

**TECHNICAL REPORT 33**

# **GEODETTIC MODEL OF AUSTRALIA 1982**

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## ABSTRACT

The success or otherwise of a Geodetic Adjustment is heavily dependent on three main factors. These are:

- The mathematical model
- The adjustment algorithms
- The data set

It is axiomatic that a poor mathematical model or an inferior algorithm will distort the results obtained from a good data set and also that a "perfect" model and algorithm cannot rectify a poor data set.

In carrying out the GMA82 adjustment, every known correction or refinement has been applied in defining the mathematical model. The adjustment algorithms have been tested and retain the rigour of the mathematical model. The remaining factor, the data set, has been the responsibility of the representatives of the various members of the National Mapping Council. They have supplied their data and within the time and economic constraints have considerably strengthened the previously existing data sets.

The resulting coordinates from the GMA82 adjustment thus represent the most probable values of the horizontal coordinates that can be determined from the current data set. It appears unlikely that any significant gross errors remain in the data set or that the results have any really significant distortions in areas where adequate redundant measurements have been made.

It is also inevitable that some small errors will be detected in the data set. Generally, these may be corrected by recomputing the appropriate Stage 3 without significantly affecting the remainder of the network.

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## 1. INTRODUCTION

At the September 1981 meeting of the National Mapping Council a Resolution was passed which called for a readjustment of the Australian Primary Geodetic Network. Resolution 400 reads:

"The Council, noting Technical Subcommittee Recommendations 281 and 282, RESOLVES to produce a GMA82 adjustment on the Australian National Spheroid including all suitable data available by 30 June 1982."

This Resolution called for a least squares adjustment of a continental network with thousands of stations and the associated terrestrial observations. These observations were to be validated, reduced from geoidal to spheroidal values and then adjusted. In addition, the following observations were to be incorporated:

Point-position Doppler values calculated using the precise ephemeris.

Multi-station Doppler relative positions calculated using the precise and broadcast ephemerides.

The satellite laser ranging distance Orroral-Yaragadee.

VLBI chord distances.

An adjustment of this nature requires special techniques. The main requirement of providing a fundamental framework across the continent transcends the requirements of the individual states and is generally only contemplated when there have been significant changes in technology and/or the primary data set.

There was no suggestion that such an adjustment would replace the adjustments normally carried out by the individual States. Its purpose is to provide a homogeneous framework to be used by the individual State as control for all scientific, mapping, cadastral and engineering surveys within that State. These densification surveys and adjustments are the responsibility of the individual States.

The impetus for the adjustment came from a growing concern regarding the adequacy of the AGD66 network as a general purpose framework now that high precision survey instruments and techniques are readily available. The capability of carrying out a continental readjustment had been demonstrated by GMA79 and GMA80 adjustments and it seemed that the time was opportune.

A Working Group comprising of a representative of each Council member was set up to assist in the collection and validation of the data. At a meeting of this Working Group in December 1981, it was agreed that the data base would be initially established on the CYBER computer at the University of New South Wales. The adjustment would be the responsibility of Professor Allman and that the results be forwarded to the Division of National Mapping for future management and promulgation.

The same Working Group meeting also indicated that the GMA80 data set was to be substantially increased and would include a multistation Doppler satellite connection between Tasmania and Victoria. A proposed multistation Doppler satellite survey of Queensland was noted.

Figure 1. GMA82 Stations

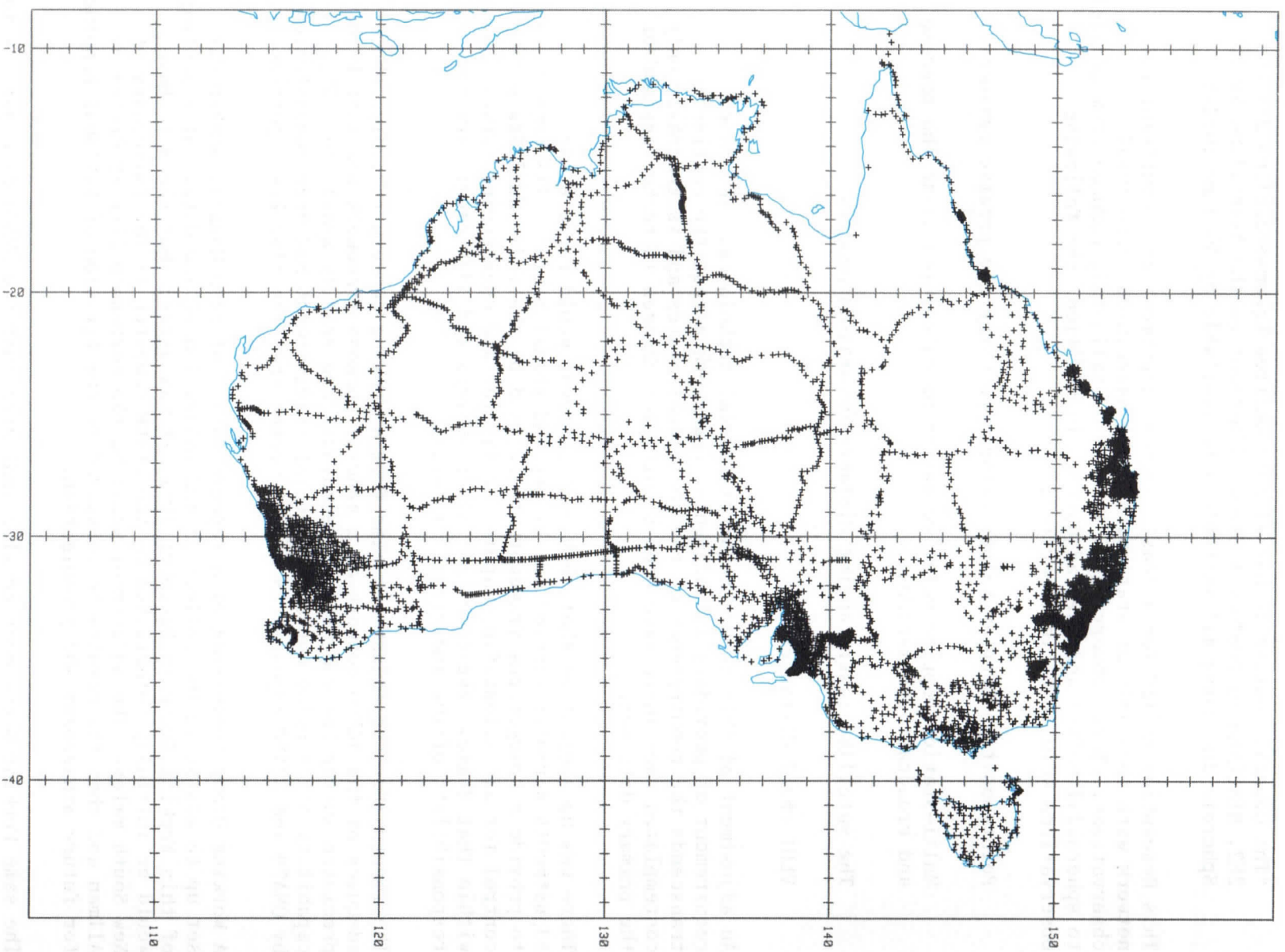
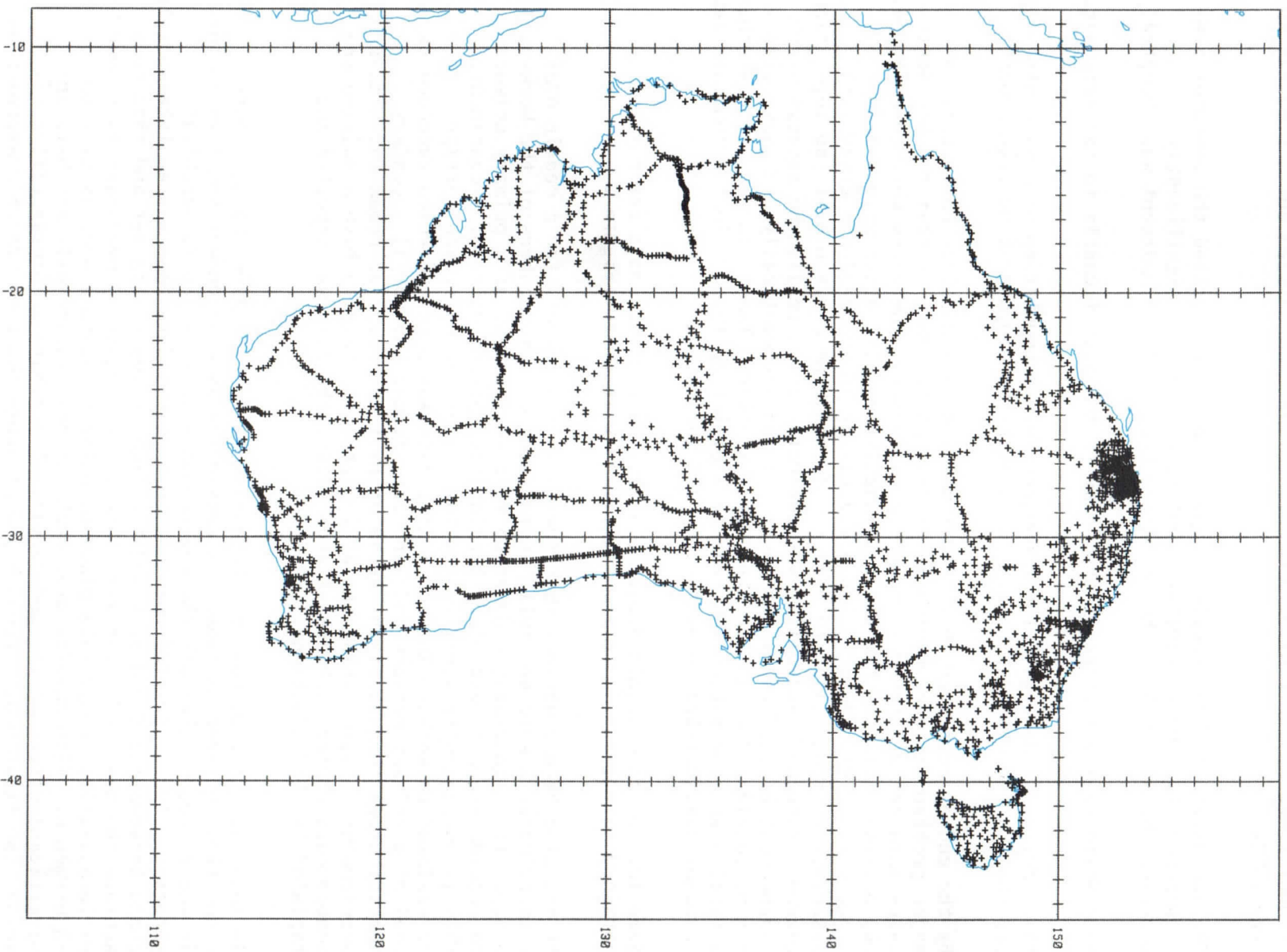


Figure 2. GMA80 Stations



By February 1982, the proposed Queensland Doppler survey had been extended to cover the eastern portion of Australia. The survey was carried out during February and March 1982 and fortuitously included one station in Papua New Guinea. The success of this project which demonstrated the ability and willingness of many government and private sector organisations to work together at short notice, led to plans to carry out a similar project to cover the rest of Australia. This was carried out during May and June 1982.

The inclusion of these Doppler projects more than doubled the computer time requirement for the readjustment but since the data significantly improved the strength of the primary network, this commitment was accepted.

Each State took advantage of the delay to add significantly to the data set.

This additional data with the consequential recomputation of many previously processed areas caused a major increase in the computer time requirement.

By the end of September 1982, the remaining data had been collected. A major problem was encountered in processing the data in that the New South Wales data set was found to comprise two separate networks which overlapped. The original data was the older triangulation with observations from groundmark to groundmark. The new data was an independent survey with observations from pillar to pillar. Although on a given hill the separation between groundmark and pillar was generally quite small, the surveys were independent of each other but connected and mathematically one mark could not be considered as an eccentric of the other. The confusion as to whether a particular observation was made to a pillar or the original groundmark led to major difficulties in setting up and validating the data sets.

Some idea of the overall increase in the size of the data set may be obtained by contrasting GMA82 (Figure 1) with GMA80 (Figure 2).

It had also become obvious that the data requirements for a continental primary network were not fully appreciated by some members of the Working Group. It is generally understood that the purpose of a primary network is to provide overall control by linking the various parts of a continent. This is subsequently densified by the use of secondary and tertiary "breakdown" networks. In each State, data was supplied which exceeded the need of a primary network and, in fact, formed the normal secondary densification throughout the areas of past and proposed urban and semi-urban development. Appreciating the desire of each state to have a unified and homogeneous control network in such areas, the data was accepted and included in the adjustment.

In some cases the purpose of the data sets in providing the necessary connections between stations was considered as a secondary function in that it was requested that all the measurements be included for archival purposes. It was pointed out that an archival data base was an intermediate phase between the field book and the final primary data set and that to achieve the most efficient data base it is frequently necessary that some rationalisation should take place by using techniques such as station adjustments. With careful use, such rationalisation will not have any significant effect on the end result. The debate is simply efficiency versus the "text book" approach and although the more academic approach has been adopted, it is felt that the data set should still be rationalised. For example, at one station from which directions have been observed to 16 other stations, the data set currently contains 29 sets of directions and a

total of 71 measurements. By using a full station adjustment, this could have been reduced to 16 measurements. These values would have been correlated and hence such a procedure is not recommended. However, some combinations of measurements would not have introduced correlation and would have greatly reduced the number of measurements required in the data set. When this situation occurs at many stations, the data set is expanded dramatically without serving any useful purpose for a continental adjustment. The requirement to provide archival records should remain the responsibility of the individual data collection agencies and should not complicate national adjustments.

