactory for a band matrix. No additional storage is required other than that used by the original matrices, and the omission of multiplications by the zero coefficients outside the band is easily arranged. Provided the surveyor has ensured that each point in the network has been soundly fixed geometrically, there is little danger of rounding errors causing instability in the solution; but it is always wise to check by iterating the adjustment.

13.2 Any symmetric matrix  $A$  can be factorised into the product of two triangular matrices  $S$  and  $S'$ , one of which is the transpose of the other. Then the original normal equations

$$A X = B$$

can be solved in two stages

$$S'Y = B, \quad S X = Y,$$

each of which requires only the simple routine of "back-substitution".

13.3 To find the triangular matrix  $S$ , the rule for multiplying matrices gives

$$a_{ii} = s_{1i}^2 + s_{2i}^2 + s_{3i}^2 \dots s_{ii}^2, \quad i=j,$$

$$a_{ij} = s_{1i}s_{1j} + s_{2i}s_{2j} + s_{3i}s_{3j} \dots s_{ii}s_{ij}, \quad i < j,$$

from which the following rules are derived:

$$s_{11} = (a_{11})^{1/2}; \quad s_{1j} = a_{1j} / s_{11}; \quad s_{ij} = 0, \quad i > j;$$

$$s_{ii} = (a_{ii} - \sum_{\ell=1}^{i-1} s_{\ell i}^2)^{1/2}, \quad i > 1,$$

$$s_{ij} = (a_{ij} - \sum_{\ell=1}^{i-1} s_{\ell i} s_{\ell j}) / s_{ii}, \quad j > 1.$$

13.4 A diagonal element  $s_{ii}$  is computed from the square root of the difference between the corresponding diagonal element  $a_{ii}$ , and the sum of the squares of all previously computed elements  $s$  in the same column.

13.5 A non-diagonal element  $s_{ij}$  is obtained by subtracting from the corresponding element  $a_{ij}$  the sum of the products, row by row, of the corresponding elements  $s$  taken from the columns  $i$  and  $j$ . This difference is then divided by the diagonal element  $s_{ii}$  for the row.

13.6 Zero elements in  $A$  are zero in  $S$ ; and once  $s_{ij}$  has been found,  $a_{ij}$  is not again required; so  $s_{ij}$  can replace  $a_{ij}$  in the computer store.

13.7 To find the intermediate column matrix  $Y$

$$Y_1 = b_1 / s_{11}$$

$$Y_i = (b_i - \sum_{\ell=1}^{i-1} s_{\ell i} Y_\ell) / s_{ii}, \quad i > 1.$$

13.8 Each element  $y_i$  is found in turn by subtracting from the corresponding element  $b_i$  the sum of the products, row by row, of the elements in column  $i$  and the previously found  $y_i$ . The difference is then divided by the diagonal term. When  $y_i$  has been found,  $b_i$  is no longer required, so that  $Y$  can replace  $B$  in the computer store.

13.9 Similarly, but working upwards from the bottom, the unknowns  $x$  are found from the rule

$$x_n = Y_n / S_{nn}$$

$$x_i = (Y_i - \sum_{\ell=i+1}^n S_{i\ell} x_\ell) / S_{ii}, \quad i < n$$

and  $X$  can replace  $Y$  in the computer store.

13.10 If  $A$  is a band matrix, the limits of  $\ell$  are reduced in accordance with the band width. See Appendix P.

### THE COMPUTER

14.1 This Variation of Coordinates programme was written for the Control Data Corporation 3600 computer, which is owned by the Commonwealth Scientific and Industrial Research Organisation in Canberra. It was installed in July 1964. Much of the program was written before the computer arrived, and the program first ran in September 1964.

14.2 The CDC 3600 is a large and powerful machine. It can perform 100,000 additions plus 100,000 multiplications in one second. It has a high-speed random-access store of 32,000 words, each of 10 floating-point decimal digits, but about one quarter of the store is permanently occupied by the monitor systems. The card reader reads almost 1000 cards per minute, and the output is printed at 1000 lines per minute, each line having up to 130 digits. There are 8 magnetic tape drives. The program uses three magnetic tapes to store data, leaving most of the high speed store free for the solution of the normal equations.

14.3 The program runs very quickly. Feeding the program into the computer takes 35 seconds from binary cards. Thereafter, from reading the first card of the data to printing the last line of the output, the following times are typical:

- (a) A traverse of 13 points, 11 angles, 3 azimuths and 12 distances : 28 seconds, the solution of the normals taking 0.4 seconds.

- (b) A small triangulation chain of 16 points with 71 angles, 2 azimuths and 11 distances : 40 seconds, the solution of the normal taking 2 seconds.
- (c) The geodetic survey of Tasmania, with 94 variable points, 385 angles, 9 azimuths and 102 distances: 5 minutes 7 seconds, the solution of the normals taking 2 minutes 47 seconds.

## THE LANGUAGE

14.4. The program is written in Fortran 3600, which is very similar to Fortran 4. Full details of this very convenient language are given in the CDC 3600 Fortran Reference Manual (Control Data Corporation, 1964).

## PROGRAM VARYCORD

15.1. The complete VARYCORD program consists of 24 parts: program VARY, 22 subroutines, and function RADFORM. The subroutines vary a great deal in complexity. DEGFORM simply converts radians into degrees, minutes and seconds. Others such as MAIN and STATPRIN contain about 200 Fortran statements. Program VARY itself is merely a dummy, which serves to call subroutine MAIN - see Appendix "A".

15.2. A block diagram is shown in Figure 2. Commentaries on each of the subroutines are given in Appendixes A to W. The appendices are not designed for general reading; their aim is to guide a programmer wishing to amend the program to the right point of attack, and warn him of any dangers that exist, and of other alterations which may be necessary as a consequence. In the appendices, it is assumed that the reader can read Fortran.

15.3. Except for subroutine BANDIT, the program does not make use of matrix algebra. With the possible exception of BANDIT, any surveyor who has carried out a simple least squares adjustment by the method of observations should be able to follow the program with the aid of the appendices.

## DATA INPUT

16.1. All data cards contain a number in column 2, in the range 1 to 9, which defines their use. The different types of cards are called 1-cards, etc. Printed data sheets (see Figures 3 and 4) are available for tabulating cards type 3 to 7. **Every card contains a section number in column 3 - 10,** so that a card found loose can be restored to its proper deck.

16.2. An unlimited number of different sections may be adjusted consecutively without pause; but the adjustment of a section cannot be iterated without the substitution of new cards in the data deck.

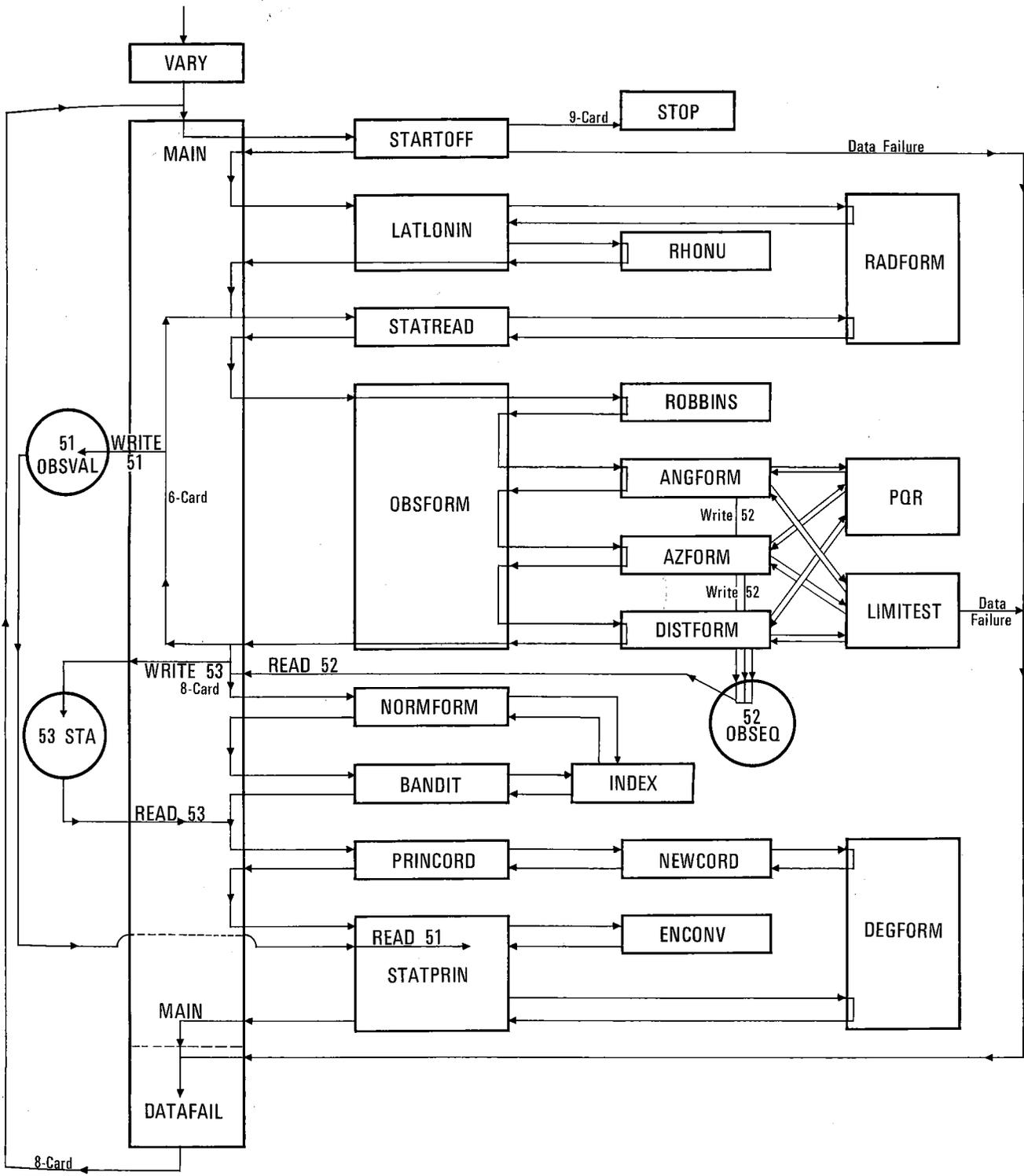


FIGURE 2: THE STRUCTURE OF PROGRAM VARYCORD, SHOWING THE SEQUENCE IN WHICH SUBROUTINES ARE CALLED.

1-CARDS

16.3 A data deck must start with a 1-card. The rest of the card is usually left blank, though entries in columns 3-55 do no harm.

16.4 A 1-card serves one purpose only: it warns the computer that a 2-card follows. A 1-card is therefore essential not only at the start of a deck, but whenever it is desired to change spheroid.

2-CARDS

16.5 A 2-card defines a spheroid. Each 2 card must be preceded by a 1-card, and be followed by a 3-card. If no 2-card is in the deck, the adjustment cannot run, and a diagnostic is printed, - see paragraph 70.

16.6 Columns 4-43 may contain the name of the spheroid, the origin, etc., which is printed on the output. Columns 47-59 must contain the a radius, and columns 60-69 must contain the inverse of the flattening. Both boxes must include a decimal point. The a radius must be in the units (FT or MS) selected in columns 77-78 of the 3-card; but if the M option is used, converting heights and distances from feet to metres (paragraph 24), the a radius must be in metres.

16.7 Cards 1 and 2 are usually yellow or blue-topped, and once punched can be used indefinitely.

3-CARDS - See Figure 3

16.8 The deck for each section must start with a 3-card, which is customarily red-topped. The 3-card need only be preceded by a 1-card and 2-card if:

- a) It is the first section in the data deck, or
- b) The spheroid has been changed.

Otherwise, 3-cards follow directly after the 8-card of the preceding section.

16.9 Column 3-10 contains the section number, which is punched in all the cards in a section, so that a card found loose can be returned to its own section. Either numeric or alphabetic characters can be used in the eight spaces reserved for the section number. For example:

123-10            or CORALSEA

16.10 Columns 11-55 contain the name of the section. This is often in the form of the name of the State in which the section lies, followed by the names of the terminal junction points; for example:

W.A.            - ESPERANCE TO ALBANY

16.11 Columns 77-78 must contain either FT or MS, depending on whether



the a radius on the 2-card is in feet or metres. The other columns 58-80 are used for a variety of options - see paragraphs 20-32.

4-CARDS - See Figure 3.

16.12 The 4-cards contain the serial numbers, latitudes and longitudes of the fixed points. They are customarily coloured green.

Columns 3-10 contain the section number, see paragraph 16.9

Columns 12-14 contain the station serial number, right justified, see paragraph 11.

Columns 21-33 contain the fixed latitudes, and columns 37-50 the fixed longitude, in degrees, minutes and seconds of arc. Unless zero, the seconds must contain a decimal point in columns 29 and 46.

16.13 No signs are tabulated. The standard version of the program, for use in Australia, requires south latitudes and east longitudes. A version is available for north latitudes and west longitudes - see paragraph 79.

16.14 At least one, and sometimes two, 4-cards are obligatory - see paragraph 8.4. All the 4-cards must precede the 5-cards, and they must be in numerical order by station serial numbers.

5-CARDS - See Figure 3.

16.15 The 5-cards contain the serial numbers and the preliminary latitudes and longitudes for the variable stations. They are identical to 4-cards, except they have a 5 in column 2, and are usually white. The 5-cards must be in their correct order by serial numbers, else a diagnostic is obtained- see paragraph 73.

16.16 New 5-cards can be automatically punched by the computer, making it easy to iterate an adjustment - see paragraph 77.

6-CARDS

16.17 The 6-cards contain a 6 in column 2. They may contain the section number in columns 3-10, but must otherwise be blank.

16.18 The six cards are customarily coloured orange. They tell the computer that data for a new station is about to be fed in, and that a 7-card with special format containing the station name and height follows.

7-CARDS- See Figure 4.

16.19 There are two types. Both contain the section number in columns



3-10, and the station serial number, right-justified, in columns 12-14.

16.20 A 7-card immediately following a 6-card contains the station name and number (limited to a total of 24 digits) in columns 16-39. The name is customarily left-justified, and the number right-justified, to give a neat appearance on the output. The height of the station, including a decimal point (see Appendix F, note 1.1) goes in columns 56-64, followed in columns 66-7 by "FT" if the height and distances are in feet, and "MS" if they are in metres. Heights are optional, and columns 56-64 may be left blank. The same units must be used for heights and distances, and must be the same as the units used for the  $a$ -radius on the 2-card; except, if the M option is used (paragraph 24) which converts feet on the input to metres in the output, the 2-card must be in metres, the 3-card may contain either FT or MS, and the 7-cards must contain FT, and height and distances in feet.

16.21 The subsequent 7-cards contain the observed quantities, if any. Arbitrarily oriented directions from the field book or abstract are entered in columns 20-31; followed by the individual weight for each direction. Laplace azimuths are entered in columns 37-48, with individual weights in columns 50-52; and distances in columns 54-64, with individual weights in columns 66-68. Decimal points must be punched. For individual weights, see paragraph 7.2. If the weight boxes are left blank, a weight of 1.0 is adopted internally. Weights greater than 9.9 can be inserted in the form 5E2, which the computer interprets as  $5 \times 10^2$ , or 500. Cards containing Laplace azimuths are customarily green topped. At least two 7-cards, one of each sort, must be included for every station, else a diagnostic is obtained - see paragraph 66.

16.22 Where no directions, azimuths or distances have been observed, the boxes are left blank. The program does not distinguish between zero and blanks so the opening direction of a round, if zero, must be entered as 360 00 00.0. If 00 00 00.0 is entered, the computer assumes this direction has not been observed. Distant points are usually tabulated in clockwise order by directions. The program does not fail if the order is varied, but observation equations are formed with different angles and slightly different results may be obtained. All 7-cards which contain directions must precede those that do not. After a gap in the sequence of directions, subsequent directions are ignored. This is easy to arrange with the original data, but care is needed if it is desired to omit a troublesome direction. It is best to leave the 7-card in, with the direction deleted, so that the adjusted value of the direction will be listed on the output; but it is necessary to move the card to the end of the station.