By December 1982, copies of the then current data sets were sent to each State for verification. Some errors were found and these were subsequently corrected.

By the end of January 1983 the data set was complete and the final validation commenced. Some errors were detected and referred to the appropriate authorities for correction.

The SLR and VLBI data was then incorporated into the Stage 2 data set and the data set re-adjusted on 18 March 1983. This gave the data set which when combined with the NSW 92-2 precise ephemeris Doppler point positions allowed the calculation of the seven transformation parameters required to transform between NSW 92-2 and the AGD (see Section 5).

The seven parameters required to transform from the broadcast ephemeris to the precise ephemeris were calculated at this time using all 53 stations for which both ephemerides were available (see Section 6).

The multi-station relative positions were then added to the Stage 2 data set and the Stage 2 adjustment commenced. The final iteration was completed on 11 April 1983 and the results showed that the mathematical modelling for the whole adjustment was most acceptable. This iteration gave the final coordinates for all the junction stations. This allowed the analysis process to proceed in parallel to the Stage 3 adjustment of the individual blocks.

A complete Stage 2 and Stage 3 recomputation following the advice on 27 May 1983 that all States had checked their data sets and found them acceptable.

By 6 June 1983, the Stage 3 re-adjustments had been completed and thus the project appeared to have been completed.

The results were tabled for consideration by Council in September 1983 and the opportunity was taken for a final check on the correctness of the data sets. A number of errors were located and subsequently corrected. A small amount of additional data was also added to the data set and the final recomputation was completed during March 1984.

The National Mapping Council formally adopted the Geodetic Model of Australia 1982 adjustment at their meeting, 2-4 October 1984. It was also agreed at the meeting to refer to the resultant coordinate set as Australian Geodetic Datum 1984 (AGD 84).

## 2. MATHEMATICAL MODEL

### 2.1 The Fundamental Model

The fundamental model adopted for the adjustment is simply the classic least squares principle, i.e. that corrections  $V$  be found such that  $VTG^{-1}V$  is a minimum. Furthermore, the values of the corrections so found must satisfy all the appropriate statistical tests. When care is taken in preparing the data set and in assigning the appropriate variance to each observation, these statistical tests should be significantly satisfied without difficulty.

The observations supplied for the adjustment were of many types. For convenience, they may be separated into terrestrial and extra-terrestrial measurements. The terrestrial observations use as datum the direction of local gravity and the plane perpendicular to it. The observations are then reduced for the effect of the elevation of the station above some equipotential surface known as the geoid. Due to local gravity anomalies, this surface is most irregular. Since it is impossible to compute on such a surface, these observations must be transformed to some appropriate ellipsoid. The ellipsoid chosen for this adjustment is the Australian National Spheroid (ANS) as gazetted in the Australian Commonwealth Gazette of 6 October 1966 and which is further described in Bomford (1967a). The definition specified in the Gazette gives the ellipsoidal parameters as:

$$\begin{aligned} a &= 6378160. \text{ m} \\ f &= 1/298.25 \end{aligned}$$

and defines the origin of the Australian Geodetic Datum by giving the coordinates of the Johnston Geodetic Station as:

$$\begin{array}{ll} \text{Latitude} & \text{S } 25^{\circ} 56' 54''.5515 \\ \text{Longitude} & \text{E } 133^{\circ} 12' 30''.0771 \\ \text{Spheroidal Height} & 571.2 \text{ m} \end{array}$$

It should be noted that this definition implies that the value of the geoid/spheroid separation  $N$  at the Johnston Origin is +4.9m and this value has been adopted for this adjustment. Previously, a value of -6.0m has been used (Fryer) in preparing contour plans of  $N$  on the ANS.

### 2.2 The Choice of Ellipsoid

The choice between a local reference ellipsoid and an earth-centred system was based on political and mathematical considerations. These factors have been discussed and documented by a number of people.

On one hand it is desirable to adopt an ellipsoid which may be used world-wide. This would take full advantage of the current generation of global measuring systems and would facilitate international exchange of data. However, such a system may have disadvantages for practical use for local surveys. One such disadvantage occurs when there is a significant slope when the ellipsoidal surface is compared to the geoid in the region. This in turn leads to a situation where a differential scale correction has to be applied to all measured distances with the amount of the correction a function of the location of the measurement. For Australia, if WGS72 were adopted then the scale correction would vary from +6ppm in south western Australia to -12ppm at Cape York. The burden of applying this correction would fall on engineers and non-geodetic surveyors: that is on the people least qualified to understand the necessity for the correction.

Alternatively, if some local ellipsoid is adopted which has a reasonable fit within the region with the geoid, then the refinement of applying the correction for the geoid/ellipsoid separation may be ignored for many applications. For the Australian National Spheroid the correction varies relatively slowly across the continent and reaches a maximum of 3ppm.

For those wishing to transform results from the local ellipsoid to some other ellipsoid, it is simply a matter of applying the appropriate transformation parameters.

The decision by the National Mapping Council to retain the Australian National Spheroid "for the present" implies that, at some time in the future, the figure may be changed to some "earth-centred" ellipsoid such as WGS72 or its successor. One important point to realise is that there are a number of "earth-centred" reference systems and that the centres and scales of the systems are not necessarily identical. At this time, the transformations between these systems have not been finalised and a number of misconceptions are apparent in the current procedures used to "transform" from one system to another.

For example, it is commonly believed that to convert point positions calculated from Doppler measurements with the precise ephemeris (NSWC 9Z2) to values on WGS72 that one merely has to apply Seppelin's formulae. However, his formulae were derived mathematically to convert values from one ellipsoid ( $a=6378145$ ,  $f=1/298.25$ ) to another concentric ellipsoid ( $a=6378135$ ,  $f=1/298.26$ ). This does not take into account the 2 to 4 metre Z axis translation or the 0.4 to 0.8 ppm scale difference. Failure to include these values will introduce significant systematic errors into the converted coordinates (Kumar).

It should be noted that this applies to the "absolute" coordinates and that the effect is minimised in "relative" positions.

Thus, until such time as the transformation parameters between a world geodetic system and spatial measuring systems (e.g. TRANSIT, GPS, VLBI, SLR, etc) have been accurately determined and internationally accepted, it would seem inadvisable to adopt an earth centred ellipsoid.

### 2.3 Datum Point

In previous adjustments of the Australian Geodetic Network, the gazetted values for the triangulation station Johnston have been adopted as the origin for coordinates. In the provisional GMA82 adjustment, a change of coordinates when compared to the AGD66 has been found for stations throughout Australia. In some of the major urban regions this appears almost as a block shift and has led to the suggestion that the values of the coordinates be changed by say 2 metres. While this may appear to reduce the differences in south eastern Australia, the differences would be increased elsewhere. This approach also ignores the fact that by changing the geodetic longitude of Johnston, the values of the Laplace corrections for all astronomic azimuths will be systematically changed by a significantly constant amount. The overall effect would be a rotation of the entire network thus introducing a further change to the latitudes and longitudes.

As pointed out in the GMA80 Report (page 17) there is very little practical advantage in rotating the network unless the change to Johnston is such that the earth's spin axis and the semi major axis of the ANS are made parallel. This would involve a change of roughly 20 metres in the longitude of Johnston and similar changes in the latitude/longitude of all other stations in the network.

This appears to be undesirable at the present time.

## 2.4 The Geoid

In order to carry out the transformation of the terrestrial observations from geoid to ellipsoid, it is necessary to be able to predict the values of the deflection of the vertical and the geoid/spheroid separation for any point given its latitude and longitude. It must be stressed that although the corrections to individual observations are quite small, if neglected there is a significant error in the overall result (Thompson).

During the last decade a number of geopotential models such as the series of GEM models have become available which give a model for the geoid in terms of spherical harmonics. These spherical harmonics may be used with routines such as those developed by Dr G. Lachapelle to predict the deflection and separation. This technique was investigated by comparing values computed using the spherical harmonic routines with the values contained in the National Mapping Deviate file. This file contains the astronomic positions and N values for 1315 stations distributed over the whole of Australia. Two different geoid models were used in the comparison, namely GEM 8 and GEM 10B. Both models are based on a geocentric ellipsoid of radius 6378139 metres and a flattening of 1/298.257. The computed separations and deflections must be transformed to the Australian National Spheroid which is not geocentric and has a radius of 6378160 metres and a flattening of 1/298.25. This transformation may be carried out by using a simple translation of origin and computing a bias term (Undno) which gives the "best mean fit" to the area being adjusted. Initially, the best translation values available were those found during the GMA80 adjustment. After processing the GMA82 preliminary data set through both Stage 1 and 2, new values for the parameters were found (Table 1). The new values are slightly different from the GMA80 values and both sets are given below in Table 1.

Table 1.  
Geoid parameters

	GMA80	GMA82
X . . . . .	113.1m	116.5m
Y . . . . .	47.7	50.3
Z . . . . .	-145.7	-138.9
Undno . . . . .	10.4	6.9

The comparison with the Deviate file values was repeated using the GMA82 parameters. The comparisons of the two geoid models and the two sets of parameters gave the results quoted in Table 2. Since there was not a significant difference in the calculated separations or deflections, the Stage 1 computation of GMA82 was not iterated. Furthermore, purely for convenience the GMA80 values were used for all subsequent computations.

It is recommended that for all future computations the GMA82 parameters be used.

Since the vast majority of all geodetic measurements are observed within a few degrees of horizontal, it can be seen that the required precision of the predicted values is more than adequate for the purpose of transforming terrestrial observations to the ellipsoid. There was little difference in the results from the two models but GEM 8 was much simpler to compute.

The routines used to calculate the deflections and separations were further tested by comparing the results from the routines developed by Dr G. Lachapelle with the results from the routines developed by Dr C. Rizos. The comparison gave maximum differences of less than 1 second of arc for the deflections and less than 10 cms for the separations. Hence for all practical purposes the routines can be considered to give the same results when used for the reduction of terrestrial observations.

Table 2.  
Comparison of Geoid Models with the Deviate File

<i>Standard deviation of differences</i>			
	<i>Deflection</i>		<i>Separation</i>
	<i>Latitude</i>	<i>Longitude</i>	
Using GMA80 Parameters—			
GEM8 . . .	3.2"	4.6"	2.8m
GEM10B . . .	3.2"	4.7"	3.2m
Using GMA82 Parameters—			
GEM8 . . .	3.2"	4.7"	2.7m
GEM10B . . .	3.2"	4.7"	3.2m

A recent geoid model JSA82 (Allman 1982) offers better precision and much easier computation as the model is defined as a table of N values rather than in terms of spherical harmonics. The tightness of deadlines has prevented the development of suitable routines and adequate testing in time to use this geoid for GMA82. However, the GEM 8 model is more than adequate at the level of precision required for the reduction of observations and has been adopted for the adjustment.

Some concern has been expressed over the values used for the geoid/spheroid separations N particularly at the SLR and VLBI stations. To investigate this a comparison has been made of all the differences in separation as determined by the values in the Deviate file and the values for the same stations computed by using the GEM 8 model. The maximum difference was 9.68 metres. An error of such an amount in the value of N at one end of a measured length would introduce a systematic error of 0.75 ppm. This systematic error would apply to all lengths measured from that station. Such an error would be significant and thus the difference in N warrants investigation.

The difference in N was found by comparing N values from two sources, neither of which could be adopted as the standard. The values from the Deviate file are the result of an analysis of astro-geodetic levelling and gravity data carried out by Dr J.G. Fryer. This analysis has been published by the Division of National Mapping as Technical Report 13. On page 12 of the report Dr Fryer states:

"9.9 The largest standard deviation in N at any junction station with respect to Johnston Geodetic Station was  $\pm 1.5$  metres. The largest standard deviations were at the junction stations on geoidal profiles around the coast line of Australia. Some caution must be attached to the quoting of the standard deviations of N at junction stations as experience has shown that values of N along a section of widely spaced astro-geodetic stations may change dramatically with the inclusion of more astronomical observations."

If his assessment is accepted then errors of up to three standard deviations, i.e. at least 4.5 metres may be expected in the values in the Deviate file.

The GEM 8 values of N are computed using spherical harmonics determined by the analysis of the orbits of artificial satellites. The precision of separations so determined is difficult to assess but comparisons of 324 Doppler point positions scattered throughout Australia, Asia and Pacific gave a standard deviation of 3.5 metres for the difference between Doppler spheroidal height and MSL height plus computed N using GEM8. The computed value of N would thus appear to have a standard deviation of about 2 metres. This value could be significantly reduced when time permits the development of the necessary algorithms to use the JSA82 geoid. This geoid is derived from GEM 10C and a correction surface based on 594 Doppler point positions and has a standard deviation of 1.6 metres for the comparison between Doppler spheroidal height and MSL plus computed N.

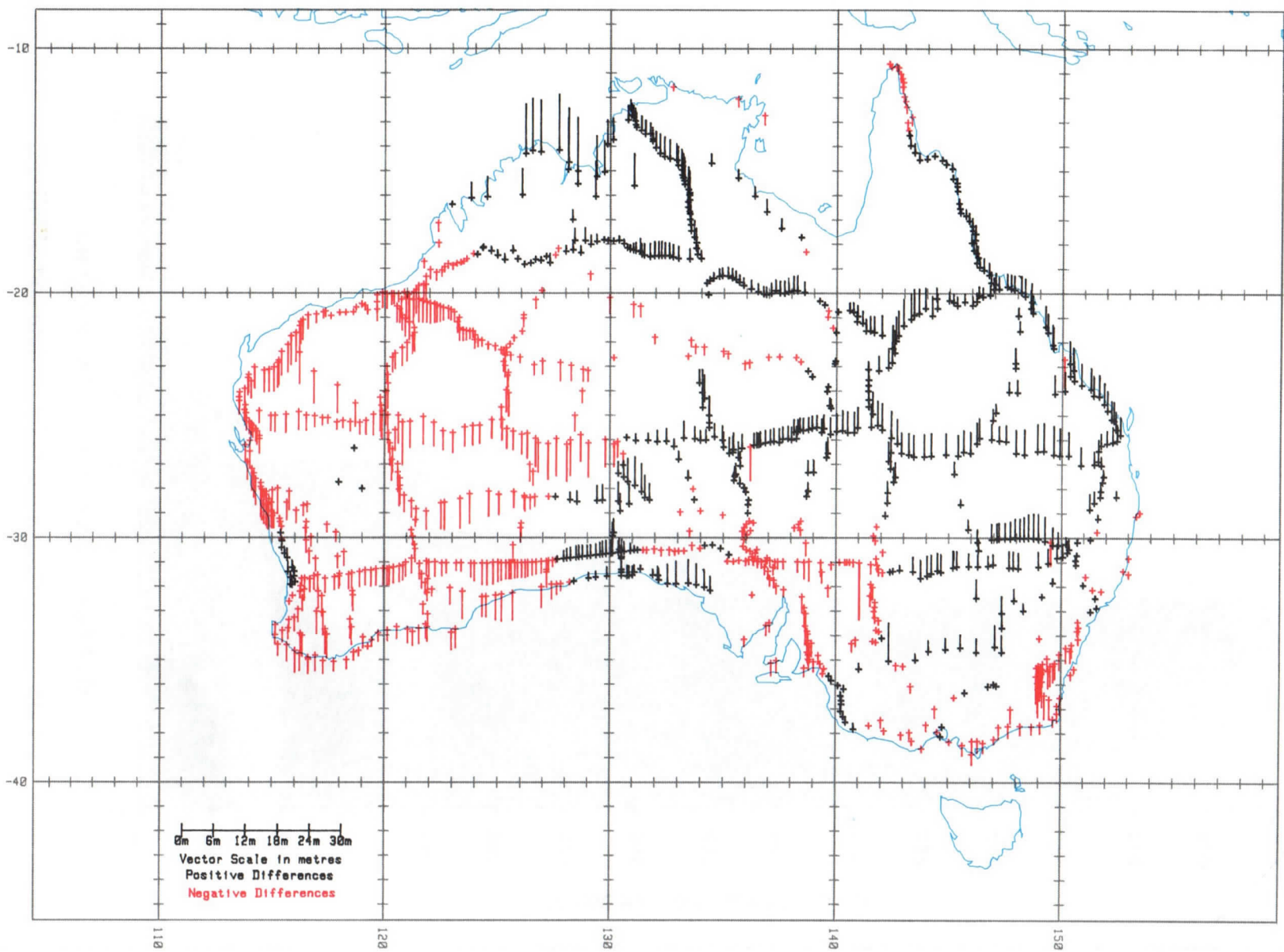
It would appear that values of N for individual stations interpolated from Fryer's plan and those computed from the GEM 8 model are of more or less the same precision. The differences referred to above are consistent with this conclusion.

The differences are plotted in Figure 3 in which the differences are taken as (GEM 8-Deviate). The positive differences are plotted as black vectors and the negative as red. In this way it can be seen that there are regional systematic differences between the two models. To further demonstrate that although there are systematic differences between the models, the use of either one will not cause major changes in the resulting coordinates, the differences have been examined to find the percentages which fall into specified classes. The results are given in Figure 4 which clearly indicates that 76.2% of the values agree to within less than 3 metres and 98.6% within 6 metres. Since neither model can be taken as standard, the choice of model is a matter of personal choice and convenience.

For GMA82, the GEM 8 model has been preferred since this allows the value of N for any desired station to be determined using a computer algorithm and the coordinates of the station. The same routines also predict the deflections of the vertical at the station. The alternative would be the manual interpolation of the Fryer plan for each station. Neither model gives values of N which are as precise as would be desirable although the standard deviation of the introduced systematic errors should not exceed 1 ppm. The fact remains, however, that both models represent current "state of the art" levels of precision and must be used until a significantly better model is developed.

The effect of adopting one model rather than the other must also be examined against the background of the overall purpose of the adjustment. For GMA82, the geoid is used to reduce field observations to the spheroid. The reduced observations are then adjusted to form a basic framework across the continent. Subsequently, additional fieldwork is carried out to give a "breakdown" or densification of control. If the same geoid is used to reduce the new fieldwork, then the same values of N are used in the reduction and the new survey will fit the framework. The GEM 8 routines used in CHAOS are identical to those in GANET and thus the primary and densification adjustments will be consistent.

Figure 3. Differences of N between Deviate and GEM8



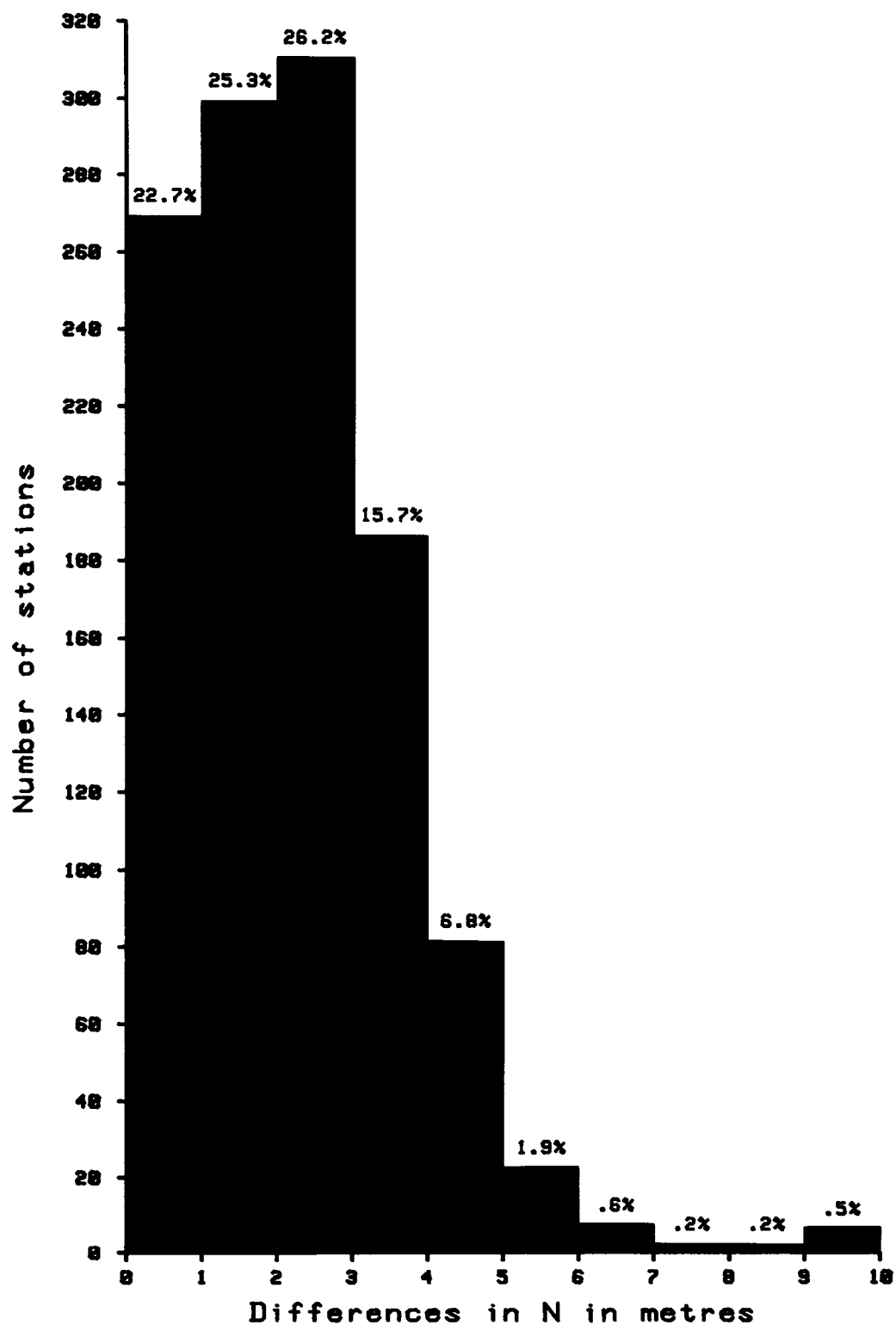


Figure 4. Histogram of N for Deviate and GEM8

The above leads directly to a discussion of the terms of reference implicit in carrying out an adjustment. Until recently, specifications adopted by most countries defined first order control as having a nominal precision of one part per million and gave the allowable limits for actual comparisons of measurements. For GMA82, the design where feasible has been to work to at least one order of magnitude better than 1 ppm. This is of particular relevance when examining the vectors shown in the Figures given in this report in that the displacement vectors shown are the result of coordinate changes between various models each of which holds fixed the coordinates of the Johnston Geodetic Station. The vectors are given in metres and their magnitude may cause some concern unless it is understood that they refer to Johnston. The vector should be examined in terms of parts per million from the Johnston Origin since this approach will give a better appreciation of the differences in the models. To enable a mental conversion from metres to parts per million, Figure 5 has been included. This Figure gives the magnitude of one part per million against the distance to the point from Johnston.

It should also be remembered that the most important requirement of an adjustment for general surveying, mapping and similar purposes is that within local regions the relative position of stations should be as tight as possible. To assess this criteria, it is necessary to difference the vectors within the region. If the vectors are of roughly the same magnitude and the same orientation, then relativity is being maintained.

Thus, the terrestrial observations for direction, distance and azimuth may be transformed from geoid to ellipsoid by using a spherical harmonic geoid model and the appropriate reduction formulae as given in Bomford's "Geodesy", in National Mapping's user manual for program GANET and elsewhere.

The extra-terrestrial observations obtained by satellite laser ranging and by VLBI yield chord distances. These are simply reduced to ellipsoidal distances and the parametric equations formed in the same manner as for terrestrial distances.

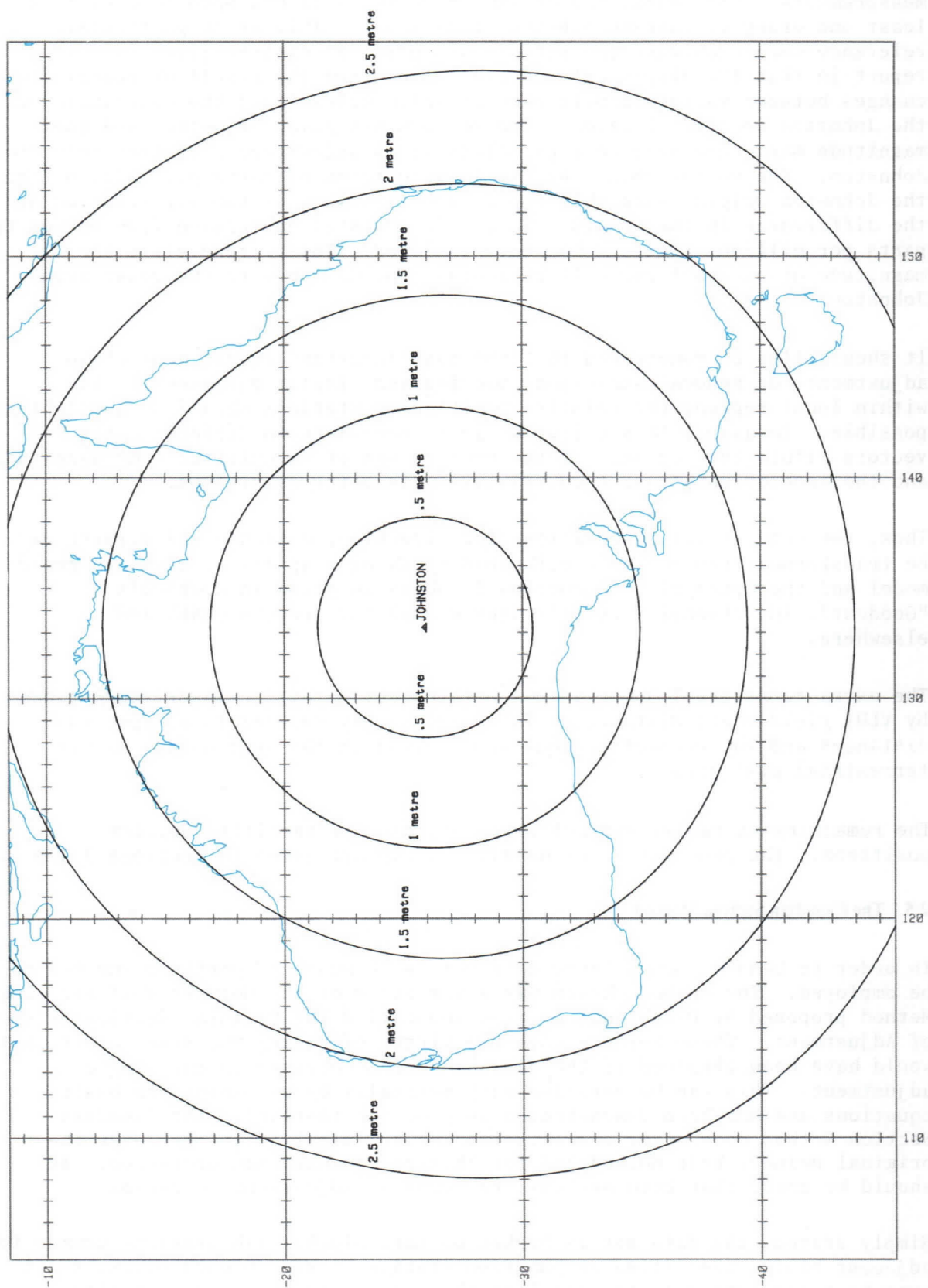
The remaining extra-terrestrial observations were satellite Doppler positions. The procedures for handling these are given in Sections 5 and 6.