16.23 Distances are customarily tabulated twice, once on the card from A to B, and once on the card from B to A. This is convenient on the output. Both entries should be identical. The computer ignores the entry from the

station with the higher serial number to the station with the lower number.

### 8-CARDS

16.24 The last 7-card at a station is usually followed by the 6-card for the next station, but the last station in a section is terminated by an 8-card.

16.25 An 8-card may be followed by a 3-card if another adjustment follows on the same spheroid; by a 1-card, 2-card and 3-card, if another adjustment follows on a different spheroid; and by a 9-card if it is the last adjustment in the data deck.

### 9-CARD

16.26 At the end of the deck, the last 8-card is followed by a 9-card, which must in turn be followed by a blue "End of file" control card, which has a 7 and an 8 both punched in column 1, and is otherwise blank. This causes control in the computer to transfer back to the monitor system, which immediately processes the next job.

### INTERSECTED POINTS

17. At an intersected point, 7-cards should be included for every ray out of the point, even though all the observation boxes are blank. The adjusted azimuth and length of each ray will then be tabulated on the output.

### MULTIPLE OBSERVATIONS AT ONE STATION

18. If more than one set of observations have been made at a station, without a common RO, or with more than one common ray, a preliminary station adjustment can be avoided. The various sets of observations should be included in the adjustment, just as if they were observations at different stations, each preceded by a 6-card, but in each case the station name, height and serial number should be the same. Each set must contain the same number of rays. This arbitrary requirement is easily satisfied by adding cards to the smaller sets which contain serial numbers for extra rays, but have the observation boxes blank. Adjusted azimuths and distances are listed in separate batches on the output, common rays having identical azimuths and distances, and adjusted angles can if required be taken out by hand.

### HEIGHT DIFFERENCES

19.1 On the data sheets, provision was made for difference in height,

plus or minus, to be tabulated in columns 70-79 of the 7-cards, in case a height adjustment were carried out at a later date. In column 80, a code letter may be entered, for example;

V for vertical traingulation

B for barometric heights

L for spirit levelling

19.2 The variation of coordinates program ignores all entries in columns 70-80, which can therefore be left blank.

### OPTIONS

20. Options are exercised by making entries in columns 58-80 of the 3-card.

### REJECTION OF OBSERVATIONS

20.1 Each observed quantity has two values, the observed value O, and the value C computed from the preliminary coordinates. Each value of O-C is tested against pre-selected limits, Z seconds for angles and azimuths, L units for distances. If too many observations are rejected, there will be insufficient left to fix all the points, and the normal equations become insoluble. A diagnostic is printed (paragraph 67.1) and the adjustment is abandoned. However, if something were hopelessly wrong with the data, for example, if the wrong 5-cards were inadvertently put in the deck, the adjustment would not be abandoned until every observation had been tested. To avoid this wasted effort, a limiting number LIMIT of bad observations is pre-selected. If the limit is attained, the adjustment is abandoned forthwith.

20.2 LIMIT is entered in columns 58-59. If left blank, an arbitrary limit of 5 bad observations is selected internally. This is satisfactory as a general rule; but when running dubious data for the first time, it is wise to set LIMIT high, at say 20, and hope to flush out all the bad observations in a single run.

20.3 The acceptance limits Z and L are set in columns 62-65 and 68-71 respectively. If left blank, they are set internally to 100" and 40 units of length. This may seem large, but if the fixed points have been moved out of sympathy with the variable points by several feet, and if lines are short, the value of O-C for angles may be big. If all the preliminary coordinates are known to be in sympathy, finer limits can be set.

20.4 It is sometimes desirable to retain all observations, however great the value of O-C; for example on the first run of an aerodist

adjustment, the observed distances may all be satisfactory, but the preliminary coordinates may be some miles in error. If a limit of 9999 is inadequate, one can enter, for example, "2E5", which the computer interprets as  $2 \times 10^5$ , or 200000.

20.5 All entries in the LIMIT, Z and L boxes must be right justified, as blanks are read as zeros.

### WEIGHTING SYSTEMS

Column 74

21.1 Column 74 must contain either a blank, a W, an X or an R.

21.2 A blank sets all general weights (see paragraph 7.1) equal to unity, implying that the standard errors of angles and distances, on the one hand, and distances on the other, are in the ratio of 1 second to 1 foot, or 1 second to 1 metre, according to the units used. This is the system recommended by Rainsford (1962). If distances are in feet, the results often appear reasonable, but the system has no theoretical justification and different results are obtained if different units of length are used. It is not recommended for general use. It is however satisfactory for an adjustment containing nothing but angles. It is only satisfactory for the adjustment of pure trilateration if the error in a distance is assumed to be independent of its length. This is approximately true of Hiran trilateration; if an occasional azimuth is included, care must be taken to give it a suitable individual weight (see paragraph 7.2).

21.3 A W weights distances inversely as the square of their length. Each observation equation for a distance is divided by  $(\text{length} \times \sin 1'')$ . This is the system suggested by Murphy (1958). It implies that at the end of a line whose azimuth and length have both been observed the error ellipse is circular, and its radius is proportional to the length of the line. This system can also give reasonable results, but it too has no real theoretical justification. In geodetic triangulation, with lines averaging about 20 miles, tellurometer distances tend to get adjustments which appear too small; very short lines received adjustments which are close to zero; and azimuths have the same weights as directions, unless individually weighted.

21.4 An X weights distances and azimuths inversely as their length. The error ellipse is still assumed to be circular, but its radius increases only as the square root of the length of the line. Observation equations for distances are divided by  $(L \sin 1'')^{\frac{1}{2}}$ , and observation equations for azimuths are multiplied by  $(L \sin 1'')^{\frac{1}{2}}$ . The X option was designed for continental adjustments, in which each "observation" consists of an azimuth or distance derived from a whole section of the geodetic survey which has previously been adjusted on its own. A section will likely contain many individual azimuths and distances. If their individual

errors are random, the error in the azimuth and distance of the whole section should be proportional to the square root of its length. The X option may also be preferred for Hiran and Aerodist trilateration, if the error in a line is assumed to be proportional to the square root of its length. Care will be needed in selecting a suitable individual weight for each azimuth included in the Hiran adjustment. The X option is not intended for use with adjustment containing angles.

21.5 The R option uses a priori standard errors. For the Geodetic Adjustment of Australia in 1966, the standard errors adopted were:

Directions 0".5; Azimuths 1".0; Distances (0.1 ft or 0.03 ms) + 3 ppm.

If the observations are uncorrelated, the R option will yield the most probable adjustment for all surveys for which the standard errors of directions, azimuths and distances are in the above proportions, even if the absolute values of the standard errors are different. For an occasional job where first order tellurometer distances run through second or lower order triangulation, it is convenient to use the R option, but to increase the weight of all distances by the WIFACTOR option (see below, 22.1) and increase the individual weight of the relatively few azimuths by hand. To alter the standard errors in the R option, it is necessary to alter the Fortran source deck for subroutine STARTOFF at (21.1 ff) - see Appendix C, 16.1. If several different R options are in use, it is convenient to have a separate binary STARTOFF deck for each.

21.6 The R option is intended for general use. Note that the standard error is for a direction, not for an angle. If the average adjustments printed at the end of the output are in the same ratio (for the average length of the measured lines) as the standard errors in the R option, then (if the observations are uncorrelated) the best possible adjustment has been obtained. But small differences in the ratios will detract very little from the quality of the adjustment, and it is best to decide on a representative set of values for an R option, and use them throughout the corresponding survey, rather than make continual small variations.

#### SPECIAL WEIGHTS FOR DISTANCES - "WIFACTOR"

Column 75-76

22.1 It is sometimes desirable to weight all distances by a constant factor relative to the angles and azimuths. If it is desired to weight distances by say 4.0 or 0.4, then enter "4." or ".4" in columns 75-76. The decimal is obligatory in both cases. A notice is printed on the output - see paragraph 40. The square root of WIFACTOR is computed in STARTOFF at (31.00 ff) and the observation equations are altered in DISTFORM at (10.0, 11.0).

22.2 This option is useful when tellurometer distances have been used to strengthen low order triangulation.

OMISSION OF EXTERNAL ANGLES - "ANGCLO"

Column 73

23.1 Any entry in column 73 causes external angles to be ignored - see paragraph 10. The option is put into effect in OBSFORM at (1.3 ff). It has so far only been used when testing the program against published examples of test figures, where external angles have nearly always been ignored. In the Geodetic Survey of Australia, external angles were always included. The option could be useful for very large adjustments; the resulting reduction of band width might be the only way of fitting an exceptionally large adjustment into the computer. When using ANGCLO, care must be taken in the tabulation on the observed directions. The external angle must lie between the last direction and the first.

CONVERSION OF FEET TO METRES - "M"

Column 68

24. If M is punched in column 67, all input heights and distances on the 7-cards are multiplied by 0.3048, converting them from feet to metres. When using this option, the a radius on the 2-card must be in metres, and columns 77-78 of the 3-card must contain MS, not FT. The option is put into effect by STATREAD (1.1 and 6.1) - see Appendix F. See also paragraph 16.20

TRANSVERSE MERCATOR COORDINATES AND CONVERGENCE

Column 72

25. If column 72 contains a T, Transverse Mercator coordinates, convergence and the zone number are computed for all points, on the Australian Grid or UTM system. The complete formulae given by Redfearn (1948) are used. The lay-out of the output is altered to accommodate the additional information (see Figure 6), and the printing of  $\Delta\phi$ ,  $\Delta\lambda$  is suppressed; their average and maximum values is only suppressed if NCPRIN = 2. The extra computation is called from NEWCORDS (0.18ff) and STATPRIN (31.1), and coordinates and convergence are calculated in subroutine ENCONV.

PUNCHED CARD OUTPUT - "PUNCORD"

Column 79

26.1 Any entry in column 79 causes new 5-cards to be punched out automatically by NEWCORDS (44.1), ready for iteration, - see paragraph 77. A 5-card is punched for every station, fixed or variable. No harm is done if the whole lot are inserted in the data deck before iteration; but some 4-cards must be left in the deck, and it is usually worth removing the corresponding 5-cards to prevent the printing of a notice - see paragraph 45. These 5-cards contain the station name in columns 51-74, which is ignored by the computer on iteration.

26.2 If Transverse Mercator coordinates are being computed any entry in column 79 causes each 5-card to be preceded by a 10-card, giving th

TM station name, TM coordinates, zone number, convergence and height. If the point lies in a zone overlap, the 10-card is followed by an 11-card giving information in the adjacent zone. See paragraph 78.

#### SUPPRESSION OF PRINTING - "NOPRIN"

Column 80

27.1 A blank gives the full output. There are two alternatives.

27.2 A "1" in column 80 terminates the adjustment after the printing of the coordinate list. The printing of the observed and adjusted quantities is omitted, and a good deal of time is saved. This option should only be used for the iteration of an adjustment for which a previous run has proved the observations satisfactory: there is no other check that bad observations are not included in the data.

27.3 A "2" in column 80 suppresses most notices - those described in Paragraphs 39, 41.1, 42, 43, 45, 50, 51.

This option can be used to produce a clean output for general publication, free from all distracting matter of interest only to the computer. See Figure 6.

#### DECK STRUCTURE

28. The usage of the CDC 3600 computer, as operated by the C.S.I.R.O. in Canberra in 1966, is beyond the scope of this report, but some guidance is necessary for those wishing to use the program for the first time, which may also be of interest to those using other machines.

There are four ways of feeding the program into the machine:

- a) Fortran cards.
- b) Binary cards.
- c) Some subroutines in Fortran, others in binary.
- d) Binary magnetic tape.

#### FORTRAN DECKS

29.1 The Fortran program takes about  $3\frac{1}{2}$  minutes to read into the computer and compile, so that the Fortran deck will seldom be used unless the Fortran has been amended.

29.2 See page F-11 of the Fortran Manual (Control Data Corporation, 1964). The deck starts with a red JOB card, containing typically

```
7
9 JOB, CHDNM*TB, VARY, 10
```

That is to say, the user's code, the program name, and the time limit in minutes.

29.3 A blue Fortran card follows, possibly containing:

```
7
9 FTN, *, X,L,P.
```

The asterisk is always required.

The X indicates "Execution", and is omitted when Fortran is being tested without any data.

The L gives a listing of the Fortran program. Listing slows up compilation by about 40%, but if the program has been amended, a listing will usually be desired.

The P is only required if a binary program is to be made. A plain "P" will produce a binary program on punched cards. To make a binary magnetic tape, on say logical unit 44, write "P=44", and insert a green EQUIP card immediately after the job card:

```
7
9 EQUIP, 44= (VARYTAPE), SV, MT, RO
```

29.4 The Fortran program and subroutines follow, followed by blue SCOPE, LOAD and RUN cards. The latter will typically contain:

```
7
9 RUN, 10, 5000
```

indicating a time limit of 10 minutes, and a print limit of 5000 lines.

29.5 The data deck follows the RUN card, starting with 1-, 2-, 3-, 4-cards, and ending with an 8-card, a 9-card, and a blue card with

```
7
8 in column 1, which terminates the deck.
```

#### BINARY DECKS

30.1 A binary deck takes about 35 seconds to read in, and is convenient for general use. See page F-9 of the Fortran Manual.

30.2 The binary program and subroutines follow immediately after the JOB card. There is no SCOPE or LOAD card. The last card of the last

binary subroutine is followed by a RUN card, which is followed immediately by the data.

30.3 Different binary subroutines may be made for different tasks, e.g., for different standard errors in the R weighting option, or for latitudes north and longitudes west, or for angles in grades instead of degrees, minutes and seconds. As a rule, two or three subroutines only will be affected, usually STARTOFF, STATREAD and STATPRIN. The special subroutines should be boldly labelled. It is an easy matter to exchange them for the standard subroutines when required. See paragraph 79.

#### MIXED FORTRAN AND BINARY DECKS

31. When the program is amended, only a few of the subroutines are usually changed. The amended Fortran decks should be placed after a FTN card, and be followed by a SCOPE and LOAD card, exactly as in paragraph 29 above. The unchanged binary subroutines then follow the LOAD card, followed in turn by a RUN card and the data, as in paragraph 30.

The order of the subroutines is immaterial. The essential points are that the deck shall contain one program, and one only; and that each subroutine shall occur once, and once only. If both the Fortran and binary versions of a subroutine are included, or both are omitted, a monitor diagnostic LOADER ERROR will be obtained, giving a clue to the offending subroutine.

#### BINARY MAGNETIC TAPE

32. The program can be read in from magnetic tape in 17 seconds, which is about twice the speed of binary cards. However, a tape takes  $3\frac{1}{2}$  minutes to prepare, and at present a complete new tape has to be made whenever a single subroutine is amended. Moreover, it is not always possible for the operators to have the tape loaded when the job comes on, and sometimes the delays extend into minutes. At present (March, 1966) it is doubtful if any real saving occurs in using magnetic tape for program VARY, and binary cards are again in use. However, improvements are continually being made, and the situation may change again for the better.

32.1 Having prepared a binary tape VARYTAPE on say logical unit 44, (paragraph 29.5) the deck structure to use it is:

7	JOB, CHDNM*TB, VARY, 10	Red
9		
7	EQUIP, 44 = (VARYTAPE), SV, MI, RO.	Green
9		
7	POST, 44, VARY	Orange
9		

7 9	LOAD, 44	Blue
7 9	RUN, 10, 5000 Data (1,2,3,4,...8,9 cards)	Blue
7 8	End of job	Blue
*	EOD	Yellow

OUTPUT

33.1 The output consists of three parts; see Figures 5 and 6.

33.2 First, there is a series of Notices ( see paragraphs 34-49) giving details of the job, the spheroid, date of computation, etc., together with details of the weighting system used, the number and type of observations equations, details of the matrix of the coefficients in the normal equations, and details of any faulty data.