## **2.5 The Canadian Section Method**

In order to handle such a large data set, some means of partitioning had to be employed. The method chosen was a variation of the Helmert Wolf Blocking Method proposed by Pinch and Peterson and called The Canadian Section Method of Adjustment. These methods have the virtue of giving the same result that would have been obtained if the data had been processed in one single adjustment. This can be verified mathematically by examining the basic equations and has been demonstrated in practice (Berlin). The Canadian Section Method does offer some significant practical advantages over the original Helmert Wolf Method and for this reason has been preferred. It should be noted that both methods are types of Adjustment in Phases.

Simply stated, the data set is broken up into blocks with stations common to adjacent blocks identified as junction stations. Within each block, there must be sufficient data to calculate the coordinates for every station within the block. The manner in which the data is subdivided is shown in Figure 6.

Figure 5. One Part Per Million from Johnston



In Case 1, stations B, C, E and F are the junction stations and occur in both blocks. The observations at stations B and C are placed in block X1. The observations at stations E and F are placed in block X2. Any observed distance or azimuth between the junction stations can be put into either block.

In Case 2, stations P and Q are the junction stations and are placed in both blocks. The directions at station P are placed in block Y1, the directions at Q are placed in Y2. The distance QR is placed in both blocks but with double variance. This has the same effect mathematically as a single entry of the observation with the original variance but allows coordinates to be calculated in each block.

In difficult cases, a fictitious or duplicated observation may be used with an extremely large variance. This allows the coordinates for each junction station to be calculated in each block without significantly altering the mathematical model.

These techniques allow the data to be subdivided into the discrete blocks as required for the section adjustment.

Having subdivided the data, each block is adjusted as a "free network". The outputs from each adjustment are the "quasi" observations of latitude and longitude (i.e. position equations) and the associated variance/covariance matrix. This procedure is designated as Stage 1 of the adjustment. When the size of a block becomes excessive the data set may be further subdivided. The adjustment of these sub-blocks is designated as a Stage 0 adjustment.

In Stage 2, the position equations for each block are put together and the adjustment continued. The result is the final adjusted coordinates for the junction stations.

Stage 3 consists of readjusting each block of data holding the coordinates of the junction stations fixed at the values found in Stage 2. The resulting coordinates and the corrections to the observations so found are identical with those that would have been obtained by a single adjustment.

The technique also allows the SLR and VLBI data and the satellite Doppler data to be processed as individual blocks by simply flagging the appropriate stations as junction stations in the terrestrial data sets.

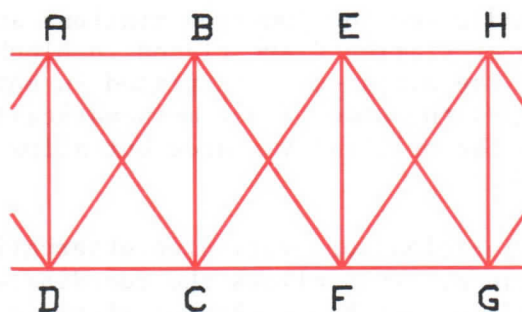
A schematic flow chart showing how Stages 1 and 2 were processed is given in Appendix A. Stage 3 was processed in the reverse order.

## **2.6 Schreiber Method of Eliminating the Orientation Unknown**

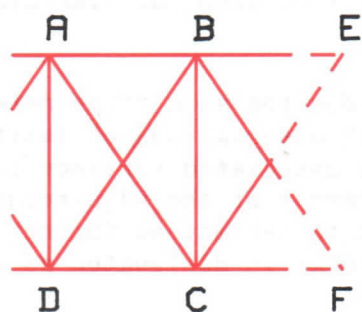
In carrying out an adjustment of a triangulation network by the parametric method there are two parameters (or unknowns) for each free station plus one orientation parameter for each set of observed directions. With multiple occupations of stations the number of unknowns increases dramatically. Since the time to invert the normal equations is a function of the cube of their size, any technique which reduces the number of parameters while retaining mathematical rigour should be adopted.

One such option is the use of the Schreiber method for the elimination of the orientation parameter. The basis of the method has been known for many years and has been published many times (e.g. Rainsford).

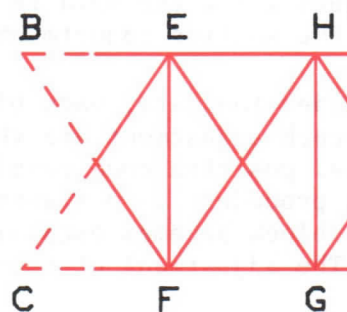
## CASE 1



BLOCK "X1"



BLOCK "X2"



## CASE 2

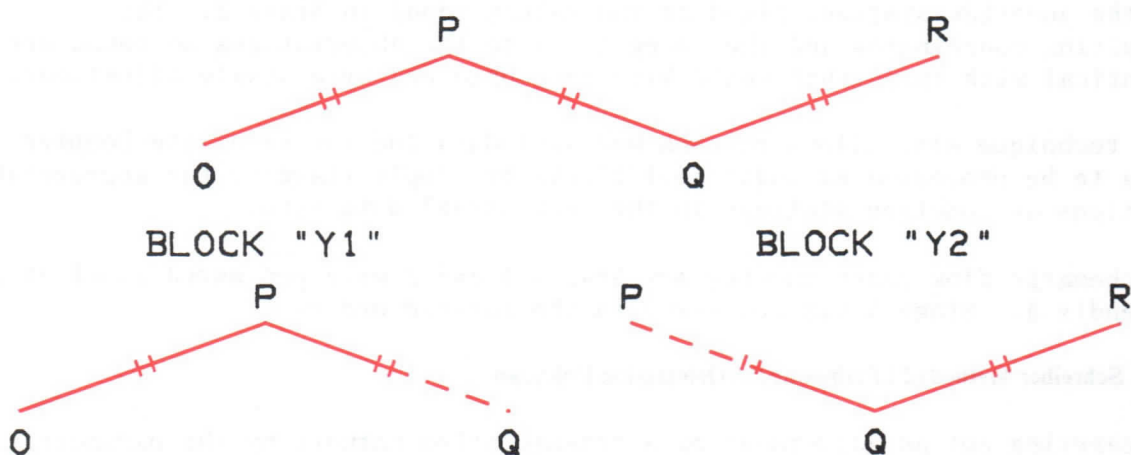


Figure 6. Subdivision of Data into Blocks

## 2.7 Inversion by Diagonal Partitioning

The amount of core storage required for the normal equations is a function of half the square of twice the number of free stations. There are a number of routines available which may be employed to reduce the core requirement for the solution of a set of symmetric simultaneous equations. These techniques rely on banding or variable height profiles to eliminate the storage of zero elements. Such techniques become inefficient when a significant portion of the inverse matrix is required. To overcome this, use is made of techniques which partition the normal equations into blocks such that two of the blocks can be in central memory simultaneously while the remaining blocks are kept in off-line mass storage devices. One such technique (proposed by D.S. Zimmerman) is the routine used for inversion by program CHAOS.

## 2.8 Program CHAOS

In carrying out a least squares adjustment many separate steps must be computed. Some of these steps may be undertaken by using preliminary programs in order to prepare the data set for final computation. After the major computation, there still remains an evaluation phase which can also be carried out using a separate program. The program called CHAOS (Canadian Horizontal Adjustment Or Simulation) is an algorithm with many options so that the "pre" and "post" processing may be carried out by the one algorithm. In general, it will take data straight from the field book and process it straight through to the final solution. The options include the input of slope distances, the computation of corrections for deflection of the vertical, geoid/spheroid separation, skew normal and the Laplace equation.

The initial version of the program (HAVOC) was developed by Mr T. Vincenty of the USAF on an IBM360. Subsequently, the program was obtained by National Geodetic Survey in Washington and early in 1975 a version was obtained by Geodetic Survey in Ottawa.

During Dr Allman's study leave at Geodetic Survey during 1975/76 his main task was to incorporate all appropriate options and diagnostics into the input routines, verify the operation of the adjustment portion of the program and to add whatever statistics he thought would be beneficial to the output routines.

On his departure from Ottawa, the program was given to a team of system analysts to improve the computer efficiency. The resulting program was named Geodetic Adjustment NETworks or GANET.

Dr Allman also brought a version back with him to the University of New South Wales where the program has been undergoing continuous upgrading. This version is now called CHAOS.

As an example of the continuous modification, the program was modified in March 1983 to output as an option some statistics relevant to Stage 3 of a section adjustment. These statistics had been computed manually for GMA80 and it seemed more appropriate to add yet another option to the program to save this manual effort.

The program CHAOS has not been developed as a production program. Many of the features, options and output statistics are research oriented. Some values are self explanatory whilst others need explanation before a correct interpretation can be made. The program, by selecting the appropriate options, can be used for many types of adjustment model although by so doing, some of the statistics change in their interpretation. In some cases, a manual calculation may be required using values obtained from the program output. Accordingly, care should be taken and advice sought before attempting to draw any conclusions from the output of the program.

## 2.9 Programs for Doppler Reductions

The School of Surveying has had the Geodetic Survey of Canada programs PREDOP 3 and GEODOP 3 since 1976. Later versions and modifications of both programs have also been supplied by Geodetic Survey. Active development of the programs has been carried out since 1977 and the periodic modifications supplied by Dr J. Kouba have been incorporated as they arrived. Other modifications have been found necessary and after testing have been incorporated. The people mainly involved in the Doppler program evaluation at the School of Surveying were:

Mr P. Shi	Research Scholar
Mr A. Obeid	Post Graduate Student
Mr G. Jeffress	Professional Officer
Mr W. Milward	Technical Officer

The programs are currently "state of the art" and have been used in reducing the Doppler data in both point position and multi-station modes and using both the precise and broadcast ephemerides. Careful checking of the resulting values against the terrestrially derived coordinates for the same stations does not reveal any anomalies and hence the resulting data has been accepted as part of the GMA82 data set.

It is anticipated that further research into the mathematical modelling used in the programs may lead to even more precise values in the future. This project which commenced in 1981 was suspended in December 1981 to allow all facilities and resources to be concentrated on GMA82. The project has now been re-activated.

## 2.10 Transformation Parameters

Seven parameters are required to rigorously transform coordinates from one 3-dimensional cartesian system into coordinates on another cartesian system with a different origin, scale and spatial orientation. If the coordinates of three stations are known exactly in both systems then the values of the seven parameters may be calculated exactly. In geodesy this is not the case as the station coordinates are the result of an adjustment of observations and thus more stations are required and the parameters are found by a least squares solution.

In addition, the nature of the transformation leads to a geometric instability when the stations are close together. Ideally, the stations should be uniformly spread around the globe or a significant portion of it. When the parameters have been calculated by using stations which are enclosed within a small area, the instability becomes apparent when stations which lie outside the area are transformed. To overcome this, the standard Bursa-Wolf model has been modified to give the Molodensky-Badekas model. It is claimed for the computation of the parameters from a group of stations in close proximity that the Molodensky-Badekas model will give more precise values for the parameters (Boucher).

For GMA82, which has common stations covering the whole continent and where the parameters will only be used to transform the coordinates of stations within those bounds, the Bursa model has been adopted.

Two programmes (PARM and DOPTRAN) have been developed to carry out the necessary computations. PARM calculates the seven parameters given the common station coordinates and variances in both systems. DOPTRAN transforms coordinates from one system to another given the values of the parameters.

### 3. TERRESTRIAL DATA SET

The terrestrial data set consisted of a complex mixture of triangulation, trilateration and traversing. The observations, spanning a period of more than a hundred years, used a variety of instruments and field techniques. Fortunately, the majority of the measurements were taken during the last thirty years and careful checking by each State has ensured that the number of possible data errors has been kept to an absolute minimum.

The terrestrial values entered into the data base were:

1. Station label, name, coordinates and elevation.
2. Direction and variance.
3. Slope distance and variance.
4. Astronomic azimuth reduced to CIO, variance and astronomic coordinate values.
5. Each observation was given a code indicating the type of instrument used and the year of observation.