33.3 Secondly, there is a list of the adjusted coordinates. This will be in one of two forms, depending on whether Transverse Mercator coordinates have been asked for or not.

33.4 Thirdly, details are given, station by station, of the observed and adjusted quantities, their differences, and their average and maximum differences. The coordinates given in the second part are repeated here, so that the information for each station is complete. This part of the output may be cut up, and the pieces attached to the relevant field station summaries.

33.5 The whole of the third part, and many of the notices in the first part, may be suppressed if desired - see paragraph 27.

NOTICES

34. Notices are described below in the order in which they occur on the output. For diagnostics, which occur when a program fails, see paragraph 55-75. The subroutine and statement which writes the notice is indicated on the right hand side of the page.

PROGRAM AMENDED ../../..

STARTOFF (13,0)

35. This is the date the Fortran source deck was last amended. The date is altered by hand when necessary. The relevant card (STARTOFF (13.0)) is blue topped for ease of reference.

## SURVEY ADJUSTMENT - LEAST SQUARES VARIATION OF COORDINATES ON THE SPHEROID

S.A. - KINGOONYA TO WUDINNA

SECTION 54 46

AUSTRALIAN GEODETIC DATUM

A = 20925721.78 FT 1/F = 298.250

UNIT WEIGHTS ACCORD WITH THE FOLLOWING STANDARD ERRORS

DIRECTIONS (SECONDS) AZIMUTHS DISTANCES (FT)

0.5 1.0 0.1 + 3.0 PPM

NORMAL SECTION AZIMUTHS

OBSERVED VALUES OF ANGLES AND AZIMUTHS REJECTED IF MORE THAN 100 SECONDS FROM VALUES COMPUTED FROM COORDINATES

OBSERVED VALUES OF DISTANCES REJECTED IF MORE THAN 40 FT FROM VALUES COMPUTED FROM COORDINATES

NO REJECTIONS

ORDER OF MATRIX = 18 BANDWIDTH = 15 BANDMAT = 168 SOLUTION TIME IN SECONDS 0.555

NUMBER OF ACCEPTABLE OBSERVATIONS 57 OF WHICH ANGLES = 48 AZIMUTHS = 1 DISTANCES = 8

STATION	SERIAL	SOUTH LATITUDE	ADJ-INIT	EAST LONGITUDE	ADJ-INIT	RHO	NU	HEIGHT FT
FIXED POINTS								
KINGOONYA	NM E 53 1	30 57 46.9738	0.0000	135 20 57.4706	0.0000	20841004.87	20944286.77	576.0
RENTON	NM E 66 2	30 57 38.6658	0.0000	135 4 25.5006	0.0000	20840997.41	20944284.27	527.0
GAIRDNER	NM E 64 11	32 17 42.9566	0.0000	135 56 12.7959	0.0000	20845358.61	20945745.11	1103.0
NOTT	NM E 65 14	32 31 20.2833	0.0000	135 48 57.4313	0.0000	20846110.69	20945997.00	1415.1

## ADJUSTED POINTS

CLUCAS	NM E 55 3	31 15 33.9698	0.0099	135 24 38.9100	0.0054	20841964.44	20944608.21	647.8
CHITANILGA	NM E 54 4	31 14 52.8480	0.0089	135 13 34.2140	0.0067	20841927.36	20944595.79	1053.0
WAULALUMBO	NM E 57 5	31 28 44.0227	0.0053	135 30 44.1727	0.0006	20842678.35	20944847.35	744.5
DINGO	NM E 58 6	31 40 2.0750	0.0020	135 28 58.2088	-0.0001	20843293.32	20945053.34	677.3
HARPER	NM E 59 7	31 39 49.3725	0.0030	135 44 17.0714	-0.0039	20843281.78	20945049.47	831.0
BOND	NM E 60 8	31 50 53.8083	-0.0005	135 37 32.3479	-0.0024	20843886.36	20945251.98	1102.7
MUNGO	NM E 61 9	32 6 20.7733	0.0020	135 16 43.3135	-0.0026	20844733.08	20945535.59	1194.0
JUMPUPPY	NM E 62 10	32 5 49.4463	0.0034	135 41 7.8087	-0.0022	20844704.41	20945525.99	1141.7
YARDEA	NM E 63 13	32 22 21.8146	0.0031	135 30 54.2279	-0.0016	20845614.89	20945830.94	1386.3

AVERAGE	0.0042	AVERAGE	0.0028
MAXIMUM	0.0099 AT 3	MAXIMUM	0.0067 AT 4

OBSERVATIONS AT KINGOONYA	SERIAL	SOUTH LATITUDE	EAST LONGITUDE	HEIGHT	SECTION				
	NM E 53 1	30 57 46.9738	135 20 57.4706	576.0 FT	54 46				
OBSERVATIONS TO CLUCAS	NM E 55 3	ADJ AZIMUTH	OBS DIRN	OBS-ADJ	LPL AZ	LPL-ADJ	ADJ LENGTH	OBS LENGTH	CBS-ADJ
		169 53 30.14	30.01	-0.13			109517.077	109517.345	0.268
CHITANILGA	NM E 54 4	200 21 44.39	44.16	-0.23			110588.570		
RENTON	NM E 66 2	270 29 9.41	9.77	0.36	9.35	-0.06	86377.049	86377.173	0.124
ORIENTATION CONSTANT		200 21 44.16	AVERAGE	0.24	AVERAGE	0.06		AVERAGE	0.196
			MAXIMUM	0.36	MAXIMUM	0.06		MAXIMUM	0.268

## SURVEY ADJUSTMENT - LEAST SQUARES VARIATION OF COORDINATES ON THE SPHEROID

S.A. - KINGOONYA TO WUDINNA

SECTION 54 46

AUSTRALIAN GEODETTIC DATUM

A= 6378160.00 MS 1/F= 298,250

STATION	SERIAL	SOUTH LATITUDE	EAST LONGITUDE	ZONE	EASTING	MS	NORTHING	CONVERGENCE	HEIGHT MS
FIXED POINTS									
KINGOONYA	NM E 53 1	30 57 46,9738	135 20 57,4706	53	533359,116		6574429,047	+ 0 10 46,96	175.6
RENTON	NM E 66 2	30 57 38,6658	135 4 25,5006	53	507043,547		6574734,790	+ 0 2 16,59	160.6
GAIRDNER	NM E 64 11	32 17 42,9566	135 56 12,7959	53	588211,309		6426437,828	+ 0 30 2,14	336.2
NOTT	NM E 65 14	32 31 20,2833	135 48 57,4313	53	576632,418		6401362,944	+ 0 26 19,32	431.3
ADJUSTED POINTS									
CLUCAS	NM E 55 3	31 15 33,9698	135 24 38,9100	53	539111,974		6541560,708	+ 0 12 47,44	197.4
CHITANILGA	NM E 54 4	31 14 52,8480	135 13 34,2140	53	521535,633		6542877,399	+ 0 7 2,37	321.0
WAULALUMBO	NM E 57 5	31 28 44,0227	135 30 44,1727	53	548658,790		6517196,813	+ 0 16 3,02	226.9
DINGO	NM E 58 6	31 40 2,0750	135 28 58,2088	53	545770,769		6496333,543	+ 0 15 12,55	206.4
HARPER	NM E 59 7	31 39 49,3725	135 44 17,0714	53	569969,541		6496589,297	+ 0 23 14,84	253.3
BOND	NM E 60 8	31 50 53,8083	135 37 32,3479	53	559194,037		6476198,355	+ 0 19 48,53	336.1
MUNGO	NM E 61 9	32 6 20,7733	135 16 43,3135	53	526294,411		6447794,058	+ 0 8 53,25	363.9
JUMPUPPY	NM E 62 10	32 5 49,4463	135 41 7,8087	53	564681,972		6448587,005	+ 0 21 51,33	348.0
YARDEA	NM E 63 13	32 22 21,8146	135 30 54,2279	53	548453,100		6418120,166	+ 0 16 32,82	422.5

KINGOONYA	NM E 53			SECTION	54 46	SERIAL	1
SOUTH LATITUDE	EAST LONGITUDE	ZONE	EASTING	NORTHING	CONVERGENCE	HEIGHT	
30 57 46,9738	135 20 57,4706	53	533359,116	6574429,047	+0 10 46,96	175,6	
TO	SERIAL	ADJ AZIMUTH	OBS	LAPLACE	ADJ LENGTH	OBS	
CLUCAS	NM E 55 3	169 53 30,14	-0,13		33380,805	0,082	
CHITANILGA	NM E 54 4	200 21 44,39	-0,23		33707,396		
RENTON	NM E 66 2	270 29 9,41	0,36	-0,06	26327,725	0,038	

RENTON	NM E 66			SECTION	54 46	SERIAL	2
SOUTH LATITUDE	EAST LONGITUDE	ZONE	EASTING	NORTHING	CONVERGENCE	HEIGHT	
30 57 38,6658	135 4 25,5006	53	507043,547	6574734,790	+0 2 16,59	160,6	
TO	SERIAL	ADJ AZIMUTH	OBS	LAPLACE	ADJ LENGTH	OBS	
KINGOONYA	NM E 53 1	90 37 39,75	-0,16		26327,725	0,038	
CHITANILGA	NM E 54 4	155 30 2,76	0,16		35012,679		

COMPUTED.././..

STARTOFF (13.0)

36. This is the date of computation. It is set automatically, by calling DATE in statement (7.0) in STARTOFF.

## GEODEPIC SURVEY OF AUSTRALIA

SURVEY ADJUSTMENT - LEAST SQUARES VARIATION OF COORDINATES ON THE SPHEROID

STARTOFF (13.0)

37. This occurs on all jobs. It could be altered by hand for overseas use if necessary.

JOB, SECTION, SPHEROID

STARTOFF (4.0,15.0)

38. These are taken directly from the entries on the 3-card and 2-card respectively.

UNIT WEIGHTS ACCORD WITH THE FOLLOWING STANDARD ERRORS -

DIRECTION (SECONDS) AZIMUTHS DISTANCES (UNITS)

STARTOFF (18.0)

39.1 The quantities tabulated vary with the different weight options in column 74 of the 3-card (see paragraph 21):

Entry in Column 74:				STARTOFF
				Statement:
1	1	1	Blank	(22.0)
1	1	L/206265	W	(18.2)
1	1	SQRT(L/206265)	X	(20.1)
0.5	1.0	0.1 + 3 PPM	R (FT)	(21.1)
		0.03 + 3 PPM	(MS)	

39.2 Each observation equation is divided through by the respective standard error, but note that the standard error listed is for a direction, which is modified internally to give the standard error of an angle.

39.3 To alter the R option, it is necessary to alter not only

ANSIGSEC, AZSIGSEC, FTCON, MSCON AND PPMCON, but also JANG, JAZ and JDIST. Four cards are involved, which are blue topped for easy reference: STARTOFF (21.1, 21.2, 23.0 and 24.0).

39.4 The whole notice is suppressed if the NOPRIN option in column 80 of the 3-card contains a "2".

WEIGHTS OF ALL DISTANCES MULTIPLIED BY... STARTOFF 35.0

40. This notice is only printed if there is an entry in columns 75-76 of the 3-card, indicating that the WTFACOR is not unity. It is not suppressed by NOPRIN = 2.

NORMAL SECTION AZIMUTHS

GEODESIC AZIMUTHS STARTOFF (37.0)

41.1 Normal section azimuths are usual. For continental adjustments, where the "observations" consist of azimuths and lengths of whole sections of survey, geodesic azimuths may be preferred. It is essential to use geodesic azimuths in this program if the "observations" in fact consist of geodesic azimuths obtained from program "ROBBINS" or from Hiran surveys.

41.2 In March 1966, geodesic azimuths are obtained with the X weighting option (see paragraph 21) and in no other way. If geodesic azimuths are used, the notice is never suppressed. With normal section azimuths, the notice is suppressed if NOPRIN = 2.

OBSERVED VALUES OF ANGLES AND AZIMUTHS REJECTED IF MORE THAN ....

SECONDS FROM VALUES COMPUTED FROM COORDINATES

OBSERVED VALUES OF DISTANCES REJECTED IF MORE THAN ....

(UNITS) FROM VALUES COMPUTED FROM COORDINATES STARTOFF (17.0)

42. The Z and L entries in column 62-65 and 68-71 of the 3-card are here reproduced (see Options, paragraph 20), unless suppressed by NOPRIN = 2. Asterisks will be obtained if the entries exceed 9999. If the Z and L entries were blank, values of 100 and 40 are internally allocated.

NO REJECTIONS

MAIN (5.0)

43. This notice is printed if none of the O-C values (see Observation Equations, paragraph 6) exceed the Z and L limits set on the 3-card (see Options, paragraph 20.1). The notice is suppressed by NOPRIN = 2; otherwise its absence indicates a faulty adjustment, and we obtain the following notice, which is never suppressed:

THE FOLLOWING OBSERVATIONS EXCEED THE LIMITS LIMITEST (4.0)

TYPE 1 = ANGLE, TYPE 2 = AZIMUTH, TYPE 3 = DISTANCE

AT	TO	AND	TYPE	O-C	
16	14	17	1	3.601+003	
16	17	18	1	-3.599+003	LIMITEST (1.0)
21	22		3	1.001+002	

44.1 This example suggests that there is an error of about 3600 seconds, or one degree, in the direction from station 16 to 17, which has thrown out both the adjacent angles. In this case, the incorrect direction will be suppressed on the print-out by ANGFORM (13.0-16.0). This keeps the average and maximum adjustments printed by STATPRIN meaningful. A direction which throws out only one of the adjacent angles is not suppressed.

44.2 The last line of the example suggests there was an error of about 100 feet or 100 metres in the distance from station 21 to 22. As distances tabulated from higher station numbers to lowest station numbers are ignored by the computer (see paragraph 16.23), an error in input distance from 22 to 21 would cause no notice to be printed. Attention would be drawn to the error by the notices in paragraphs 52 and 51.

44.3 Errors of this sort occur not only when the observations are faulty, but when the preliminary coordinates are inaccurate. If the latter is likely, the limits L and Z should be set wide on the initial run, and tightened on iteration. If the faulty value of O-C only slightly exceeds the values of Z and L set on the 3-card, a cure is to increase these limits. It may be necessary to obtain new 5-cards by setting PUNCORD=1 in column 79 of the 3-card, and then to iterate.

ORDER OF MATRIX = ..... BAND WIDTH = ..... BANDMAT = ..... SOLUTION TIME  
 IN SECONDS ..... NUMBER OF ACCEPTABLE OBSERVATIONS ..... OF WHICH  
 ANGLES = ..... AZIMUTHS = ..... DISTANCES = .....

MAIN (8.0)

45.1 The order of the matrix, MATSIZE, which is twice the number of variable points, is formed in MAIN at (31.2). It is limited to 200 by the size of the arrays CONSTRHS and OUTPUT. A limited increase should be possible, especially for adjustments of narrow band width.

45.2 The band width, BANDWID, (see paragraph 11), is limited to one less than the order of the matrix. The band width required by each individual observation (THISBAND) is computed in ANGFORM (5.0 to 5.4), AZFORM (0.7, 0.8) and DISTFORM (0.7, 0.8) and put into BANDWID if it exceeds the previous maximum value.

45.3 The number of elements KMAX in the band matrix is computed in MAIN (5.1). It is currently limited to 10000 by the size of the array BANDMAT. Some increase is possible, but how much is not known.

45.4 The solution time of the normal equations is the time taken by subroutine BANDIT; see MAIN (202.1 to 202.3).

45.5 The number of acceptable observations is accumulated in NUMBEQ, and the number of each type in NANG, NAZ and NDIS, by the concluding statements in ANGFORM, AZFORM and DISTFORM. These accumulators are initially set to zero by MAIN (1.1). There is no limit to the number of observations, as they are processed one by one.

45.6 The notice is suppressed by NOPRIN = 2.

COORDINATES OF ... LISTED TWICE. POINT HELD FIXED LATLONIN (20.0)

46. If both a 4-card and a 5-card are included for the same station, the 5-card is ignored. When adding new 4-cards to a deck, it is thus not necessary to remove the corresponding 5-cards, of which a complete set can be left in the deck at all times. However, a tidier output results if redundant 5-cards are removed.

ALL OBSERVATIONS AT ... IGNORED AS NO COORDINATES LISTED STATREAD (13.0)

47. If the 6-card and 7-cards for a station are left in a deck which contains no corresponding 4-card or 5-card, the observations are ignored. This can be convenient when testing alternative adjustments, but before a formal adjustment either the coordinates should be put in, or the observations taken out.

AT .. OBSERVATIONS TO .. IGNORED AS NO COORDINATES LISTED STATREAD (18.0)

48. Observations to a station for which there is no 4-card or 5-card are ignored. This is convenient when one wants to test the effect of leaving out a station; all the observations can be left untouched, and the 5-card temporarily removed.

AT .. SINGLE DIRECTION TO .. IGNORED ANGFORM (12.0)

49. At least two directions are required at a station before an angle observation equation can be formed. If inadvertently only a single direction has been tabulated, or the others have been removed, the single arbitrarily oriented direction is meaningless, and is ignored.

AVERAGE ..... AVERAGE ..... PRINCORD (6.0, 7.0)

MAXIMUM ..... AT ..... MAXIMUM ..... AT ...

50.1 The average and maximum values of  $\Delta\phi, \Delta\lambda$  are printed at the end of the list of adjusted values, unless suppressed by NOPRIN = 2. The stations at which the maxima occur are indicated. The main use of these values is in deciding whether to iterate an adjustment - see paragraph 77.

ORIENTATION CONSTANT .. .. . AVERAGE ... AVERAGE ... AVERAGE ...  
 MAXIMUM ... MAXIMUM ... MAXIMUM ...

STATPRIN (19.0-20.0)

51.1 The average and maximum adjustment to each observed quantity is printed after every station, unless suppressed by NOPRIN = 2.

51.2 In the input, directions are arbitrarily oriented. In the output, adjusted azimuths are listed. To compare the two sets, the input directions are rotated through an angle called the Orientation Constant, to make the best mean fit on the adjusted azimuths. ORIENCON is computed by STATPRIN (6.12 ff). Thus:

$$\text{OBS DIRN} = \text{Input direction} + \text{Orientation Constant}$$

The value of an input direction can be deduced from the output by reversing this formula. It is, however, much more convenient to look at the actual data deck, or a listing of it, whenever possible.

51.3 These average and maximum values are formed without regard to sign.

WHOLE ADJUSTMENT

```

AVERAGE ...      AVERAGE ...      AVERAGE ...
MAXIMUM ...      MAXIMUM ...      AT.. AND..      MAXIMUM ... AT ...
ABSOLUTE
AVERAGE ...      AVERAGE...      AVERAGE ...
    
```

STATPRIN (22.0 ff)

52.1 The first two lines give the average and maximum values, for each type of observation, without regard to sign, for the whole adjustment. The stations at which maxima occur are indicated.

52.2 This notice is an important guide to the quality of the observations. If the maxima are low, say 1.5" for directions and azimuths and 1.5 feet for distances on first order work, the adjustment is likely to be free from serious error, unless a series of large corrections occur together. The maximum values are easily located and checked.

52.3 This notice is never suppressed. No adjustment should be accepted until it has been read, and a check has been made to see that there have been no rejections.

52.4 The absolute averages are formed with regard to sign, and should be close to zero, in free adjustments. They will only be exactly zero if all the observations of each type have equal weights. This will only occur with distances if the W option (column 74 of the 3-card - see paragraph 21) is left blank.

TIME FOR THIS SECTION ..... SECONDS MAIN (13.0)

53.1 This is the time which elapses between statement 1.0 and 100.0 in MAIN. It excludes the time taken to process the 1-card and 2-card, and to set various accumulators to zero at the start of a data deck. The time at any instant is obtained from the computer clock by a statement such as

TIMENOW = TIMEF ( DUMMY )

53.2 The notice is suppressed by NOPRIN = 2.

ASTERISKS IN A NUMERICAL FIELD

54. If for example the computer tries to print the number 0.04 in a format F3.1, instead of printing "0.0" it prints "\*\*\*\*" as a warning that the 4 has been dropped. For many survey purposes, this is a nuisance; if the adjustment to an angle is less than 0".005, surveyors would rather have 0".00 printed than asterisks, which causes alarm to those not acquainted with their significance. In the VARYCORD program, steps have been taken to program round this difficulty by such statements as

$$\text{IF (ABSDLAT.LE.5E-05) SECDLAT} = 0.$$

but it is conceivable that asterisks may still occur in some unforeseen combination of circumstances. Note that it is the absolute value of the function that must be less than or equal to the set limit, otherwise all negative values are set to zero also.

DIAGNOSTICS

55.1 Diagnostics give information about an adjustment which has failed. Notices give information about an adjustment, which while not necessarily satisfactory, has not terminated abnormally. Notices are listed separately in paragraphs 34-54.

55.2 There are two sorts of diagnostic, those written by the monitor system, and those written by the Varycord program. Diagnostics about errors in the Fortran are not here considered - see the Fortran Reference Manual (Control Data Corporation, 1964), Appendix H.

Monitor Diagnostics

56. These occur when the program loses control and the monitor automatically takes over. The program is designed to make loss of control rare, but no way of preventing loss of control by format errors in the data is available in March 1966. After a monitor diagnostic, the program cannot regain control, and subsequent adjustments are not run. Adjustments suspected of containing format errors should therefore be placed at the end of the job.

Input Format Errors

57. See Figure 7. The diagnostic indicates that a format error was found on tape 60, the standard input tape, when the format statement shown was read from subroutine STATREAD. The arrows indicate that an asterisk was found in an F4 format. This occurred on the 7-card

DATA ERROR DURING INPUT CONVERSION ON TAPE - 60  
INVALID CHARACTER ON INPUT

FORMAT,., 036764  
(I2,13X,I3,2(F4,F3,F7,F3),F12,F4)  
↑

DATA,., 032400  
7 8 13 1 2 360 0 0,00 1,0 213 51 52,26 0,0 56942,862\*  
↑

\*\*\*

A = 6060606060606060 Q = 0000002100032357  
ERROR DETECTED IN ROUTINE I/O TSH,  
CALLED FROM STATREAD + 00162  
CALLED FROM MAIN + 00242  
CALLED FROM VARYCORD + 00010

FIGURE 7: DIAGNOSTIC FOR INPUT FORMAT ERROR

\*\*\* NEG ARG,

A = 5775177777777777 Q = 7777777777777777  
ERROR DETECTED IN ROUTINE SQRTF  
CALLED FROM DISTFORM + 00057  
CALLED FROM OBSFORM + 00077  
CALLED FROM MAIN + 00250  
CALLED FROM VARYCORD + 00010

\*\*\* NEG ARG,

A = 5775177777777777 Q = 7777777777777777  
ERROR DETECTED IN ROUTINE SQRTF  
CALLED FROM DISTFORM + 00057  
CALLED FROM OBSFORM + 00077  
CALLED FROM MAIN + 00250  
CALLED FROM VARYCORD + 00010

\*\*\* NEG ARG,

A = 5775177777777777 Q = 7777777777777777  
ERROR DETECTED IN ROUTINE SQRTF  
CALLED FROM DISTFORM + 00057  
CALLED FROM OBSFORM + 00077  
CALLED FROM MAIN + 00250  
CALLED FROM VARYCORD + 00010

FIGURE 8: DIAGNOSTIC FOR SQUARE-ROOT OF NEGATIVE NUMBER.

for section 8-13, on the ray from station serial 1 to station serial 2. The faulty card can be found at once. It is often wise to consider if subsequent cards in the same deck are likely to have similar errors. In this case, the 7-card concerned contains a Laplace azimuth, and was probably punched by the computer. The asterisk was found in the individual weight for the distance probably because the weight had been set to "1E2" or some similar value, which the computer cannot punch in an F format. Subsequent 7-cards containing Laplace azimuths may well contain asterisks too, and should be checked. On re-running, it is wise to put this adjustment last.

#### RECOVER TIMESUP

58. The time limit entered on the JOB or RUN card has been attained before the job is complete. Either the limits can be increased or jobs kept smaller. The last adjustment attempted is likely to be incomplete.

#### RECOVER PRINT LIMIT

59. The print limit entered on the RUN card was insufficient for the size of the job. Limits should be kept reasonably small. Otherwise a faulty job may run on and waste a lot of money; and the operators run short jobs before they run long ones.

#### RECOVER OPERATOR TERM

60. The job has been terminated by the operator. This will seldom be done without good cause, such as a torn card, a dropped deck, or such a long delay without visible action that the operator believes the program has gone into a loop, and it would be kinder to the customer to terminate the job than to let it run on to the time limit. If the operator terminates the job unnecessarily (as has occurred when unusually large number of normal equations were being solved, keeping the machine busy but silent for about four minutes), a free re-run should be requested.

#### NEGATIVE ARGUMENT

61.1 If the computer is asked to compute the square root of a negative number, it prints out a diagnostic showing the subroutine in which the mistake occurred. See Figure 8. The machine tried to take the square root of three negative numbers in DISTMOD. This has occurred

when weights have been inserted for distances on 7-cards punched by the computer, i.e., those that contain Laplace azimuths.

If the individual weight box was originally blank, -0.0 was punched, to distinguish the blank from an entry of 0.0. When the weight was altered to 2.0, without deleting the negative sign, this diagnostic was obtained.

61.2 The adjustment does not stop. The square root is taken as zero, and computation proceeds, but in survey calculations the results will nearly always be incorrect. It is therefore essential to re-run the job with the faulty cards corrected.

UNCHECKED END OF FILE

62. This diagnostic is printed if the computer finds a  $\bar{7}$  end-of-file card without having reached the instruction STOP, as would occur if the 9-card were omitted. The last adjustment computed should be checked, but is likely to be satisfactory, if the 9-card is missing. If the 9-card is present, some other irregularity at the end of the deck has likely occurred.

LOADER ERROR

63. Loader errors occur if both the Fortran and binary decks for a subroutine are inadvertently included in the deck, or if both are omitted. They may also occur when the program loses control due to an unknown cause - see the next paragraph.

CONTROL STATEMENT FORMAT ERROR

64.1 This diagnostic occurs, usually without any further clue, when the program loses control. Known causes have included the following.

64.2 Attempted division by zero, causing exponential overflow. This occurs with some weighting systems if the distance computed from the preliminary coordinates is zero. This could occur if two 5-cards erroneously contained the same coordinates, or if faulty serial numbers implied an observation from a station back into itself. The program diagnostic "Serial Numbers Faulty at ....." (paragraph 72) has been incorporated in the program to overcome the latter difficulty.

64.3 Subscripts exceeding the declared dimension of an array.

For example, the dimension of the array SINLAT is declared in COMMON to be 140. If the program looks for SINLAT (141), it will likely look at the first element in the next array, COSLAT (1). This may contain garbage, and send the program out of control.

64.4 For further assistance with monitor diagnostics, see C.S.I.R.O., Computing Research Station publication EPA 4 (1966).

PROGRAM DIAGNOSTICS

65. When a program diagnostic is written, the program retains control. A switch occurs to entry point DATAFAIL in subroutine MAIN, which reads through the rest of the cards in the faulty deck until an 8-card is encountered, when the next adjustment, if any, is processed normally.

Written by MAIN:

"NO 7 CARDS AT . . STATIONS. ADJUSTMENT TERMINATED". Statement 23.

66. This indicates that a 4-card or 5-card has been included for a station where a 6-card and 7-card have been omitted. Even for intersected points, where no observations have been made, a 6-card and at least two 7-cards, one of each sort, must be included in the deck; otherwise the station name is nowhere recorded, and the print-out fails. It is advisable to include a 7-card for every ray into a station, even if there are no outward observations, containing the station numbers for the ray but with the observation boxes left blank.

"MATRIX SINGULAR. NO COEFFICIENTS IN COLUMN . ." Statement 11.

67.1 For the causes of singular matrixes, see paragraph 12. Columns 1 and 2 refer to the first variable station, 3 and 4 to the next variable station, and so on. Thus if there is no coefficient in column 31, there were insufficient observations to fix the 16th variable station. If the first or last column has no coefficient, it suggests that either there was no redundancy to adjust, or there were insufficient fixed points. This diagnostic has also been obtained when more than 15 observation cards have been included at a station.

"MATRIX PARAMETERS FAULTY. ADJUSTMENT TERMINATED". Statement 20.1

67.2 This diagnostic is called by subroutine BANDIT at statement 0.8. It indicates that the order of the matrix or the bandwidth is zero or negative, or the bandwidth is greater than or equal to the order. As the order and the bandwidth are now calculated internally, it is unlikely that

this diagnostic will occur in future.

"PARITY ERROR ON SCRATCH TAPE . . ."

Statement 26.

68. Three scratch tapes are used for the temporary storage of information, which are checked for parity errors when they are read. If a parity error occurs the tape is automatically re-read three times. If the parity error persists, this diagnostic is printed. A free re-run should be requested with new tape. In 1967, the tapes have been replaced by a drum, and this diagnostic is unlikely to occur.

Written by STARTOFF

"2-CARD MISSING"

Statement 12.

69. This indicates that either a 2-card has been omitted, or the 1-card and the 2-card have been placed in the wrong order. More generally, the diagnostic may occur if there is some error in the card order at the beginning or end of an adjustment.

Written by LATLONIN

"NO FIXED POINTS"

Statement 8.

"NO VARIABLE POINTS"

Statement 13.

70.1 This indicates that the 4-cards or the 5-cards have been left out of the deck.

"LATLONGS NOT FOLLOWED BY 6 CARD".

Statement 15.

70.2 The last 5-card is normally followed by the 6-card for the first station. The diagnostic indicates that either this 6-card has been omitted, or there is a stray card among the 4-cards or 5-cards.

"5-CARDS OUT OF ORDER AT ... AND ..."

Statement 23.

71. The 5-cards must be in numerical order by station serial numbers. There is no objection to gaps at the beginning or in the middle of the sequence. But if a higher number precedes a lower number, the adjustment is terminated, and this diagnostic is printed giving both the offending serial numbers.

72. These four diagnostics written by LATLONIN are followed by:

"SECTION . . . . . SKIPPED".

Statement 16.

Written by STATREAD.

"SERIAL NUMBERS FAULTY AT . . ."

Statement 5.

73. This indicates that the same station serial number has occurred twice on the same card, i.e., that an observation has apparently been made

from a station back into itself. Errors in station numbers may cause monitor diagnostics, as the computer searches the store for functions of the non-existent station, and the program loses control. A distance of zero can also cause an "exponential overflow" monitor diagnostic, terminating the complete job, when a division by zero is attempted by DISTFORM (12.1).

"ERROR IN CARD ORDER AT . . . SECTION . . . SKIPPED" Statement 17.

74. The 7-cards at the indicated station either contain, or are followed by, a card other than a 6-card or an 8-card. The most usual case is a 7-card in which the "7" has not been punched.

#### Written by LIMITEST

". . . BAD OBSERVATIONS, ADJUSTMENT TERMINATED." Statement 2.

75.1 The number of rejected observations which the program will try to tolerate, together with the rejection levels, is fixed by entries on the 3-card - see "Options", paragraph 20.1. Too many rejections will cause the matrix to be singular.

75.2 The diagnostic indicates that the limiting number of bad observations has been attained. The bad observations will be tabulated immediately above - see Notices, paragraph 44.

#### INSPECTION OF OUTPUT

76. On receiving an adjustment from the computer, the following points should always be checked:

- (a) Correct spheroid?
- (b) Correct weighting system?
- (c) Any rejected observations?
- (d) Maximum values of  $\Delta\phi, \Delta\lambda$ ; is iteration required?
- (e) Average and maximum adjustments to observations; do they accord with the expected quality of the work?
- (f) Stations at which maximum adjustments occur; does the data need checking?