In line with overseas experience, coastal Tellurometer distances in Australia displayed a systematic scale difference when compared with Geodimeter measurements. Appendix B shows the corrections expressed in parts per million that were applied to Tellurometer measurements in coastal regions prior to the value being entered into the data base.

Each station was identified by a unique 8-character label. The station name was also entered into the data base. This was not necessary for the computer program but greatly simplifies the manual checking of data.

For computer efficiency, the data set was subdivided into 35 blocks for processing by the Canadian Section Method (Section 2.3). The locations of the individual blocks are given in Figure 7 and the details of each block are given in Table 3.

As part of the Stage 1 and 3 adjustment process, the following reductions were carried out each time the data was processed. In other words, to ensure the integrity of the data set the fundamental data base was always used and intermediate results were not stored. This greatly facilitated the correction of errors during the validation phase of the project.

1. The directions were reduced to the ellipsoid using the values for the deflection of the vertical computed using GEM 8.
2. The distances were reduced to the ellipsoid using the MSL elevations as provided with the N values computed using the GEM 8 model.
3. The astronomic azimuths were corrected using the Laplace equation and the deflections implied by the astronomic positions.

In assigning variances to the field observations it has long been recognised that there are two basic components: namely the internal and external. Many authors (e.g. Forstner, Pfeifer, Hoar) have dealt with methods of assessing the internal and external components of the total variance. The internal component is the precision with which the measurement may be repeated whilst the external component is a complex function of systematic effects such as refraction, phase of targets and long and short wave length periodic errors. This external component can only be determined by the analysis of many networks and by using evidence from other sources.

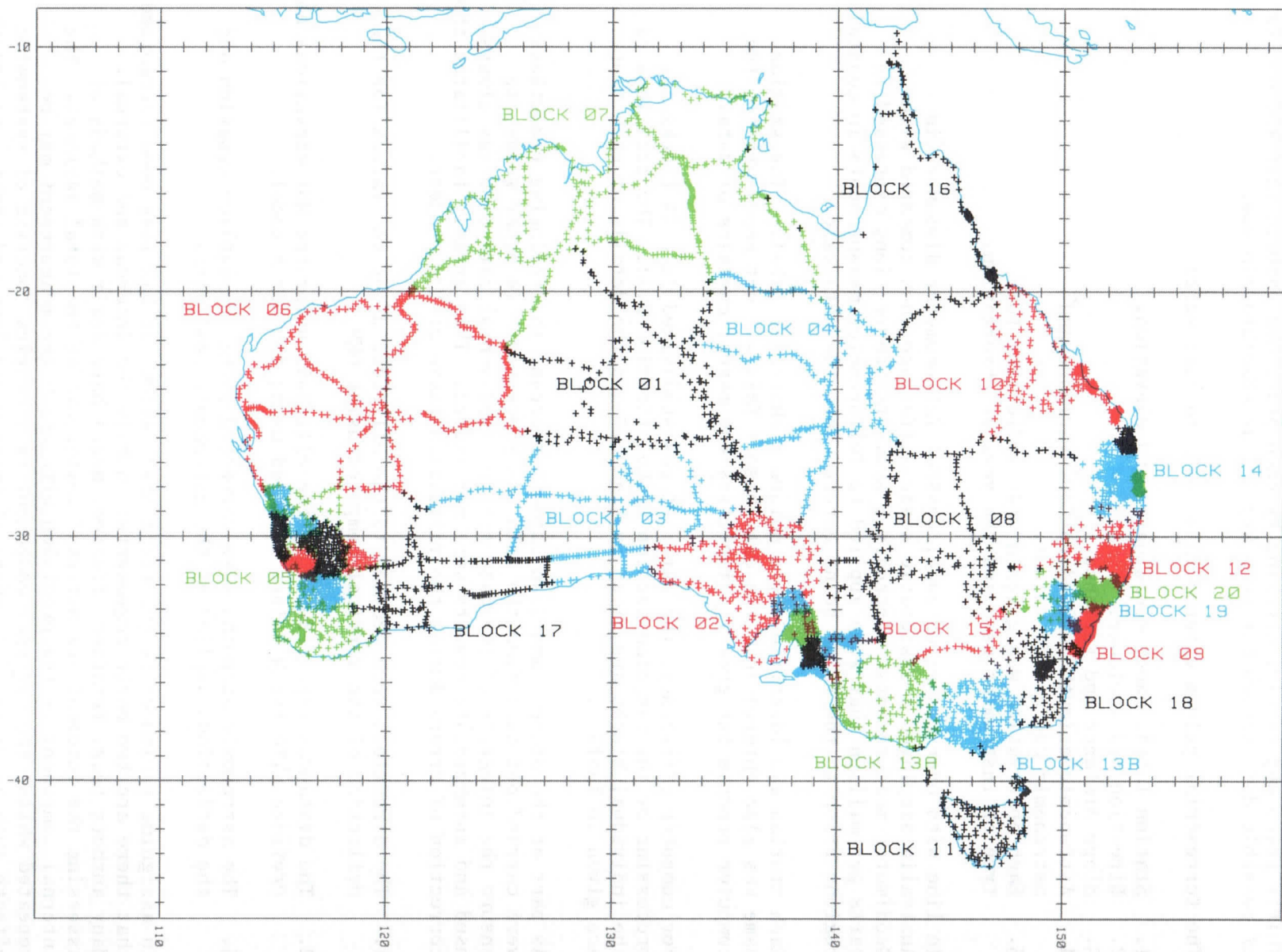


Figure 7. GMA82 by Blocks

Table 3.  
GMA82 Block Details

<i>Block</i>	<i>Stations</i>	<i>Directions</i>	<i>Distances</i>	<i>Azimuths</i>	<i>Total</i>
01	184	801	193	51	1 045
02	318	1 288	289	154	1 731
02A	90	331	164	2	497
02B	110	101	257	6	364
02C	69	84	166	4	254
02D	110	62	283	1	346
02E	59	29	143	4	176
03	205	416	242	130	788
04	180	382	230	65	677
05	297	937	551	98	1 586
05A	153	788	442	2	1 232
05B	127	621	276	1	898
05C	129	468	215	1	684
05D	66	264	140	1	405
05E	60	198	89	1	288
05F	52	177	81	1	259
05G	81	398	246	1	645
06	261	627	298	79	1 004
07	393	924	411	121	1 456
08	173	614	209	93	916
09	306	3 976	529	6	4 511
10	273	388	972	80	1 440
11	126	620	287	18	925
12	158	2 319	418	40	2 777
13A	151	1 316	226	27	1 569
13B	275	3 867	549	23	4 439
14	362	345	1 723	38	2 106
14A	110	36	465	1	502
14B	105	10	439	1	450
15	50	169	39	6	214
16	194	225	619	99	943
17	161	374	289	93	756
18	179	926	332	31	1 289
19	201	2 517	297	1	2 815
20	276	3 465	397	12	3 874
Total	5 498	30 063	12 506	1 292	43 861

The values adopted in general for the minimum variance to be assigned to observations made in accordance with the standard specifications laid down by the National Mapping Council are given in Table 4. An exception was made for the observations supplied by the State of Victoria where the variances were used as supplied.

For observations which did not conform to the Council specifications conservative values were assigned by the appropriate authority after some consultation with Dr Allman. The results show (Table 9) that the values assigned were slightly over-conservative but since this would not significantly alter the coordinates, it is not proposed that they should be amended or that the adjustment should be repeated.

Table 4.  
Minimum Variances for Standard Observations

<i>Observation Type</i>	<i>Variance</i>
Directions (1st Order)	0.5 sec <sup>2</sup>
Azimuths (Laplace)	2 sec <sup>2</sup>
Geodimeter Distance	(5mm + 1ppm) <sup>2</sup>
Tellurometer MRA1 & 2	(5cm + 5ppm) <sup>2</sup>
Tellurometer MRA4	(3cm + 3ppm) <sup>2</sup>
Doppler PP (PE)	2.25 m <sup>2</sup>

#### 4. SATELLITE LASER RANGING AND VERY LONG BASELINE INTERFEROMETRY

This data set consisted of baseline measurements obtained from:

- (a) approximately 500 000 laser ranging measurements to the LAGEOS satellite made between 1 January 1980 and 31 December 1980 from the Orroral Valley (Smithsonian Astrophysical Observatory) site near Canberra and the Yarragadee (NASA MOBLAS) site east-south-east of Geraldton in Western Australia, and
- (b) approximately 200 Very Long Baseline Interferometry (VLBI) measurements of extragalactic radio sources made during two 24-hour observing sessions on 26 April and 3 May 1982 and one 12-hour night-time observing session on 28 April 1982 from the NASA Deep Space tracking facility at Tidbinbilla (DS 43) near Canberra, the CSIRO Radio Telescope at Parkes, the University of Tasmania's Radio Observatory near Hobart, the University of Sydney's Fleurs Observatory (X3) near Sydney and the LANDSAT tracking station at Alice Springs. The position of the stations is given in Figure 8.

The laser baselines were computed at the University of New South Wales by Drs A. Stolz and E. Masters and Mr B. Hirsch. The data was made available by NASA under the Crustal Dynamics Program. The laser project is supported by the Australian Research Grants Scheme.

The VLBI measurements were organised by the staff of the CSIRO Division of Radiophysics. A significant portion of the geodetic data processing was carried out in the United States as facilities are not available in Australia. The work was carried out at the Jet Propulsion Laboratory and the Massachusetts Institute of Technology and at the University of New South Wales by Mr B. Harvey and Dr A. Stolz. JPL provided special equipment and expert personnel to supervise the observational program. This aspect was partially supported by NASA under the Crustal Dynamics Program. The Division of National Mapping distributed time at all five observatories and cooperated with other National Mapping Council members in establishing connections between the antennas and the Australian Primary Network. Staff of CSIRO, Division of National Mapping, Sydney University, University of Tasmania, University of New South Wales and the Australian National University assisted with operations at the observatories.

The measurements given by the VLBI observations were chord distances between the electrical centres of the antennas. These measurements were reduced to chord distances between groundmarks for the adjustment process. The values and their precision are given in Table 5.

On completion of Stage 1 for all the terrestrial data, a Stage 2 adjustment of the terrestrial data was carried out. This gave an opportunity to compare the unadjusted chords with those deduced from the pure terrestrial data. The differences are shown in Table 5 and exhibit the remarkable agreement that can be obtained by careful mathematical modelling and rigorous adjustment processes. To compare the measurements which have such a wide range of precision, use should be made of the standardised difference which is also given in Table 5.

It is important to note that Table 5 refers to values before adjustment and that the VLBI measurements are from electrical centre to electrical centre.

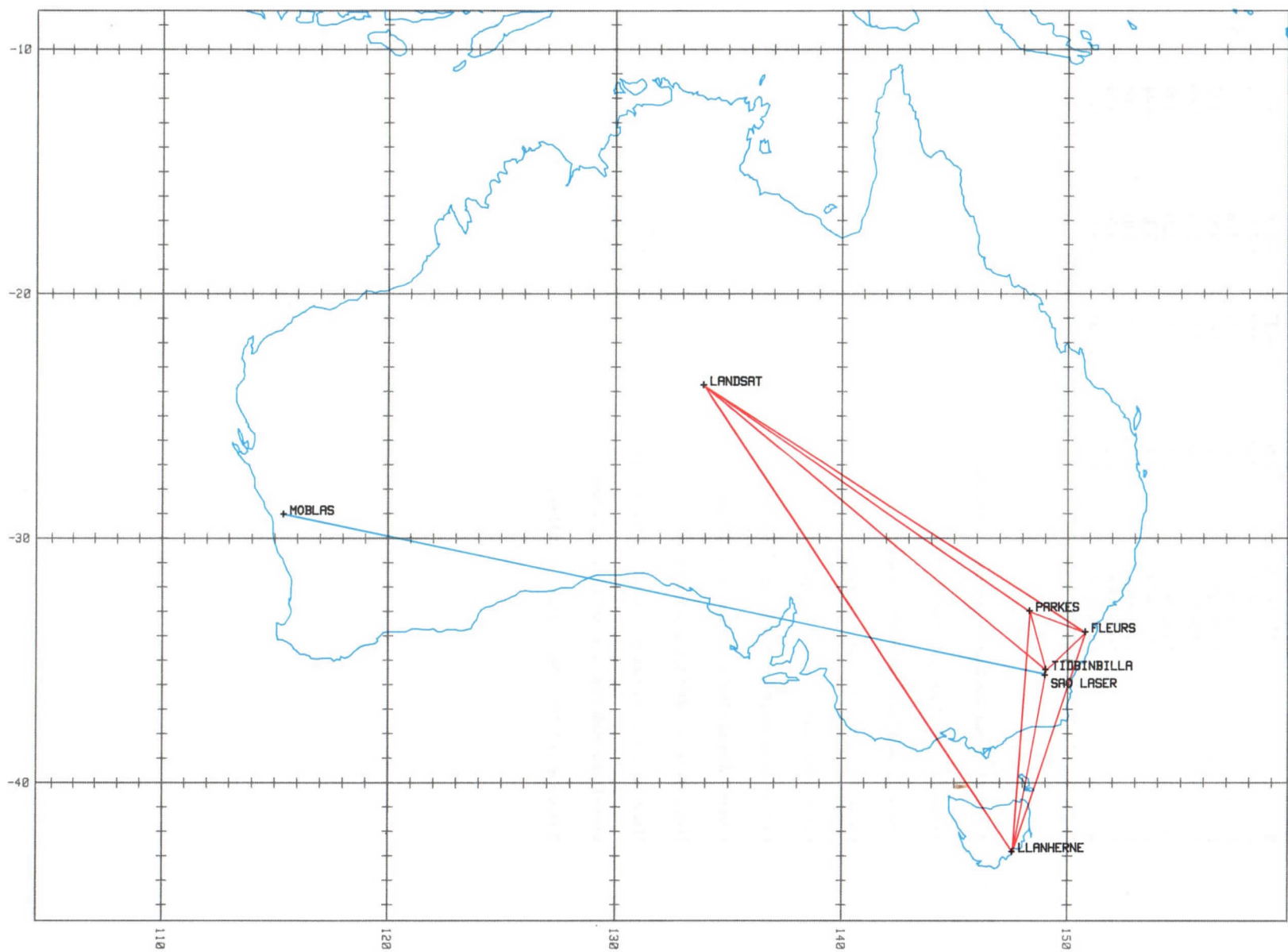


Figure 8. SLR and VLBI Measurements

Table 5.  
SLR and VLBI Measurements

<i>Line</i>	<i>Measured Chord Distance</i>	<i>Standard Deviation</i>	<i>Difference from Terrestrial</i>	<i>Parts per Million</i>	<i>Standardised Difference</i>
A . . . . .	3 196 329.00m	0.16m	0.37m	0.12	2.32
B . . . . .	236 681.19	0.16	-0.27	-1.16	-1.71
C . . . . .	251 340.47	0.16	-0.60	-2.38	-3.74
D . . . . .	1 035 709.1	6.5	-3.37	-3.26	-0.52
E . . . . .	1 979 707.5	1.9	1.04	0.52	0.55
F . . . . .	274 751.78	0.06	-0.32	-1.18	-5.40
G . . . . .	1 938 997.1	1.7	2.35	1.21	1.38
H . . . . .	835 297.0	6.4	-2.80	-3.36	-0.44
I . . . . .	1 733 991.1	1.7	1.79	1.03	1.05
J . . . . .	1 093 516.8	6.5	-3.50	-3.20	-0.54
K . . . . .	2 445 610.4	6.5	-0.47	-0.19	-0.07

A from SAO Laser Tracking Stn to Moblas, Yaragadee

B from Fleurs X3 (Sydney) to Tidbinbilla DSS

C from Fleurs X3 to Parkes Radio Telescope

D from Fleurs X3 to Llanherne, Hobart

E from Fleurs X3 to Landsat, Alice Springs

F from Tidbinbilla DSS to Parkes Radio Telescope

G from Tidbinbilla DSS to Landsat, Alice Springs

H from Tidbinbilla DSS to Llanherne, Hobart

I from Parkes Radio Telescope to Landsat, Alice Springs

J from Parkes Radio Telescope to Llanherne, Hobart

K from Landsat, Alice Springs to Llanherne, Hobart

## 5. SATELLITE DOPPLER POSITIONING

The point position Doppler data set comprised the measured values of the NSW 92-2 point positions calculated using the precise ephemeris for 191 measurements taken at 156 stations (Figure 9). The positions of 128 of these stations were supplied by the Division of National Mapping and the Royal Australian Survey Corps. The remaining positions were the point positions of 53 of the multi-station Doppler stations. These positions were calculated by Mr G. Jeffress using the programs PREDOP and GEODOP and the precise ephemeris. The precisions of the positions were assessed by the number of passes and the use of the Standard Specifications of The National Mapping Council.

Before the positions could be incorporated into the data set it was necessary to transform the values from the NSW 92-2 reference system onto the AGD datum. This required a knowledge of the seven transformation parameters which were determined by a least squares fit of the NSW 92-2 values to the corresponding AGD values determined by the terrestrial and VLBI data set. The values of the parameters so determined were labelled the initial values and were used to transform the Doppler positions onto the AGD. After a preliminary Stage 2 adjustment, the values for the parameters were recomputed to give the final values. It is realised that the above method is iterative. However, experience with GMA80 shows that the process converges in the first iteration.

The validity of using seven transformation parameters rather than four or five has been demonstrated before (Technical Report #29). The parameters themselves have been carefully evaluated as a number of authors (Kouba, Ashkenazi, Wolf) have found significant systematic differences in the modelling of space-based measurements against terrestrial measurements. As part of this evaluation, the displacement vectors from the terrestrial and VLBI solution to the transformed point positions have been plotted (see Figure 10). Examination of this plot does not reveal any significant systematic error in that the vectors appear to be completely random. Hence the parameters have been adopted.

The parameters adopted to transform from NSW 92-2 positions using the precise ephemeris to GMA82 are given in Table 6.

It should be noted that the sign convention adopted for these parameters is the standard geodetic convention, i.e. that when looking along the positive axis towards the origin, anti-clockwise rotations are positive.

Table 6.  
Parameters for NSW 92-2 to GMA82

<i>Parameter</i>	<i>Initial Value</i>	<i>Final Value</i>	<i>Standard Deviation</i>
Dx . . . . .	116.47m	116.00m	1.2m
Dy . . . . .	50.25	50.47	1.2
Dz . . . . .	-138.87	-137.19	1.5
Rx . . . . .	0.21 sec	0.23 sec	0.04 sec
Ry . . . . .	0.36	0.39	.04
Rz . . . . .	-0.47	-0.47	.04
Bs . . . . .	-0.75 ppm	-0.699 ppm	.07 ppm

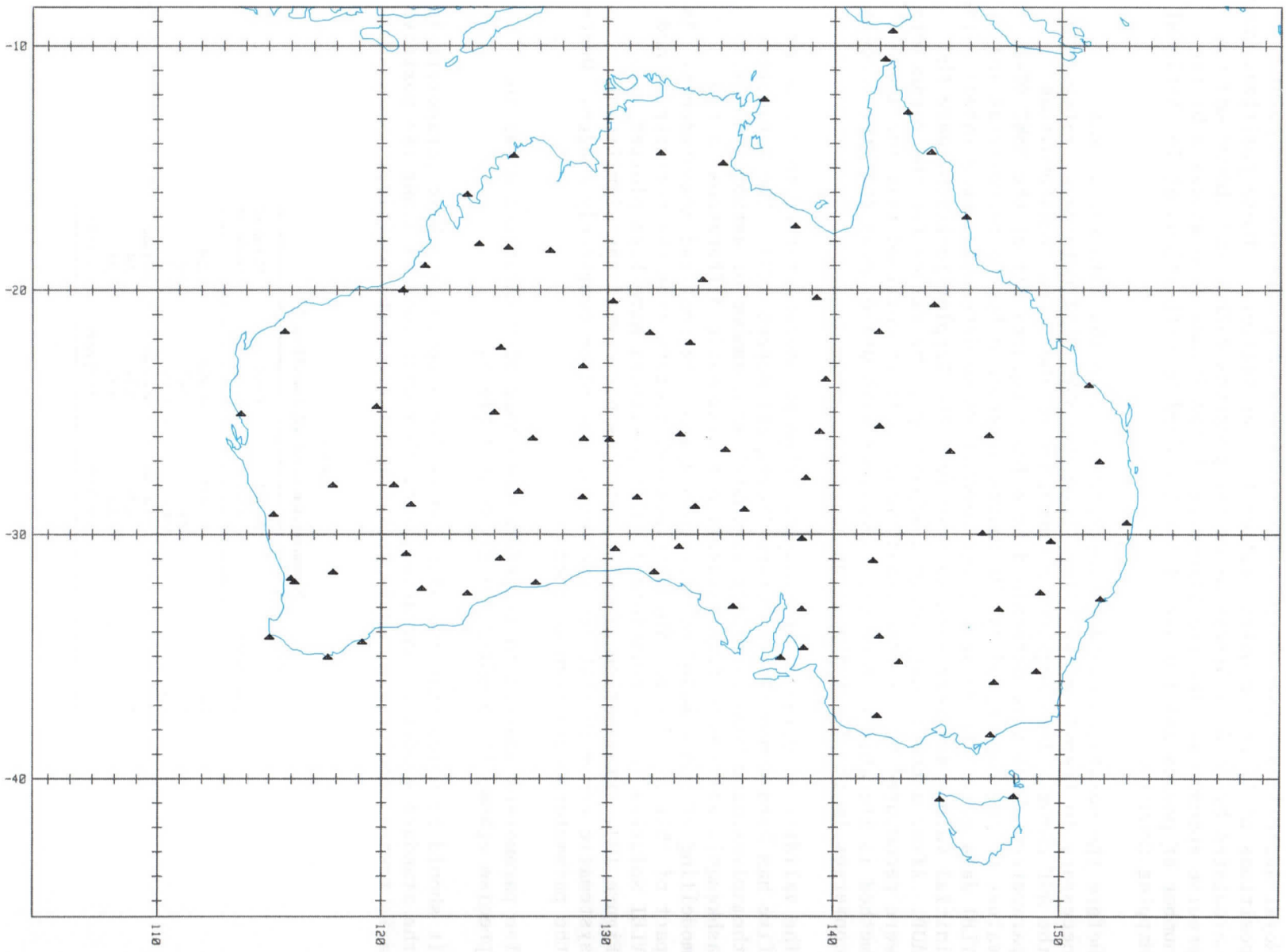


Figure 9. Doppler Point Positions

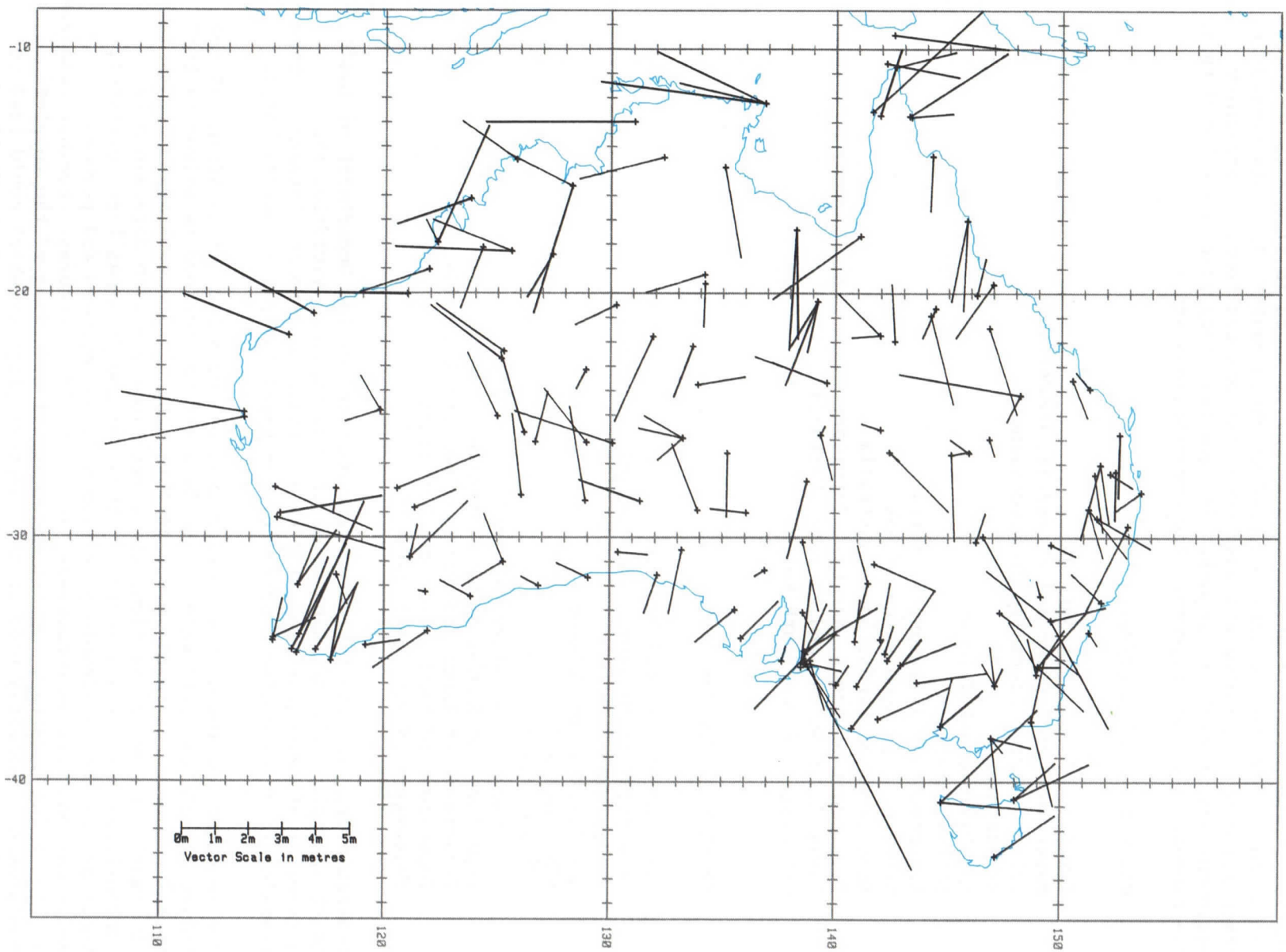


Figure 10. Displacement Vectors . . . Terrestrial + VLBI to Point Positions

## 6. MULTI-STATION SATELLITE DOPPLER POSITIONING

To strengthen the relative position of stations across the continent, a number of figures were observed in which up to thirteen MAGNAVOX 1502 Doppler satellite receivers were simultaneously deployed at primary stations. The distribution of the stations is shown in Figure 11 in which the colours show the stations occupied simultaneously. Two main periods of observation were employed and many organisations generously contributed equipment and/or field personnel. In each case they also contributed their own expenses for the projects. The organisations were:

### PHASE A February/March 1982

Division of National Mapping  
 Department of Mapping and Surveying, Queensland  
 Private Survey Companies in Queensland  
 Mt Isa Mines  
 NSW Division of Telecom  
 ESSO Australia  
 Department of Aviation, Victoria  
 Western Lands Commission of NSW  
 Department of Lands, South Australia  
 Department of Lands, Northern Territory  
 Division of Survey and Mapping, Victoria  
 University of New South Wales

### PHASE B May/June 1982

Division of National Mapping  
 Department of Mapping and Surveying, Queensland  
 Mt Isa Mines  
 NSW Division of Telecom  
 ESSO Australia  
 BHP Australia  
 Department of Aviation, Victoria  
 Department of Lands, South Australia  
 Department of Lands and Surveys, Western Australia  
 Department of Lands, Northern Territory  
 University of New South Wales

Subsequently, additional figures were observed by the Department of Lands, South Australia, the Division of Survey and Mapping, Victoria, the University of Melbourne, and also by the Lands Department, Tasmania. Three translocations were also observed by the Royal Australian Survey Corps.