#### ITERATION

77.1 The formulae in paragraph 5.2 include the distance and the azimuths between the stations. The values used in the computer are those computed

from the preliminary coordinates. If the preliminary coordinates are incorrect, the azimuths and distances will be incorrect, and the formulae will be inexact. It is then necessary to iterate. Convergence is quick, and the following empirical table may be used as a practical guide:

Maximum error in preliminary coordinates:	Maximum error in adjusted coordinates:
20".	3".
3".	0".03
0".5	0".0004
0".25	0".0001

77.2 The computer can punch out the adjusted coordinates on 5-cards, ready for immediate substitution in the data deck - see paragraph 26. Completely automatic iteration has not been programmed, as it is always desirable to inspect the output, as in paragraph 76, before iterating. The name of the station is punched out in columns 66-80 of the 5-cards; this is ignored by VARYCORD, but could be useful if the cards are required for other purposes.

#### RECORDING OF ADJUSTED COORDINATES ON CARDS

78. If there is an entry in PUNCORD and Transverse Mercator coordinates are computed, then a 10-card is punched, prior to each 5-card, giving the TM coordinates in the following format:

Column 1 - 2: 10  
 3 - 26: Station name  
 27 - 29: UTM Zone  
 34 - 43: Eastings in metres  
 46 - 57: Northings in metres  
 60 - 71: Convergence  
 74 - 79: Height.

If the point lies in a zone overlap, an 11-card is also punched, giving coordinates in the adjacent zone. The format is identical except that columns 1 - 2 contain 11, and the height is omitted.

78.2 Taken together, 10-, 11- and 5-cards contain all the adjusted coordinates for a point ready for sorting and listing by some other program.

### ALTERNATIVE VERSIONS

79.1 As the number of options gets large, the incorporation of additional options gets increasingly difficult. At some stage it becomes almost impossible to ensure that every possible combination of options works correctly. It is easier to make alternative versions of the program for different tasks.

79.2 In May 1967, the following versions exist. In each case, only a few subroutines are affected. Fortran listings can be supplied on request.

### VARYTWO

80.1 All the write statements, which are normally routed through logical unit 61, are duplicated through logical unit 41. Program VARYTWO can be used as follows:

1. Eight part listings can be obtained, four parts from each of two printers, by inserting the following EQUIP cards:

```
*EQUIP, 41=LP,4P
*EQUIP, 61=LP,4P
```

If fewer than 8 copies are required, either of the printers can be equipped with one or two part paper, and if one of the printers is not required, logical unit 41 (but not logical unit 61) can be by-passed:

```
*EQUIP, 41=BY
```

2. The two printers can be used to produce different versions of the output, by inserting a special version of subroutine STATPRIN, which produces normal output with TM coordinates on logical unit 41, but a listing with the observations and corrections suppressed on logical unit 61 (see paragraph 27.4). Thus one can obtain up to four copies of one version and up to four copies of the other with a single pass through the computer.
3. The output can be listed on magnetic tape, from which further copies can be obtained with program TAPELIST, more cheaply than by recomputation with program VARYCORD. A listing of what is put on the tape is obtained from the other logical unit:

```
*EQUIP, 41=(NGA/66/1),MT,SV
*EQUIP, 61=LP
```

Subroutine STARTOFF contains two extra statements, which print the section number in a special format immediately before each section, and an end-of-file mark immediately after the last section, which enable program TAPELIST to select any particular section for listing, instead of listing the whole tape.

The affected subroutines are:

```
VARYTWO (Main program)
MAIN
STARTOFF
PRINCORD
NEWCORDS
STATPRIN
```

### VARYPLAN

80.2 This version can only be used with the Transverse Mercator option (see paragraph 25). In addition to coordinates and convergence, the point scale factor is printed for each station, and the following are listed for each ray:

Grid Bearing (= Adjusted azimuth plus convergence)

Plane Bearing (=  $\text{Arctan}(\Delta E^2 + \Delta N^2)$ )

Plane Distance (=  $\text{SQRT}(\Delta E^2 + \Delta N^2)$ )

The arc-to-chord correction is the plane bearing minus the grid bearing; the line scale factor is the plane distance divided by the adjusted length. Neither is listed. The affected subroutines are:

```
VARYPLAN (Main program)
NEWCORDS
STATPRIN
ENCONV
```

### VARYGRAD

80.3 The input directions and output azimuths are in grads and three decimals (100 grads equal one right angle). This version is used for adjusting photogrammetric radial triangulation, using angles read on a Wild RT 1 radial triangulator. The affected subroutines are:

```
VARYGRAD (Main program)
STATREAD
STATPRIN
```

VARYNW

80.4 The standard program is designed for use in Australia, and unsigned latitudes are taken to be south, and unsigned longitudes to be east. For occasional use in the northern and western hemispheres, unsigned latitudes and longitudes can be fed in, and correct results will be obtained, provided all input azimuths are changed by  $180^{\circ}$ , i.e., measured clockwise from south. All output azimuths will similarly be measured from south, but all coordinates and adjusted quantities will be correct. Program VARYNW works with north latitudes, west longitudes, and azimuths clockwise from north. The affected subroutines are:

```
VARYNW (Main program)
MAIN
LATLONIN
NEWCORDS
STATPRIN
```

It would not be difficult to make a version for use with latitudes north and longitudes east.

PROGRAM VARY

## Appendix A

This is the master program. The rest are subroutines. It is simply a dummy, which calls subroutine MAIN. The main program may not have an alternative ENTRY point, and it is convenient for MAIN to have a second entry point, DATAFAIL.

SUBROUTINE MAIN

## Appendix B

This is the main subroutine, whose chief function is to call the other subroutines in succession as required. See the block diagram, Figure 2.

0.1 EQUIVALENCE. See the Fortran manual, page 4-7. The band matrix of the coefficients in the normal equations BANDMAT shares storage with the arrays ISTA . . . . SECLON listed in the first COMMON statement, which are either destroyed (5.1 to 6.0) or temporarily stored on magnetic tape, before the normal equations are formed. Similarly, the column of the constants on the right hand side of the normal equations, CONTRHS, shares storage with the solution OUTPUT.

0.2 COMMON. See the Fortran manual, page 4-4. All the COMMON blocks of storage used anywhere in the program are for convenience tabulated here, whether they are used by subroutine MAIN or not. The gaps in the serial numbers of the COMMON blocks are not significant. The dimension of the arrays are declared in the COMMON statements, avoiding duplicate DIMENSION statements. To increase the size of any array, for example, the number of fixed points, it is necessary to alter the present dimension 40 and 140 to say 60 and 160 in all COMMON statements. It will also be necessary to alter some DO Limits.

0.5 TYPE INTEGER. It is necessary to declare all integers which do not begin with the letters I, J, K, L, M or N. Similarly, many subroutines contain a TYPE REAL statement, declaring those floating-point quantities which, by mischance or for convenience, start with the letters I, J, K, L, M or N.

0.6 Various constants are defined, which remain in store for all adjustments.

1.0 This is the starting point of each separate adjustment. The time is registered, and various counters, accumulators and arrays are set to zero.

2.0 Subroutine STARTOFF is called to read and process the 2-card and the 3-card, followed by subroutine LATLONIN, to process the 4-cards and 5-cards.

2.2 Before anything is written on the scratch tapes 51, 52 and 53, they are checked to ensure that they are in the REWIND state.

3.0 Subroutine STATREAD is called to read and process the 6-card and 7-cards for one station only. On return, the signal N is checked. If zero, the program continues normally. If negative, there is a jump

to 17.0, skipping the formation of observation equations, but letting the adjustment proceed; this occurs for example when no coordinates have been listed for the station. If N is positive, the adjustment is terminated (due typically to an error in the card order), and there is a jump to 99.0, DATAFAIL, to get rid of the remaining data cards for this section before the next adjustment is attempted.

18.0 Subroutine OBSFORM is called to form all the observations equations at this one station.

18.1 The arrays of observed quantities IOBSVAL, OBSVAL at this station are written in binary on scratch tape 51. These arrays are formed by STATREAD, but the writing on tape is delayed until after the observation equations are formed, so that bad observations may be deleted. The number of stations NUMBOBS is increased by one, and the signal N re-set to zero.

17.2 If a 6-card follows, data for another station is waiting. There is a loop back to 3.0, and this loop is repeated until all the data has been read, and all the observation equations formed. After the last station, there will be an 8-card, and the program then continues.

17.3 There must be 7-cards for all stations for which there are 4- or 5-cards, even if all the observations are blank, (e.g., an intersected point). NUMBOBS is checked against the sum of NUMFLX and NUMVAR, and if there is a discrepancy, a diagnostic is printed, there is a jump to DATAFAIL, and the adjustment is terminated.

22.0 The arrays ISTA and STA, containing the fixed data for each station, are written in binary on scratch tape 53. The limit MAXAT, the highest station serial number, has been formed by LAPLONIN at 7.0.

31.1 Scratch tape 52, containing the observation equations formed in ANGFORM, etc., is rewound.

31.2 The order MATSIZE of the matrix of the coefficients in the normal equations is twice the number of variable stations. The formula for calculating BANDWID in subroutines ANGFORM, etc., can sometimes give a value equalling or exceeding MATSIZE. If this occurs, BANDWID is set to MATSIZE - 1, as the band width is defined to exclude the principal diagonal. BMINUSM is a parameter required by subroutine BANDIT, which solves the normal equations.

31.5 If no value of O-C has exceeded the arbitrary limits set on the 3-card, "NO REJECTIONS" is printed unless suppressed by NORPIN = 2.

5.1 KMAX is the number of elements in the band matrix. If the order of the matrix is N, and the band width exclusive of the principal diagonal is M, then

$$KMAX = (M + 1) (2N - M) / 2$$

5.2 The arrays BANDMAT and CONSTRHS which are to contain the normal

equations are cleared to zero. This storage has previously been used for the arrays in the first two COMMON statements.

6.1 The observation equations written by ANGF<sub>FORM</sub>, etc., on tape 52, were rewound at 31.1 above, and are now read back in, one at a time, for processing by subroutine NORM<sub>FORM</sub>. A parity check is made on reading. If it fails, there is a jump to 98.0 below, a diagnostic is printed, and the adjustment terminated. Otherwise NORM<sub>FORM</sub> is called for each observation, until the final coefficients of the normal equations have been formed in BAND<sub>MAT</sub> and CONSTR<sub>HIS</sub>.

202.5 The counter for the number of observed angles, NANG, is also used internally in ANGF<sub>FORM</sub> as an on-off signal, by incrementing it by 5000. It is here necessary to subtract 5000 from NANG if it exceeds that figure. This device currently sets an artificial limit of 5000 on the number of observed angles that can be incorporated in one adjustment, but the restriction could easily be removed.

202.6 Details of the matrix are printed unless suppressed by NOPRIN =2.

8.1 On return from BANDIT, the signal N is tested. If N is negative, BANDIT was called with impossible parameters; if positive, the matrix was singular, with zero coefficients in column N. In either case, a diagnostic is printed, and there is a jump to 99.0, DATAFAIL. If N is zero, the program proceeds, rewinding scratch tapes 51 and 53, which now contain the arrays IOBSVAL, OBSVAL and ISTA, STA respectively.

10.3 The whole of arrays ISTA and STA are read in and parity checked. If the check fails, a diagnostic is printed at 98.0 and the adjustment is terminated at 99.0

28.1 Headings are printed for the output of adjusted latitudes and longitudes. Subroutine PRINCORD is called to print the adjusted coordinates.

34.1 The adjustment will be terminated at this stage if NOPRIN = 1; otherwise subroutine STATPRIN is called to print the adjusted and observed values of all the observed quantities, station by station. STATPRIN reads scratch tape 51; if a parity error occurs it returns with signal N set to 1. A diagnostic is printed at 98.0 and the adjustment is terminated at 99.0.

100.0 The time for this adjustment is printed in seconds unless NOPRIN = 2. This ends this adjustment and there is a loop via 99.0 to 1.0 above, and thence to STARTOFF, to see if another adjustment follows.

98.0 This is the printer for the diagnostic indicating that a parity error has occurred on scratch tape ITAPE.

99.0 This is the entry for DATAFAIL and terminates the adjustment. Cards are read in and discarded until an 8-card is found, whereupon control reverts to 1.0 above, and the next adjustment, if any, is begun. If there is no further adjustment, a card of type 9 is read by STARTOFF and the programme is terminated with all the data cards cleared through the card reader.

SUBROUTINE STARTOFF

STARTOFF reads in and processes the 1-, 2-, 3- and 9-cards. It computes some functions of the spheroid, and prints notices regarding the options.

0.4 DIMENSION. The size of most arrays is declared in COMMON. A DIMENSION statement is here used to declare the size of two arrays which occur in this subroutine only. Both are titles written in alpha-numeric characters. A word contains eight characters, so these arrays of 5 and 6 words contain 40 and 48 characters respectively.

1.0 A card is read on the standard input card reader, logical unit 60, using FORMAT statement 2.

2.1 If it was a 9-card, there is a jump to STOP, and control reverts to the monitor system. This is the normal termination of a run.

6.0 If it was a 3-card, there is a jump to 7.0 below. On the first adjustment of any run, and optionally on later ones, the first card read will be a 1-card, which will be followed by a 2-card, defining a spheroid.

9.1 If a 1-card is not followed by a 2-card, a diagnostic is printed, and there is a jump to DATAFAIL, terminating this adjustment.

10.0 When a 2-card is read, various functions of the spheroid are computed, and stored in COMMON/4/ for use by other subroutines. There is a loop back to 1.0. This time a 3-card should be read, containing the section number and name, followed by the options.

7.0 When a 3-card is read, the DATE is called, and the initial notices are printed, followed by the job and section names from the 3-card, and the spheroid data from the last 2-card. Format statement (13.0) contains the date the program was last amended, and is blue topped.

16.1 Weights are now set according to whether the weight option (column 74 on the 3-card) contains a blank, W, X or R, and the corresponding notice is printed, unless suppressed by NOPRIN = 2. The values of the standard errors in the R option are set in statements 21.1, 21.2, 23.0 and 24.0. These cards are blue topped.

31.1 The weight factor option for distances (column 75-76 of the 3-card) is processed, unless the entry is blank, indicating a factor of 1. Otherwise, the square root of the factor is set in RTWTFAC, and a notice is printed.

35.1 With the weight option X, geodesic azimuths are used, and a notice is printed. Otherwise, normal section azimuths are used, and a notice is

printed unless suppressed by NOPRIN = 2.

39.1 If the limits Z and L on the 3-card have been left blank, they are set to 100" and 40 units of length. If the number of permissible rejects LIMIT has been left blank, it is set to 5. A notice is printed showing the values used, unless suppressed by NOPRIN = 2.

17.1 At the END of the subroutine, control automatically reverts to the calling program MAIN.

LATLONIN reads in the preliminary coordinates for all points, and computes various functions of the latitudes.

0.8 Three counters and the array INDIC are set to zero.

2.0 A card is read. It should be a 4-card (for a fixed point), a 5-card (for a variable point) or a 6-card (if all the coordinates have been read in). There is a jump to 4.0, 5.0 or 6.0 accordingly. Any other type of card gives a jump to 9.0. Latitudes are put in DEGPHI etc., and longitudes in DEGLAM, etc., as DEGLAT and DEGLON are dimensioned arrays used for the adjusted quantities.

4.0 When a 4-card is read, NUMFIX is incremented, the station serial number AT is registered in the array IFIX, and the signal INDIC(AT) is set to 888, the arbitrary signal for a fixed point. There is a jump to 7.0.