The reduction of the data was undertaken by the School of Surveying at the University of New South Wales. The data set so obtained is unique in that normal economic restraints preclude obtaining simultaneous measurements at a substantial number of stations whose positions are also determined by conventional terrestrial surveys. When the analysis has been completed, knowledge of the obtainable precision will be improved and perhaps modifications will be recommended to the field procedures. Further research is also being undertaken on the mathematical modelling of the Doppler measurements currently used in the computer programs which could lead to improved precision in the computed relative positions of the Stations.



Figure 1. Multi-Station Doppler Figures

The relative positions for each station within a figure were calculated using the programs PREDOP and GEODOP (Shi) and the broadcast ephemeris. The preliminary results for the figures observed during February 1982 indicated that there were some external systematic errors present. The field sheets for the stations observed during this period also indicated that conditions were unusual. Further investigation revealed that during the period there was unusually high sun spot activity and the resulting solar radiation was affecting the precision of the broadcast ephemeris. This was confirmed by the particularly large corrections to the "along track", "across track" and "out of plane" parameters for each satellite orbit. Fortunately for this period and for most of the later multi-station figures, the precise ephemeris was available and hence the data was reprocessed using the precise ephemeris. The resulting values and the full variance/covariance matrices were incorporated into the Stage 2 data set so that the final coordinates for the junction stations could be found.

The displacement vectors representing the differences in coordinates between GMA82 and those computed in the individual figures are given in Figure 12. The initial reaction to Figure 12 is that the multi-station Doppler will not make any contribution at all to the strength of a geodetic network. However, it must be remembered that the strength of a multi-station Doppler lies in the "relative" position rather than the "absolute". Thus, the differences in the vectors within each figure are the best indication of the high quality of the multi-station Doppler measurements.

Although there has been insufficient time for a conclusive investigation, the preliminary results indicate that for geodetic application and using the current generation of computer software, the precise ephemeris should always be used in computing relative positions.

As both the precise and broadcast ephemerides were available for the 53 stations occupied for periods of 5 to 15 days during the observation of the multi-station figures, this data was used to calculate the point positions and hence the seven parameters to convert "broadcast" into "precise" coordinates. These parameters were then compared with those found by Jenkins and the results are given in Table 7.

Recently there have been a number of suggestions (Burford, and others) that the transformation parameters should be derived from local measurements. Whilst this looks nice from a mathematical point of view, consideration of the effects of the observational errors involved in a small local sample should suffice to convince one that the statistics of a larger sample spread uniformly across Australia will yield a sounder based set of parameters. This is even more important when the broadcast ephemeris is used since the effects of the time dependent biases will be minimised. When locally determined parameters are extrapolated over a period of time the results of the transformation must be inferior. The above remarks do not apply to the case where a minimum of at least three known stations are occupied simultaneously with the new stations. Such an approach would minimise the effects of the biases but is not a cost-effective method of determining coordinates.

Figure 12. Displacement Vectors . . . Terrestrial + VLBI to Multi-station Doppler Positions

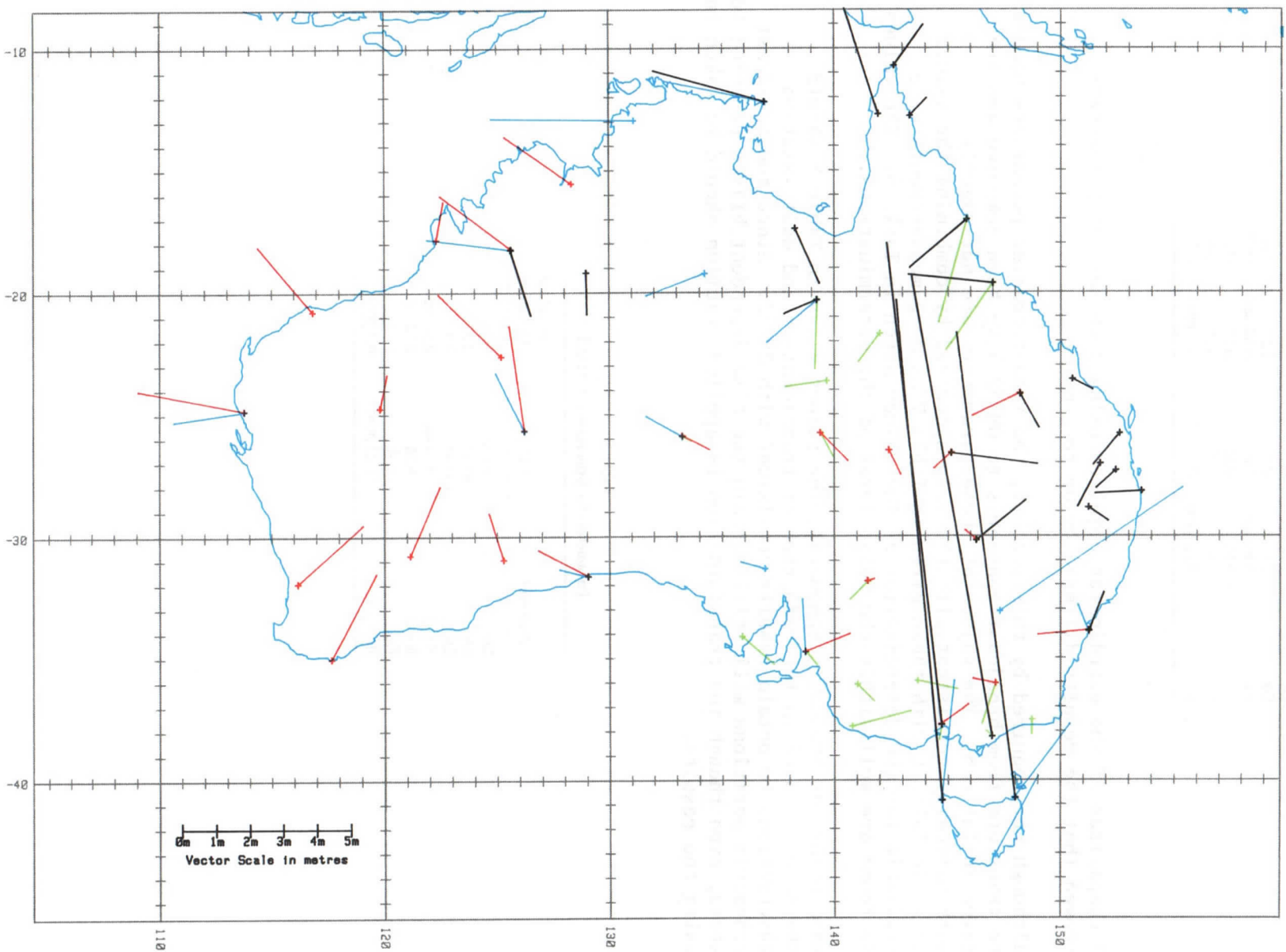


Table 7.  
Parameters for Broadcast to Precise Positions

<i>Parameter</i>	<i>Jenkins 1979</i>	<i>Allman 1983</i>
Dx . . . .	-0.8m ± 0.5	-6.5m ± 1.2m
Dy . . . .	0.2 0.5	-1.3 1.2
Dz . . . .	-2.6 0.5	-1.5 1.4
Rx . . . .	0.05 sec 0.02	-0.04 sec 0.03
Ry . . . .	-0.02 0.02	0.12 0.04
Rz . . . .	0.01 0.02	0.15 0.04
Bs . . . .	0.22 ppm 0.2	-0.4 ppm 0.1

A comparison of the coordinates obtained using both sets of parameters showed that the results differed by up to 3 metres.

Although not required by this project, the transformation parameters needed to transform from broadcast ephemeris to GMA82 have been computed and are given in Table 8. The values of these parameters are not exactly mathematically consistent with the values derived by combining the values given in Table 6 with those given in Table 7 as more measurements were available for the determination of the values given in Table 8. The values so found are well within the noise level of the determinations.

When using the broadcast ephemeris, the values given in Table 8 should therefore be used in the reduction of translocation and multi-station observations to obtain results consistent with GMA82. Since the broadcast ephemeris positions will still contain the time dependent biases referred to above, even though the transformation is applied, caution should be taken in using the results.

Table 8.  
Parameters for Broadcast to GMA82

<i>Parameter</i>	<i>Value</i>	<i>Standard Deviation</i>
Dx . . . . .	108.65m	3.5m
Dy . . . . .	49.43	3.2
Dz . . . . .	-137.49	3.9
Rx . . . . .	0.24 sec	0.11 sec
Ry . . . . .	0.54	0.12
Rz . . . . .	-0.31	0.12
Bs . . . . .	-1.22 ppm	0.3 ppm

## 7. STATISTICAL TESTING AND EVALUATION

In order to evaluate an adjustment, many investigations must be carried out. The time constraints set by the necessity of tabling this report have precluded a number of the investigations being completed in time for inclusion in the report. Sufficient tests however have been carried out to indicate the quality of the adjustment. These tests are described below.

It must be stressed that the tests which involve station position have been carried out using only the junction stations from Stage 2. For stations which are not junction stations, the ellipse values will be larger and may be approximated by compounding the values of the station ellipse taken from the relevant block with the ellipse values for the closest junction stations from Stage 2. This is particularly relevant for stations in those blocks which are primarily combinations of traverses.

### 7.1 A Posteriori Variance Factors

To test the quality of the mathematical modelling used in an adjustment, it is usual to carry out a variance ratio test for the whole model. For larger adjustments, discrete sections of the data set may also be examined using the variance ratio test. For GMA82, where the data set has been partitioned already into blocks such an approach has much to recommend it. The values of the a posteriori variance factors and the degrees of freedom for the various sections of data are given in Table 9.

Since the individual blocks have quite different degrees of freedom, they cannot be compared directly. Instead, the comparison is carried out by using the F-test factor which is defined as the variance ratio divided by the appropriate value for the degrees of freedom determined for the F-distribution. The values so found for each block of data are given as a histogram in Table 10. Statistically the values should be less than unity.

In the histogram, the black squares refer to the contributions of the terrestrial observations within the nominated block. The green squares refer to the contributions of the point position Doppler values (PT), the individual multi-station Doppler figures (EI, etc) and the total multi-station data set (MUL). The blue squares refer to the full terrestrial data set (FUL) and to the complete data set (TOT).

The histogram indicates that the hypothesis regarding the modelling of the observation variances is generally acceptable although there are indications that the variances in some of the smaller blocks were conservative. In a few other cases, it is apparent that the variances were optimistic.

### 7.2 Error Ellipses for the Junction Stations

As part of the Stage 2 output, point error ellipses were calculated for the junction stations. The numerical values for these ellipses were calculated relative to the Johnston Origin. By inspection the mean value is found to be 29 cm and that the median value is 25 cm.

Although some information may be gleaned by inspection of the numerical values, it is very difficult to get an overall impression of the trends in precision. To overcome this, the ellipses for each station were plotted in Figure 13 so that a visual impression could be gained.