5.0 When a 5-card is read, NUMVAR is incremented, and the serial number AT is registered in IVAR. If INDIC(AT) is blank, it is set to  $(2 \times \text{NUMVAR})$ . Thus the column numbers in the normal equation matrix for each variable point are INDIC(AT) and INDIC(AT) - 1. The program is currently limited to 100 variable points, 200 columns, so the arbitrary 888 label for fixed points is well clear. If INDIC(AT) was not blank, coordinates for this point have already been read in, and there is a jump to 19.0

7.0 For both fixed and variable points, there is a check to see if AT is the largest value so far read; if so it is registered in MAXAT, which is needed by MAIN at 22.0

7.1 Function RADFORM converts the latitudes and longitudes to radians. With the standard input for use in Australia, signs are altered so that latitudes become positive north, and longitudes positive west. The relevant cards are green topped; it is easy to change the signs for other conventions in the input. Special binary subroutines should be made. See paragraph 80.4

7.3 The sine and cosine of the latitude are computed, and subroutine RHONU called to compute Rho and Nu. Then there is a loop back to 2.0 to read the next card.

19.0 If the same point is listed twice, it is held fixed with the coordinates read in first, and a notice is printed. This is sometimes convenient: if we want to hold a point fixed, we can insert a 4-card, but it is not essential to remove the corresponding 5-card.

6.0 When all the preliminary coordinates have been read in, there is a check to ensure that there is at least one fixed and at least one variable point. If there is no fixed point, the matrix is singular; and if there are no variable points, we are wasting our time. If all is well, there is a RETURN to the calling subroutine MAIN. Otherwise, a diagnostic is printed, and there a jump to 14.0.

9.0 If LATLONIN encounters a card which is neither a 4-, 5- or 6-card, a diagnostic is printed.

14.0 All diagnostics are followed by the legend "SECTION .....  
...SKIPPED". The adjustment is terminated and DATAFAIL called.

SUBROUTINE RHONU

RHONU computes  $\rho$  and  $\nu$  at station IAT, given  $A$ ,  $E^2$  and  $\text{SINLAT}(IAT)$ . To obtain  $\text{SINLAT}$ , the whole of the bulky BANDMAT block has to be made available in COMMON.

The formulae used are:

$$\rho = a(1 - e^2) / (1 - e^2 \sin^2 \phi)^{3/2}$$

$$\nu = a / (1 - e^2 \sin^2 \phi)^{1/2}$$

By first computing

$$\text{BOT} = (1 - e^2 \sin^2 \phi)^{1/2}$$

the computation of both  $\rho$  and  $\nu$  can be programmed in three statements, without further exponentiation.

At the end, control returns to the calling subroutine, usually  $\text{LATLONIN}$ . In some versions of the programme where  $\rho$  and  $\nu$  for the adjusted points are included in the trig list, RHONU is called again later by  $\text{NEWCORDS}$  at 0.17.

STATREAD reads in all the observations at one station.

0.9 Counters and signals are set to zero.

20.0 A card is read. The first 6-card has already been read by LATLONIN and the initial 7-card containing the station name and height is expected. If a 7-card is not found, there is a jump to 16.0, and the adjustment is terminated.

1.1 If the M option is in use, heights in feet have to be converted to metres. Heights are read and printed in A format, to ensure that blank heights are printed as blanks, not as zero. Thus it is necessary to DECODE before multiplying by 0.3048, and to ENCODE again afterwards, and then convert any zeros to blanks.

2.0 INDIC(AT) is tested to see if any coordinates have been read for this station. If not, all the observations at this station are ignored; there is a jump to 7.0, and the signal NOCORD is set to 1.

2.1 Before reading any observations, it is necessary to clear the array RADIRN to zero, because RADIRN (NUMLINE + 1) is tested against zero in ANGFORM (0.7). It is for this reason that the DIMENSION of RADIRN is 17, not 16. It is not necessary to clear the corresponding arrays RADAZ and OBSDIST.

12.0 NUMLINE is increased by one, and a 7-card read, which should be of the second sort, containing observations.

6.1 If the M option is in use, distances are multiplied by 0.3048.

6.2 If NOCORD has been set to 1, indicating that observations at this station are to be ignored, another card is read forthwith. This loop is continued until all 7-cards at this station have been read, and a 6-card or an 8-card has been found. Then NOCORD is set back to zero, the signal N is set to -1 (indicating that observation equations should not be formed at this station), and control reverts to the calling subroutine MAIN.

22.0 If the card read at 12.0 was not a 7-card, there is a jump to 3.0. This may be the normal termination of the station by a 6-card or an 8-card, or it may be an abnormality.

22.1 If a serial number by oversight exceeds 140 (the maximum dimension of numerous arrays in COMMON) the program would lose control,

and the whole run be terminated. Again, if by mischance the serial number AT equals TO, a computed distance would later be obtained equal to zero. On dividing by zero in DISTFORM (12.1 and 14.1), an "exponential overflow" failure would occur, and again the whole run would be terminated. To avoid abnormal termination, the values of AT and TO are here tested, and if unsatisfactory, a diagnostic is printed, and DATAFAIL called. Any subsequent adjustments will run normally.

8.0 INDIC(TO) is tested to see if there are any coordinates listed for the distant point. If not, there is a jump to 9.0 below.

10.0 Directions and azimuths are converted to radians using function RADFORM, and stored in COMMON /14/.

10.3 If no individual weight has been listed for a direction, it is here given an individual weight of 1.0. For azimuths and distances, the equivalent step is more conveniently taken in subroutines AZFORM(7.0) and DISTFORM(7.0).

19.0 AT and TO are stored in IOBSVAL, and the observed quantities and individual weights are stored in OBSVAL, ready to be written on scratch tape 51 by MAIN (18.2), after the observation equations have been formed. Then there is a loop back to 12.0 above, to read the next card.

9.0 If the distant station has no coordinates listed, observations to it are ignored, a notice is printed, NUMLINE is decreased by 1, and there is a loop back to 12.0 to read the next card.

7.0 If the observation station has no coordinates listed, all observations listed there are ignored, a notice is printed, NOCORD is set to 1, and there is a loop back to 12.0 to skip through the remaining cards at the station.

3.0 When a card other than a 7-card is read, observations at this station are concluded, but NUMLINE is too big by 1. RADIRN(NUMLINE) is set to zero, in case the last card had a fraudulent entry (see 2.1 above). NUMLINE is then decreased by 1, STATION (an array of three 8-letter words), NUMLINE and HEIGHT are stored in the array ISTA, and RADLAT and RADLON are stored in STA, ready to be written on scratch tape 53 by MAIN (22.0) when all the stations have been read.

3.9 When all the data at this station has been read in, the last card read should be either a 6-card (if another station follows) or an 8-card (if this was the last station in this adjustment). Control then returns to the calling subroutine MAIN.

16.0 If any other type of card is read, a diagnostic is printed, and N is set to 1 before returning to MAIN. This in effect calls DATAFAIL, and this adjustment is terminated.

SUBROUTINE OBSFORM

OBSFORM controls the formation of observation equations by subroutines ANGFORM, AZFORM and DISTFORM. It is called immediately after STATREAD and deals with observation at only one station at a time. Hence the arrays in common /14/, /19/ and /20/ are of dimension (16) only.

0.3 For each of the NUMLINE usable observation cards at this station, the serial number TO of the distant station is obtained from the array JTO; P (which is checked by ANGFORM AT (0.6)) is set to zero; subroutine ROBIN is called and the forward and reverse azimuths and distance there computed are stored in the arrays COMFWDAZ, COMREVAZ and COMPDIST; and the sines and cosines of the azimuths are computed and stored.

1.2 Starting again at the first line, TO is again obtained from JTO, and the signal ITYPE (required by ANGFORM, etc.) is set to zero.

1.4 If ANGCL0 is blank, closing angles of a round are to be included as observations, and ANGFORM is called if there is an observed direction RADIRN. If ANGCL0 is not blank, closing angles are to be ignored, and ANGFORM is only called if there is an observed direction on the next line. Being wary of small round-off errors, RADIRN is not tested against a precise zero, but against  $5 \times 10^{-8}$  radians, about 0".001. It is because it is not easy at this point to distinguish blanks from zeroes, that the opening direction of a round may not be tabulated as 0° 00' 00.", and 360° 00' 00." has to be entered instead.

4.0 If there is an azimuth observation, AZFORM is called.

4.1 If there is an observed distance, DISTFORM is called, but only if the serial number TO is greater than AT. It is convenient to list distances on the data sheets at both terminal stations, but one entry is superfluous, and the entry at the station with the larger serial number is ignored.

2.0 There is a loop back to (1.2), and all NUMLINE cards are processed in turn. When all observation equations at this station have been formed, control reverts back to the calling subroutine MAIN.

SUBROUTINE ROBIN

ROBIN computes the forward and reverse azimuth and the distance between two points, in the normal section, using Robbin's (1962) formulae. Normal section azimuths are subsequently converted to geodesic azimuths if required.

- 0.7 The difference in longitude DELON is computed and tested.
- 2.0 If DELON is zero, the line lies in the meridian, and the latitudes are tested to see if the line is going north or south. If the difference in latitude is also zero, the azimuths and the distance are all set to zero, and there is an immediate RETURN.
- 14.0 If the line is running due south, FWDAZ and REVAZ are set to 0.
- 15.0 If the line is running due north, FWDAZ and REVAZ are set to Pi.
- 1.0 If the line is not in the meridian, and is going eastwards, FWDAZ is set to 0 and REVAZ to Pi.
- 3.0 If the line is going westwards, FWDAZ is set to Pi and REVAZ to 0.
- These temporary settings are modified in 9.2 below.

7.0 The sine and cosine of the difference in longitude are computed, and J and K set equal to the station serial numbers AT and TO respectively, for finding entries in the arrays in COMMON block /BANIMAT/.

7.3 The following are then computed:

$$\text{TANPSI (1)} = (1 - e^2) \tan \phi_2 + e^2 v_1 \sin \phi_1 / v_2 \cos \phi_2$$

$$\text{FOUR (1)} = \cos \phi_1 \text{TANPSI (1)} - \sin \phi_1 \cos \Delta \lambda$$

$$\text{AZ (1)} = \arctan (\sin \Delta \lambda / \text{FOUR (1)})$$

The values of J and K are then reversed, so that J = TO and K = AT, and the loop repeated to find the reverse azimuth.

8.1  $\text{PSI} = \psi_2$  is computed from TANPSI (1) and the absolute value of AZ(1) checked to see if it lies within an arbitrary limit of 0.3 radians from the meridian. If it does, then

$$\text{TWO} = \text{FOUR (1)} / \cos \text{AZ(1)}$$

$$\text{if not, TWO} = \sin \Delta \lambda / \sin \text{AZ(1)}$$

ANGFORM forms an observation equation from the observed and computed values of an angle, at station serial number AT, to TO and AND. Angles are formed by taking directions in successive pairs and subtracting. Unless the ANGULO option is exercised by making an entry in column 73 of the 3-card, the exterior or closing angle of a round, between the last and the first directions listed, is used to form an observation equation. There are then as many observation equations as there are directions, and the choice of the first direction, as well as its orientation, is arbitrary. If external angles are to be omitted, then the directions must be so listed that the external angle comes after the last direction and before the first. Angles are considered positive clockwise from TO to AND. Directions are normally listed in clockwise order, but the program does not fail if a station is out of order, so long as there is no blank or zero direction in the sequence.

0.6 The coefficients in the observation equations are formed in a separate subroutine PQR, which is now called if P(LINE) is zero, indicating that this is the first direction at a station. The values in the array P are reset to zero before each station by OBSFORM (0.5).

0.7 There is a check to see if the direction RADIRN (LINE) is the last direction at this station or not. Normally ANDIRN is the direction on the next card, but if the direction on the next card is zero, then the closing angle is formed with the direction on the first card. It is for this reason that lines over which directions have been observed must be listed consecutively first. If there is a gap in the sequence, the system for computing angles from directions fails.

3.0 If RADIRN(LINE) is not the last direction listed, AND is found from the array JTO, PQR is called, and ANDIRN, etc., are set, using (LINE + 1) in each case.

4.0 If RADIRN(LINE) is the last direction, there is first a check to see if it is the only direction, as a single direction has no meaning. If a single direction is found, a notice is printed, and control reverts to OBSFORM. More usually, AND is found from the array JTO, and ANDIRN, etc., are set, using the values for line 1. Subroutine PQR for line 1 has already been called at 0.6 above, so that P(LINE + 1) is now set equal to P(1), and so on. The dimensions of the arrays P, Q and R have to be 17, not 16.

5.0 The band width THISBAND for this particular observation equation is calculated. Unless there are gaps in the sequence, the band width is equal to twice the range of the serial numbers, plus one. If there is a gap or if a fixed point intervenes in the sequence, the band width will be less, but the program foregoes this possible advantage. If THISBAND is the largest value so far obtained, it is stored in BANDWID. It is not

ANGFORM forms an observation equation from the observed and computed values of an angle, at station serial number AT, to TO and AND. Angles are formed by taking directions in successive pairs and subtracting. Unless the ANGCL0 option is exercised by making an entry in column 73 of the 3-card, the exterior or closing angle of a round, between the last and the first directions listed, is used to form an observation equation. There are then as many observation equations as there are directions, and the choice of the first direction, as well as its orientation, is arbitrary. If external angles are to be omitted, then the directions must be so listed that the external angle comes after the last direction and before the first. Angles are considered positive clockwise from TO to AND. Directions are normally listed in clockwise order, but the program does not fail if a station is out of order, so long as there is no blank or zero direction in the sequence.

0.6 The coefficients in the observation equations are formed in a separate subroutine PQR, which is now called if P(LINE) is zero, indicating that this is the first direction at a station. The values in the array P are reset to zero before each station by OBSFORM (0.5).

0.7 There is a check to see if the direction RADIRN (LINE) is the last direction at this station or not. Normally ANDIRN is the direction on the next card, but if the direction on the next card is zero, then the closing angle is formed with the direction on the first card. It is for this reason that lines over which directions have been observed must be listed consecutively first. If there is a gap in the sequence, the system for computing angles from directions fails.

3.0 If RADIRN(LINE) is not the last direction listed, AND is found from the array JTO, PQR is called, and ANDIRN, etc., are set, using (LINE + 1) in each case.

4.0 If RADIRN(LINE) is the last direction, there is first a check to see if it is the only direction, as a single direction has no meaning. If a single direction is found, a notice is printed, and control reverts to OBSFORM. More usually, AND is found from the array JTO, and ANDIRN, etc., are set, using the values for line 1. Subroutine PQR for line 1 has already been called at 0.6 above, so that P(LINE + 1) is now set equal to P(1), and so on. The dimensions of the arrays P, Q and R have to be 17, not 16.

5.0 The band width THISBAND for this particular observation equation is calculated. Unless there are gaps in the sequence, the band width is equal to twice the range of the serial numbers, plus one. If there is a gap or if a fixed point intervenes in the sequence, the band width will be less, but the program foregoes this possible advantage. If THISBAND is the largest value so far obtained, it is stored in BANDWID. It is not

possible for the band width to exceed one less than the order of the matrix, and a check that the computed value of BANDWID is less than or equal to MATSIZE-1 is made in MAIN at (31.2), after all the observations have been formed.

5.4 The observed value of the angle is ANDIRN, calculated above, minus RADIRN(LINE). If negative, it is increased by  $2\pi$  radians.

5.6 Similarly, the value computed from the preliminary coordinates is ANDFWDAZ minus COMFWDAZ(LINE), which is also increased by  $2\pi$  if negative. The difference OMINC is calculated, and a check made to ensure that it is a small angle near zero, not  $360^\circ$ . The latter could occur if distances were short and preliminary coordinates inaccurate. OMINC is converted to seconds.

5.12 The signal ITYPE is set to 1, indicating an angle.

5.14 If OMINC exceeds the limit Z (see paragraph 20.1, and Appendix C, 39.1), subroutine LIMITEST is called. On return, NANG is tested. NANG serves two purposes: it is a counter for the number of angle equations formed, but by adding 5000 it also serves as a signal. A faulty direction causes angles to fail in pairs. NANG indicates whether it is the first or second angle, flanking a faulty direction, which has called LIMITEST. If it is the second angle, the 5000 is removed and OBSVAL(1,LINE), which contains the offending direction is set to zero before returning. If it is the first angle, NANG is increased by 5000, and there is an immediate return to OBSFORM without forming an observation equation. This keeps LIMITEST failures out of the average and maximum adjustments computed by subroutine STATPRIN.

14.0 Blank individual weights have been converted to 1.0 in STATREAD at (10.3). If the directions to TO and AND both have individual weights of 1.0, the individual weight of the angle is  $\frac{1}{2}$ . Otherwise the individual weight of an angle is calculated from the formula  $(w_1 \times w_2) / (w_1 + w_2)$ , and the square root of the weight is put in ROOTWF.

18.0 If the R weighting option is in use, the observation equation has to be divided through by ANSIGSEC. See STARTOFF (21.1). This is conveniently done by dividing ROOTWF by ANSIGSEC.

18.1 The coefficients of the observation equations are found using P, Q and R from subroutine PQR - see Appendix M for the definitions of P, Q and R. Using subscripts 1, 2 and 3 for AT, TO and AND, the observation

equation is:

$$\begin{aligned}
 & (P_{13} - P_{12}) \Delta\phi_1 - (R_{13} - R_{12}) \Delta\lambda_1 \\
 - & \quad Q_{12} \Delta\phi_2 - R_{12} \Delta\lambda_2 \\
 + & \quad Q_{13} \Delta\phi_3 + R_{13} \Delta\lambda_3 = 0-C
 \end{aligned}$$

See "Observation Equations", paragraph 6. The six coefficients and the constant term OMLINC are stored in the array OBSEQ.

9.0 The coefficients and the constant term are multiplied by ROOTWT unless ROOTWT equals one.

8.0 The values in the array INDIC for AT, TO and AND are stored in the array IOBSEQ. The numbers in INDIC give a guide to the column number in the normal equation matrix - see Appendix D, note 5.0.

8.3 The arrays IOBSEQ and OBSEQ are written in binary on scratch tape 52. The counter NUMBEQ for the total number of observation equations, and NANG for the number of angle equations, are both increased by one, and control returns to the calling subroutine OBSFORM.

AZFORM forms an observation equation from the observed and computed value of an azimuth. The change in the observed Laplace azimuth due to the adjustment in longitude of the observation station is automatically allowed for.

0.5 If ITYPE = 1 (see ANGFORM (5.12)), ANGFORM will already have called PQR, computed the bandwidth, and stored AT and TO in IOBSEQ. If the azimuth has been observed along a line over which there is no observed direction, ITYPE will still be zero (see OBSFORM (1.2)), and these tasks are now done here.

1.0 In an azimuth equation, only two stations are involved, so IOBSEQ (3) is set to zero.

1.1 The difference OMINC between the observed and computed azimuth is put in the range  $-\pi$  to  $+\pi$ . This avoids trouble when the observed azimuth is say  $00^{\circ} 00' 01''$ , and the computed azimuth is say  $359^{\circ} 59' 58''$ .

1.4 The signal ITYPE is set to 2, indicating an azimuth.

1.5 If OMINC exceeds the limit Z (see paragraph 20.1 and Appendix C, note 39.1), subroutine LIMITEST is called. On return, OBSVAL (2, LINE) which contains the offending azimuth is set to zero, deleting it from the output. Control then return to OBSFORM.

7.0 If the individual weight of the observed azimuth has been left blank, implying a weight of 1.0, ROOTWT is set to 1.0. Otherwise, ROOTWT is set to the square root of the individual weight.

12.0 If the weight option has been set to X in column 74 of the 3-card, requiring weights of azimuths and distances to decrease with the square root of the length of the line, FACTOR is set to  $\text{ROOTWT} \times \text{SQRT}(L \cdot \sin 1'')$ .

14.0 Alternatively, if the weight option has been set to R, FACTOR is set to  $\text{ROOTWT}/\text{AZSIGSEC}$ , - see STARTOFF (21.1).

9.0 The observation equation is

$$P_{12} \Delta \phi_1 - (R_{12} - \sin \phi_1) \Delta \lambda_1 + Q_{12} \Delta \phi_2 + R_{12} \Delta \lambda_2 = O - C$$

The coefficients and the constant term are stored in OBSEQ. As only two stations are involved in an azimuth equation, OBSEQ(5) and (6) are set to zero.

5.0 All terms in the observation equation are multiplied by  
FACTOR = 1.

4.0 The arrays IOBSEQ and OBSEQ are stored in binary on scratch  
tape 52. The counter NUMBEQ for the total number of observation equations,  
and NAZ for the number of azimuth equations, are both increased by one,  
and control returns to OBSFORM.

SUBROUTINE DISTFORM

## Appendix K

DISTFORM forms an observation equation from the observed and computed values of a distance.

0.6 If a direction or an azimuth has been observed over the same line, ITYPE will already have been set to 1 or 2 by ANGFORM or AZFORM, and the calculation of the band width and the insertion of INDIC(AT) and INDIC(TO) in IOBSEQ can here be omitted.

1.0 In a length equation, only two stations are involved, so IOBSEQ(3) is set to zero.

1.1 The signal ITYPE is set to 3, indicating a distance. The difference between the observed and computed distance is put in OMLINC.

1.2 If OMLINC exceeds the limit L (see paragraph 20.1 and Appendix C, note 39.1) subroutine LIMITEST is called. On return, OBSVAL(3,LINE) containing the faulty distance is set to zero, deleting it from the output. Control then returns to OBSFORM.

7.0 If the individual weight of the distance has been left blank, implying a weight of 1.0, ROOTWT is set equal to RIWIFACT, - see Appendix C, note 31.1. Otherwise RIWIFACT is multiplied by the square root of the individual weight.

12.0 If the weight option has been set equal to W in column 74 of the 3-card, FACTOR is set equal to  $\text{ROOTWT} / L \cdot \sin 1''$ .

13.0 If the weight option has been set to R, FACTOR is set to ROOTWT divided by the vector sum of 0.1 ft + 3 parts per million of the distance, or whatever values have been entered in STARTOFF (23.0) and (24.0).

14.0 If the weight option has been set to X, FACTOR is set to  $\text{ROOTWT} / (L \cdot \sin 1'')^{\frac{1}{2}}$

15.0 If the weight option is blank, FACTOR is set to ROOTWT.

9.0 The observation equation is:

$$\begin{aligned}
 & -\rho_1 \cos A_{12} \sin 1'' \cdot \Delta \phi_1 - \nu_2 \sin A_{21} \cos \phi_2 \sin 1'' \cdot \Delta \lambda_1 \\
 & -\rho_2 \cos A_{21} \sin 1'' \cdot \Delta \phi_2 + \nu_2 \sin A_{21} \cos \phi_2 \sin 1'' \cdot \Delta \lambda_2 = O-C
 \end{aligned}$$

The coefficients and the constant term are stored in the array OBSEQ. As only two stations are involved in a length equation, OBSEQ (5) and (6) are set to zero.

5.0 All the terms in the observation equation are multiplied by FACTOR.

4.0 The arrays IOBSEQ and OBSEQ are written in binary on scratch tape 52. The counter NUMBEQ for the total number of observations equations, and NDIET for the number of length equations, are both increased by one, and control returns to OBSFORM.

SUBROUTINE LIMITEST

## Appendix I

LIMITEST is called when the difference between the observed and the computed value of a quantity exceeds the specified limits L or Z (see paragraph 20.1, and Appendix C, note 39.1).

0.3 If LIMITEST is being called for the first time, headings for the diagnostics are printed.

4.1 The counter LIMITEEX is increased by one. LIMITEEX is set to zero at the start of each adjustment by MAIN at 1.1.

4.2 If ITYPE is not 1, LIMITEST has been called by either AZFORM or DISTFORM, so AND is set to zero.

4.3 The station serial numbers and ITYPE are printed, thus identifying the faulty observation, followed by the offending value of OMINC, which is either in seconds of arc, or in units of distance. OMINC is printed in floating-point "E" format, as its magnitude is unknown.

1.1 If the counter LIMITEEX is less than the value of LIMIT, control returns to the calling subroutine. But if the LIMIT has been attained, a diagnostic is printed and DATAFAIL called, terminating this adjustment

SUBROUTINE PQR

## Appendix M

PQR forms the coefficients P, Q and R for the angle and azimuth observation equations. The subroutine has to be called with AT, TO and LINE as parameters, not in COMMON, because ANGFORM often calls PQR for the subsequent line, for which the values of P, Q and R currently in COMMON are incorrect.

0.5 The coefficients are:

$$P_{12} = p_1 \cdot \sin A_{12} / L$$

$$Q_{12} = p_2 \cdot \sin A_{21} / L$$

$$R_{12} = r_2 \cdot \cos A_{21} \cos \phi_2 / L$$

Computed azimuths and distances have to be used, since observed azimuths are frequently not available for distance equations, and vice versa. It is for this reason that preliminary coordinates must be reasonably accurate and iteration is sometimes necessary. See paragraph 77.

NORMFORM adds an observation equation, one at a time, into the matrix of the coefficients of the normal equations. The values in IOBSEQ, OBSEQ for each observation equation in turn are read from scratch tape 52 by MAIN at 6.2 NORMFORM forms squares and products, and adds them in to the appropriate coefficients of the normal equation matrix.

0.6 The six column numbers ISIX in the normal equation matrix which corresponds to  $\Delta\phi, \Delta\lambda$  for the three stations in the observations equations are first calculated. In the matrix, the first two columns are allocated to for the first variable station, the next two columns to the next variable station, and so on. Thus, in general, each observation equation deals with six columns,  $\Delta\phi, \Delta\lambda$  for AT, T0 and AND respectively. However, station AND will not exist for azimuth and distance equations; and any of the three points may be fixed, having no unknown values  $\Delta\phi, \Delta\lambda$ . In these cases, IOBSEQ(L) will be 0 or 888 respectively (see ANGFORM 8.0 and LATLONIN 5.0). Otherwise,

$$\text{IOBSEQ}(L) = \text{INDIC}(\text{AT}) = 2 \times (\text{serial number of each variable station}).$$

A DO loop is entered with L = 1 to 3, corresponding to AT, T0 and AND. If IOBSEQ(L) equals 0 or 888, ISIX(2L) and ISIX(2L-1) are set to zero. Otherwise,

$$\text{ISIX}(2L) = \text{IOBSEQ}(L) \quad \text{and} \quad \text{ISIX}(2L-1) = \text{IOBSEQ}(L) - 1$$

1.10 The subroutine now enters a pair of nested DO loops. Each column ISIX is taken in turn. If it is zero, it is skipped. Otherwise the inner DO loop is entered, and again each column ISIX is taken in turn, and again, if zero, it is skipped. Otherwise, a sum or product is formed as follows.

1.5 It is first necessary to ensure that coefficients when formed are all in the upper right half of the symmetrical matrix. The lower of the two column numbers in ISIX now under consideration is placed in ILO, and the higher in IHI. On the main diagonal, ILO and IHI will be equal.

8.1 Subroutine INDEX is called with ILO and IHI to obtain the corresponding serial number in the linear array for the band matrix BANDMAT.

8.2 The square or product OBSEQ(I) x OBSEQ(J) is added to any previously existing value in BANDMAT(K).

5.1 On the conclusion of the inner DO loop, the product OBSEQ(I) x OBSEQ(7) is added into the relevant element of CONSTRHS. OBSEQ(7) is the O-C term, and CONSTRHS is the column matrix of constants on the right hand side of the normal equations.

4.1 On the conclusion of the outer DO loop, control returns to the calling subroutine MAIN, and subsequent observation equations will be processed the same way until all NUMBEQ observations have been processed, and the matrices BANDMAT and CONSTRHS are complete.

INDEX takes the row and column numbers I, J of an element in a matrix and computes the equivalent serial number K in a linear array corresponding to the upper half of a band matrix - see Figure 9:

J	=	1	2	3	4	5
I	=	1	2	3	4	5
		1	2	3	.	.
		2	4	5	6	.
		3		7	8	9
		4			10	11
		5				12

Figure 9: Values of K for the elements I, J in a band matrix of order  $N = 5$  and band width  $M = 2$ .

0.1 The values of N, M and M-N must be in the COMMON block INVERT.

0.2 Two parameters  $IMINUS1 = I - 1$  and  $IATEST = (M-N) + (I-1)$  are computed. In figure 9,  $M-N = -3$ ; IATEST takes the values -3 to +1 as I goes from 1 to 5.

0.3 From the first few lines, until IATEST becomes positive, the value of K is:

$$\begin{aligned} K &= (M+1) \times (I-1) + J - (I-1) \\ &= M \times (I-1) + J \end{aligned}$$

0.4 Towards the end, however, this value of K must be modified, as the last few lines of the band matrix are not M+1 in length, but gradually diminish to 1. Accordingly, when IATEST is positive,

$$\begin{aligned} K &= K - (\text{Sum of the numbers from 1 to IATEST}) \\ &= K - \sum_{i=1}^{IATEST} i \\ &= K - IATEST \times (IATEST + 1) / 2 \end{aligned}$$

SUBROUTINE BANDIT

BANDIT solves the normal equations by the Choleski or Square Root method: see paragraph 13. The coefficients in the band matrix are stored in a linear array A, and the constants on the right hand side of the equations in the array B. An element in row I and column J of the band matrix is stored in element K of the linear array A. Subroutine INDEX (Appendix O) is used to find K given I and J.

BANDIT is called with five parameters: the order of the matrix, the band width, the arrays A and B, and an error signal IERR.

0.4 The order and band width are set in N and M, and M-N calculated. All three are stored in the COMMON block INVERT for use by subroutine INDEX. If any impossible combination occurs, IERR is set to -1, and there is an immediate return to MAIN, where a diagnostic is printed and the adjustment terminated.

0.6 The triangular matrix S is now found, such that its product with its transpose S' is equal to the band matrix stored in A. A DO loop is entered for each of the N rows of the matrix, and the following functions of the row number I are calculated:

I+1;

I-1;

JMAX, the maximum value of (I+M) or N;

IMAX, the maximum value of (I-M) or 1.

0.11 For each row in turn, the diagonal element DIAG is found from the general formula

$$s_{ii} = \sqrt{a_{ii} - \sum_{l=1}^{i-1} s_{li}^2}, \quad i > 1$$

In a band matrix, the limits of  $l$  for the summation are from IMAX to I-1. The sum is accumulated in TEMP.

1.3 If the square root of the diagonal term is zero or negative, IERR is set to the row number, and there is an immediate return to MAIN, where a diagnostic is printed and the adjustment terminated. Normally the square root is calculated, and  $s_{ii}$  replaces  $a_{ii}$  in A(I,I).

1.6 Each non-diagonal term in row i, from columns (I+1) to JMAX

is now found from the general formula

$$s_{ij} = (a_{ij} - \sum_{\ell=1}^{i-1} s_{\ell i} s_{\ell j}) / s_{ii}, \quad j > i$$

In a band matrix, the limits of  $\ell$  are once again from IMAX to (I-1). Each new  $s_{ij}$  replaces  $a_{ij}$  in A(I,J).

4.1 When the original matrix A has been completely replaced by S, the first back substitution is done, to replace the original column matrix B containing the constant terms by the temporary column matrix Y, derived from the matrix equation

$$SY = B$$

The general formula is

$$y_i = (b_i - \sum_{\ell=1}^{i-1} s_{\ell i} y_{\ell}) / s_{ii}, \quad i > 1$$

A DO loop is entered for each of the N rows. IMAX is set to the maximum of I-M or 1. In a band matrix, the limits of  $\ell$  are then from IMAX to I-1, and each  $y_i$  replaces  $b_i$  in B(I).

6.1 A second back substitution gives the unknowns  $\underline{x}$ :

$$S\underline{x} = Y$$

The general formula is

$$x_i = (y_i - \sum_{\ell=i+1}^N s_{i\ell} x_{\ell}) / s_{ii}, \quad i < N$$

There is no DO loop. Initially J is set equal to N, and decreased by one for each row. In a band matrix, the limits of  $\ell$  for each row are from IPLUS1 = J + 1 to IMAX, the minimum value of J + M or N. Each computed value of  $x_i$  replaces  $y_i$  in B(I).

8.4 When J is reduced to zero, the solution is complete, and control returns to subroutine MAIN.

SUBROUTINE PRINCORD

PRINCORD controls the printing of adjusted coordinates. The actual printing is done by subroutine NEWCORDS.

2.0 A heading is printed for the fixed points. The change in coordinates for the fixed points is set to zero, both in seconds and in radians.

2.2 A DO loop is entered for the NUMFIX fixed points. The serial number of each fixed point is obtained from the array IFIX. NUMFIX and IFIX were set by LATLONIN (4.0). NEWCORDS is then called to print the coordinates of the fixed point.

1.0 When the last of the fixed points has been processed, a heading is printed for the variable points, and four accumulators are set to zero.

4.2 A DO loop is entered for the NUMVAR variable points, and the serial number of each variable station is obtained from the array IVAR. The adjustments  $\Delta\phi, \Delta\lambda$  in seconds are obtained from the array OUTPUT. They are converted to radians, and subroutine NEWCORDS is called.

4.10 The sums of the absolute values of  $\Delta\phi, \Delta\lambda$ , in seconds, are accumulated in SUMDLAT, SUMDLON. If the current absolute value is the largest so far obtained, it is stored in MAXDLAT or MAXDLON.

5.1 When all the variable points have been processed, the average and maximum values of the adjustments are printed, unless suppressed by NOPRIN = 2.

7.1 At the end, control returns to the calling subroutine MAIN.

SUBROUTINE NEWCORDS

## Appendix R

NEWCORDS prints the list of adjusted coordinates in one of two forms, depending on whether Transverse Mercator coordinates have been requested or not.

0.7 The unadjusted latitudes and longitudes in radians are in STA(AT,1) and STA(AT,2), to which the adjustments in radians are added. The adjusted values are converted to degrees, minutes and seconds, and stored in arrays in COMMON, as they are also printed out by subroutine STATPRIN later. For Australian use, the signs of the adjustments and the adjusted quantities are converted to positive south and positive east before printing.

0.13 If the adjustments are less than 0".00005, they are set exactly to zero. Otherwise asterisks instead of zeros are produced on the output - see paragraph 54.

0.15 The sine and cosine of the latitude are calculated, and Rho and Nu computed.

0.16 If TM coordinates are not required, there is a jump to 6.0 below, skipping the greater part of this subroutine.

0.17 If TM coordinates are to be computed, the longitude is converted to degrees and made positive east. Subroutine MERIDIST is called with RADLATM, and returns with the meridian distance in EM. The zone number is calculated using the parameters for the Universal Transverse Mercator projection stored in MAIN at (0.6ff). The longitude from the central meridian in degrees, DEGDELON, is calculated. The signal LAPZONE is set to -1, +1 or zero, depending on whether the point lies in a western or eastern zone overlap, or in neither.

11.0 The longitude from the central meridian is converted to radians, RADELON. Subroutine ENCONV is called to compute eastings, northings and convergence: EAST, NOR and RADCONV.

11.1 At this stage it is impossible for LAPZONE to equal 2, but if the point lies in a zone overlap, it will be set to 2 by statements 41.0 and 43.0 below, causing a loop back to statement 11.0.

32.0 False eastings and false northing are computed, and the coordinates, convergence and zone number are stored in COMMON for use by subroutine STATPRIN. On this first run in the central zone, FN2(AT), the false northing in the overlapping zone, is set to zero as a signal, which is read by STATPRIN at (75.1). There is a jump to 34.0 below.

34.0 The sign of the convergence is calculated and put in EWC in H format. The sign convention is

$$\text{Grid Bearing} = \text{True Bearing} + \text{Convergence}$$

The convergence is converted from radians to degrees, minutes and seconds.

37.2 The coordinates of the station are printed. If there is an entry in PUNCORD in column 79 of the 3-card, the name of the station, the eastings, northings, convergence, zone and height are punched out on a 10-card, and there is a jump to 31.

31.0 If LAPZONE is not zero, indicating that the point lies in a zone overlap, the zone number and longitude from the central meridian are changed to the adjacent zone, LAPZONE is set to 2, and there is a loop to 11.0 above. Coordinates and convergence are then computed in the adjacent zone, printed by statement 39, and punched (unless PUNCORD is blank) by statement 4.1. There is then a jump to 44.0 below.

6.0 If Transverse Mercator coordinates are not computed, the alternative output is printed, - the station name, serial number, latitude, longitude, Rho, Nu and height.

44.0 If PUNCORD is not blank, latitudes and longitudes are punched out on a 5-card, in the exact format required for immediate reinsertion in the data deck, enabling the adjustment to be iterated, see paragraph 77. Note however, that if the coordinates of an azimuth station are changed by changing a 5-card, it is essential also to change the corresponding 7-card punched by STATPRIN (57.0) else the Laplace equation will be upset. Note also that 5-cards are punched for all points, whether they were fixed or variable. If the same points are to be held fixed on iteration, the old 4-cards can be used and the new 5-cards thrown away; but if previously fixed points are to be allowed to vary, the new 5-cards are available.

2.1 At the end, control returns to the calling subroutine PRINCORD.

STATPRIN prints out the adjusted and observed values of all the directions, azimuths and distances. The coordinates of each station are also printed, together with the differences between the observed and adjusted quantities, and the average and maximum differences. Two formats are available, depending on whether TM coordinates are required or not. At the end, the average and maximum adjustments for the whole section are printed. Subroutine STATPRIN is not called if NOPRIN (column 80 on the 3-card) is set to 1; the output then terminates after the initial trig list is printed.

0.9 Various counters and accumulators are set to zero.

1.2 This is the start of a major DO, containing the greater part of the subroutine, one loop for each of the NUMBOBS stations. NUMBOBS is calculated in MAIN at (18.2).

1.3 A counter and accumulator is set to zero.

1.4 IOBSVAL and OBSVAL, for one station only, are read in from scratch tape 51, and checked for parity errors. If an error has occurred, N is set to 1, and there is a return to MAIN; a diagnostic is printed and DATAFAIL called, terminating this adjustment.

32.0 The serial number AT of the current station is found from IOBSVAL(1,1).

32.1 If Transverse Mercator coordinates are not required, there is a jump to 72.0 below.

74.0 The name of the station is printed followed by a line of headings.

76.0 The sign of the convergence is calculated, and the convergence is converted to degrees, minutes and seconds.

78.1 The latitude, longitude, zone, easting, northing, convergence and height are printed in one line. If the point lies in a zone overlap, coordinates for the adjacent zone are printed on the next line. Then there is a jump to 38.0 below.

72.0 If Transverse Mercator coordinates are not required, the alternative style of coordinates and headings are printed.

38.0 A short preliminary DO is entered, one loop for each line at this station. It locates the adjusted and observed values, and computes the orientation constant, which rotates the observed directions to a best mean fit on the adjusted azimuths. NUMLINE is found from ISTA(AT,4), where it was stored by STATREAD(3.5).

- 36.2 The observed quantities are extracted from the array OBSVAL, where they were stored by STATREAD (19.0).
- 36.6 Subroutine ROBIN is called to compute the azimuth and distance between the adjusted coordinates at the ends of the line.
- 36.10 If there is no observed direction on this line, there is a jump to 7.0, the end of the DO. Otherwise, the difference between the adjusted azimuth and the observed direction is calculated. Precautions are taken to see that each of these values is similar to the others, and does not differ by  $360^\circ$ . The values are accumulated in SUMORIEN, and counted in NUMDIRN.
- 8.0 At the end of the DO, the mean orientation constant is calculated, put in the range  $0^\circ$ - $360^\circ$ , and converted to degrees, minutes and seconds.
- 8.4 Three arrays of counters and accumulators are set to zero.
- 12.1 A similar DO is now entered, one loop for each line. The serial number TO of the far station is obtained again from IOBSVAL (2,L), and the adjusted azimuth of the line is converted to degrees, minutes and seconds.
- 12.4 If there is no observed direction, the array KANG or IANG is filled with blanks, the former if Transverse Mercator coordinates are being listed, the latter if not. There is then a jump to 14.0 below.
- 26.0 If there is an observed direction, the orientation constant is added to it, it is put in the range  $0^\circ$ - $360^\circ$ , and converted to degrees, minutes and seconds. The former are thrown away, and only the seconds retained, SECANG.
- 26.3 The difference between the adjusted azimuth and the oriented direction is put in DEANG, which must be close to zero, not  $360^\circ$ . The absolute value of DEANG is found, and if less than  $0.005$ , set precisely to zero, to avoid asterisks being produced on the output. SECANG and DEANG are encoded in A-format in IANG, so that if there are no observations, blanks can be printed instead of zeros.
- 62.1 The counter NUM(1) for directions is increased by one, ABSDEANG is accumulated in SUM(1), and DEANG in SIG(1). If ABSDEANG is the largest value so far obtained, it is put in AMAX(1).
- 14.1 The observed direction on this line has now been dealt with, and there is a check to see if there is an observed azimuth. If not, blanks are encoded into KAZ or IAZ, and there is a jump to 15.0 below.
- 28.0 Unless AT is a fixed point, there will have been an adjustment to its longitude, which alters the Laplace azimuth by  $-\Delta\lambda \sin\phi$ .

34.0 The revised Laplace azimuth is converted to degrees, minutes and seconds.

34.1 If PUNCORD (column 79 on the 3-card) is not blank, a new 7-card is punched, containing the revised Laplace azimuth, ready for insertion in the data deck if it is necessary to iterate. If distances in feet have been converted to metres by the "M" option, it is necessary to convert back to feet before punching.

35.0 The difference DEAZ between the revised Laplace azimuth and the adjusted azimuth is put in the range  $-\pi$  to  $+\pi$ , converted to seconds, and if less than  $0''.005$  put equal to precisely zero, to avoid the printing of asterisks on the output. DEAZ and possibly SECASTRO are encoded in one of two A-formats, depending on whether TM coordinates are required or not.

49.0 The counter NUM(2) for azimuths is increased, ABSDEAZ is accumulated in SUM (2) and DEAZ in SIG(2). If ABSDEAZ is the largest value so far obtained in it put in AMAX(2).

15.0 The observed azimuth on this line has now been dealt with, and there is a check to see if there is an observed distance. If not, blanks are encoded in KDIS or IDIS, and there is a jump to 16.0 below. If there is an observed distance, its treatment is similar to that for an observed azimuth.

16.0 The adjusted and observed values are printed out in format statements 42 or 17, depending on whether TM coordinates are required or not. The observed quantities are printed in A-format from the arrays IANG or KANG, etc.

13.0 This completes the processing of a line. If it is the last line at a station, the average and maximum adjustments at that station are encoded in A-format in IAV, and IMAX and printed, together with the orientation constant, unless suppressed by NOPRIN = 2. The counters, accumulators and maxima are transferred to the arrays NTOT, TOT and ABM, and the locations of the maxima are stored in the array MAXAT. Control then reverts to 1.2 above to process the next station.

3.0 After all NUMBOBS stations have been processed, the average and maximum adjustments for the whole section are computed and printed, together with the serial numbers of the stations at which the maximum occur. Control then reverts to the calling subroutine MAIN.

SUBROUTINE ENCONV

ENCONV computes eastings, northings and convergence, given latitude, longitude from the central meridian, and meridian distance. It uses Redfearn's formulae from ESR No. 69, in full, but rearranged in nesting form:

$$\text{If } y = a + bx + cx^2 + dx^3$$

$$\text{write } y = a + x (b + x (c + dx))$$

which requires a smaller number of additions and multiplications.

1.3 NBR is  $v/\rho$ , computed from the formula:

$$\frac{v}{\rho} = \frac{1 - e^2 \sin^2 \phi}{1 - e^2}$$

T is  $\tan \phi$ , and W is the longitude in radians from the central meridian, positive east.

The computations of eastings, northings and convergence then follow Redfearn's formulae, term by term.

ENCONV is called by NEWCORDS at (11.1). The meridian distance is put into COMMON by subroutine MERIDIST, which is called by NEWCORDS at (0.21).

SUBROUTINE MERIDIST

MERIDIST computes meridian distance from the equator, positive north, negative south, from the formula given in GEODESY, G. Bomford, 1962, page 497. Note that formula (8.39) in the 1962 edition contains a mis-print; the formula should start:

$$m = a( A_0 \phi - A_2 \sin 2\phi \dots )$$

E has been calculated in STARTOFF (10.1). The even powers of E are calculated first, followed by the sines of the even multiples of  $\phi$ . Then the program follows the formula term by term.

MERIDIST is called by NEWCORDS at (0.21).

SUBROUTINE DEGFORM

DEGFORM converts positive radians into degrees, minutes and seconds. Negative angles are best dealt with by setting an indicator to "-", or "S" or whatever the negative angle indicates, printing it separately in an A1 format, and making the negative radians positive, before calling DEGFORM. One of the few inconveniences of Fortran 3600 is that numbers less than 0.5 in the last decimal place are printed as asterisks, instead of as zero - see paragraph 54. To avoid spoiling the appearance of the output, seconds less than 0.00005 are here set equal to zero.

Appendix W

FUNCTION RADFORM

RADFORM converts positive degrees, minutes and seconds to radians. Negative angles are best dealt with as in subroutine DEGFORM. This computation can be done in one Fortran statement, and is therefore a FUNCTION sub-program, in contrast to a subroutine. It is used by writing, for example:

RADIANS = RADFORM (DEGREES, MINUTES, SECONDS)

instead of using the CALL statement. See the 3600 Fortran Manual, pages 7-1, 7-7, 7-8.

NOTATION

Symbols which are defined and used in only one paragraph or appendix are not necessarily redefined here. Only algebraic symbols are here defined; not Fortran variables.

$\phi$	=	Latitude, usually positive north.
$\lambda$	=	Longitude, usually positive west, but positive east when converting to TM coordinates.
$A_{12}$	=	Azimuth, usually in the normal section, from station 1 to station 2.
$L_{12}$	=	Length of the line between station 1 and station 2.
$\rho, \nu$	=	radii of curvature of the spheroid in the meridian and prime vertical.
$a, b$	=	semi-axis, major and minor of the spheroid.
$m$	=	meridian distance.
$R$	=	approximate radius of the earth considered as a sphere.
$e$	=	$((a^2 - b^2)/a^2)^{\frac{1}{2}}$
$e'$	=	$((a^2 - b^2)/b^2)^{\frac{1}{2}}$
$\Delta x$	=	a small change in x, where x is any quantity.
$x_1$	=	the value of x at station 1.
$P_{12}$	=	$(\rho_1 \sin A_{12})/L$
$Q_{12}$	=	$(\rho_2 \sin A_{21})/L$
$R_{12}$	=	$(\nu_2 \cos \phi_2 \cos A_{21})/L$
$S_{12}$	=	$- \rho_1 \cos A_{12} \sin 1''$
$T_{12}$	=	$- \rho_2 \cos A_{21} \sin 1''$
$U_{12}$	=	$+ \nu_2 \cos \phi_2 \sin A_{21} \sin 1''$
$\sigma$	=	Standard error.
$\sigma_A, \sigma_Z, \sigma_L$	=	Standard errors of an angle, and azimuth, a distance.
$k$	=	some constant.
$w$	=	weight.

- (O-C) = the difference between an observed quantity and its value computed from some preliminary coordinates.
- N = the order of a matrix.
- M = the band width of a band matrix, excluding the principal diagonal.
- $a_{i,j}$  = the element in row  $i$  and column  $j$  of any matrix  $A$ .
- $a_k$  = the element in position  $k$  in a linear array  $A$ .

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