Table 9.  
Data Set, a Posteriori Variance Factor, Degrees of Freedom

<i>Block</i>			<i>Terrestrial data only</i>			<i>Block</i>		
<i>Block</i>			<i>Block</i>			<i>Block</i>		
01	0.444	501	05C	0.415	242	13A	1.012	729
02	0.972	808	05D	0.532	152	13B	1.100	2 282
02A	0.838	241	05E	0.637	150	14	0.884	1 278
02B	0.583	376	05F	0.899	77	14A	0.588	266
02C	0.415	242	05G	0.827	269	14B	0.657	236
02D	0.532	150	06	0.959	228	15	1.257	74
02E	1.105	45	07	1.316	281	16	1.223	451
03	1.180	189	08	0.630	410	17	0.936	261
04	0.800	146	09	0.761	2 970	18	0.797	698
05	1.109	597	10	0.643	775	19	0.689	1 716
05A	0.455	600	11	0.977	509	20	0.731	2 456
05B	0.583	86						

<i>Other Data Sets</i>		
VLBI and SLR	0.447	11
Full Terrestrial Data Set	0.824	21 878
Point Position Doppler	0.910	266
Full Multi-station Doppler Data Set	0.945	176
Figure E1	0.661	18
Figure E2	0.520	16
Figure E3	0.636	24
Figure E4	1.360	20
Figure E5	0.512	8
Figure E6	1.078	22
Figure E7	3.235	12
Figure E8	0.478	4
Figure W1	0.446	22
Figure W2	0.923	26
Figure W3	0.336	4
Total Data Set	0.826	22 331

Examination of the plot reveals some interesting features. In previous adjustments of networks around the world, the size of the ellipse increases as a function of the distance from the origin. In GMA82 this is not so. Comparison of Figures 8, 11 and 13 gives the reason for the dramatic change in the ellipse pattern in that the SLR, VLBI and particularly the multi-station Doppler have substantially strengthened the terrestrial network. In areas where the terrestrial network is dense and includes a number of multi-station Doppler points, the cross bracing is obvious. When this is coupled with the further strengthening of the SLR and VLBI measurements (e.g. around Sydney) the end result is very tight ellipses.

There are two very large ellipses in the plot. These are the ellipses at Townsville and Hiran 30. The explanations are quite straight forward in both cases. Townsville is the Royal Australian Survey Corps station referred to in Section 6 and was "fixed" by a simple translocation and a point position. Hiran 30 is located in the north-west corner of Queensland and the only observations to the station are Hiran measurements. It should therefore represent the weakest station in the entire network and the error ellipse components can be taken as indicative of the largest probable errors in the network.

In an attempt to give a broader picture, the distribution of the length of the semi-major axes is given in Table 11. This does not take into account the geographic distribution of the stations and so the length of the semi-major axis has been expressed in terms of parts per million of the distance of the station from the Johnston Origin. This information is also given in Table 11.

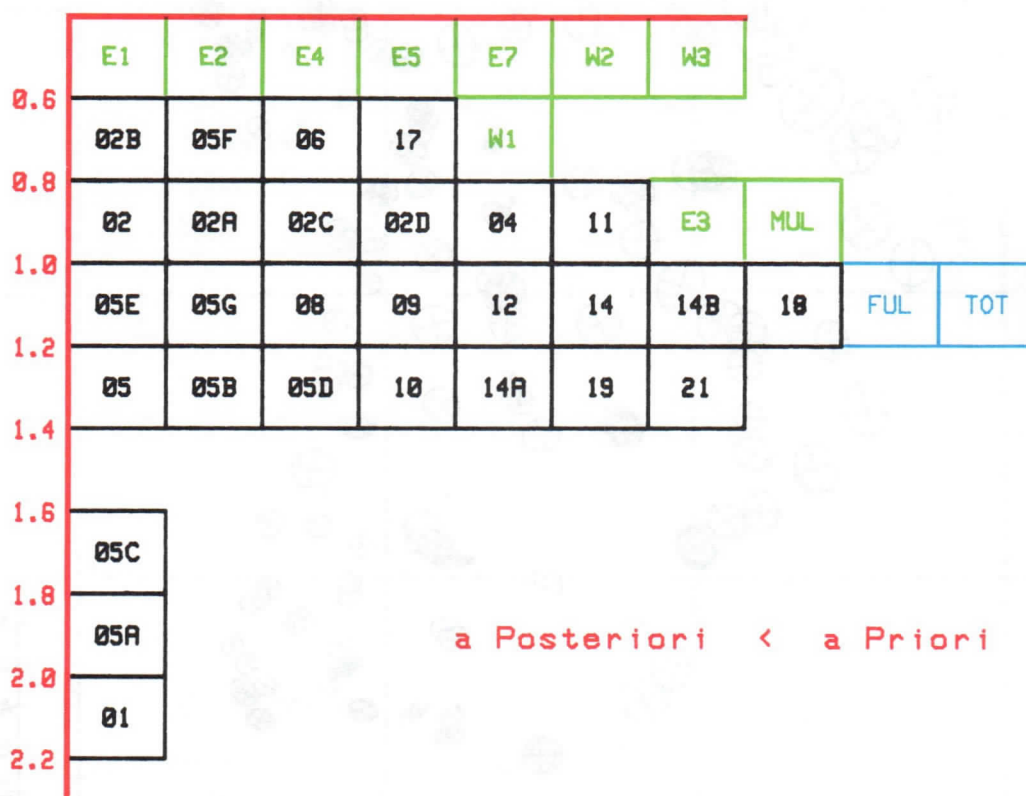
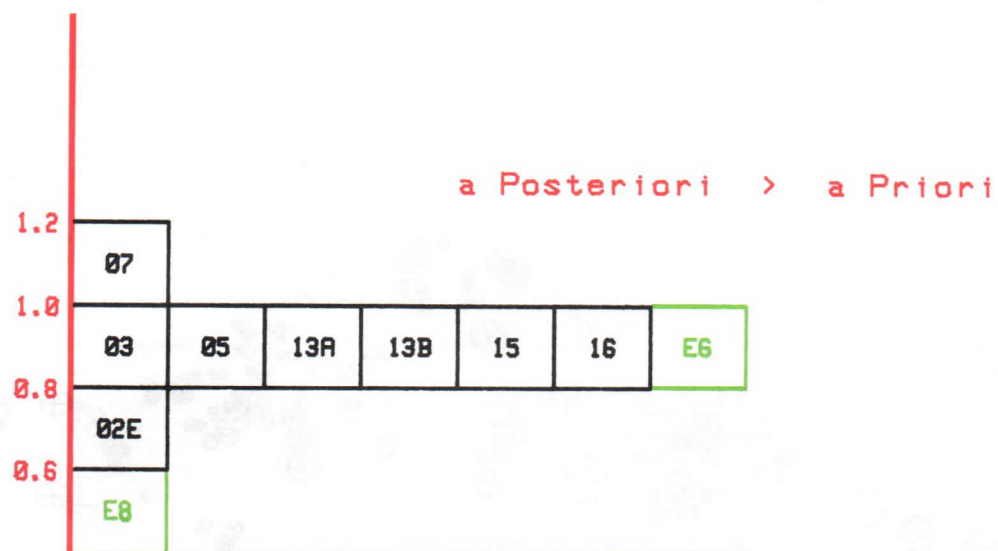


Table 10. F Test Factor

Figure 13. Error Ellipses for GMA82 Junction Stations

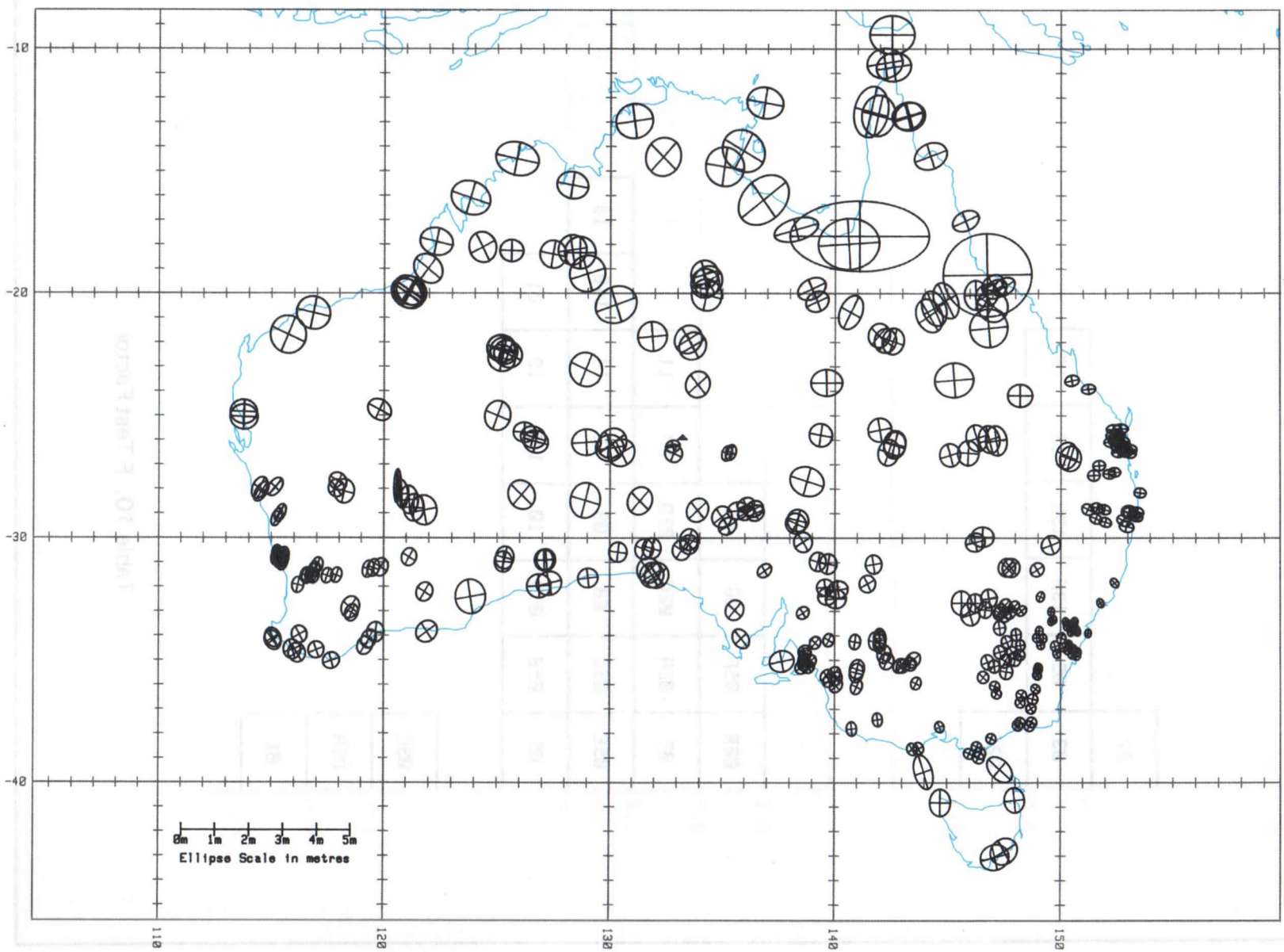


Table 11.  
Analysis of the Error Ellipses

<i>Semi-major Axis</i>	<i>Occurrences</i>	<i>ppm</i>	<i>Occurrences</i>
10 cm	1	0-0.1	114
20	116	0.2	45
30	109	0.3	45
40	58	0.4	25
50	49	0.5	17
60	17	0.6	8
70	5	0.7	4
80	1	0.8	2
90	1	0.9	7
100	1	1.0	3
100	3	1.1	1
		1.2	1
		1.3	0
		1.4	4
		1.4	3

### 7.3 Displacement Vectors GMA82/AGD66

In order to compare the coordinates of the GMA82 junction stations with the corresponding values in the AGD66 data set, displacement vectors were calculated from the GMA82 values to those in AGD66. The vectors are plotted in Figure 14 and an inspection of the vectors clearly shows the existence of regional shifts. However if the regional shifts were applied in a small area, there would remain some large resultant vectors. This would occur in areas where the plotted vectors are not parallel or of the same length. Where vectors cross or are noticeably different in length, then the resultant vector indicates the difference that would be found in both the length and azimuth of the computed chords between two stations as calculated from each adjustment. Since GMA82 contains a greater quantity and greater variety of data, it is probable that the GMA82 coordinates would be much sounder. The vector differences may thus be taken as the "distortions" in the AGD66. These differences in vectors within a small region reflect the difficulties currently encountered in fitting new surveys to the existing network.

It is appreciated that such an analysis could be carried out for all the stations common to each data set but it seems unnecessary as the sample clearly indicates the differences across the whole of Australia.

### 7.4 Displacement Vectors GMA82/GMA80

The displacement vectors from GMA82 to GMA80 are plotted in Figure 15. Inspection of the plot shows that there are regional shifts which are generally consistent with the overall bracing caused by the introduction of the extra-terrestrial measurements. The magnitude of the vectors is generally less than 1 ppm of the distance from the Johnston Origin.

### 7.5 Displacement Vectors GMA82/ Doppler Point Positions

To confirm that there were no systematic biases remaining in the combination of Doppler point position data with the terrestrial data, the displacement vectors between the GMA82 values and the observed Doppler positions were plotted in Figure 16. The obvious random nature of the vectors clearly shows that there is no significant systematic bias. The magnitude of the vectors is also consistent with the specified point position variances.

## 7.6 Displacement Vectors GMA82/Multi-station Doppler

The multi-station Doppler figures give the relative positions of the stations within the figure. In order to examine the residuals, it is first necessary to apply the block shift to each figure as computed in Stage 2. The displacement vectors may then be computed. This has been done and the results are plotted in Figure 17. In the Figure, the colour of the vector indicates the relevant multi-station figure (see also Figure 11).

It is immediately apparent that the vectors are at least one order of magnitude smaller than the corresponding vectors for the point position values. Although the figures covered much greater areas than currently recommended, the mean vector is only 59 cm. This amount represents less than 1 part per million across each figure.

Comparison of the error ellipses (Figure 13) with the location of the multi-station Doppler positions (Figure 11) clearly indicates the overall strengthening effect on the network of the multi-station Doppler.

Figure 14. Displacement Vectors . . . GMA82 to AGD66

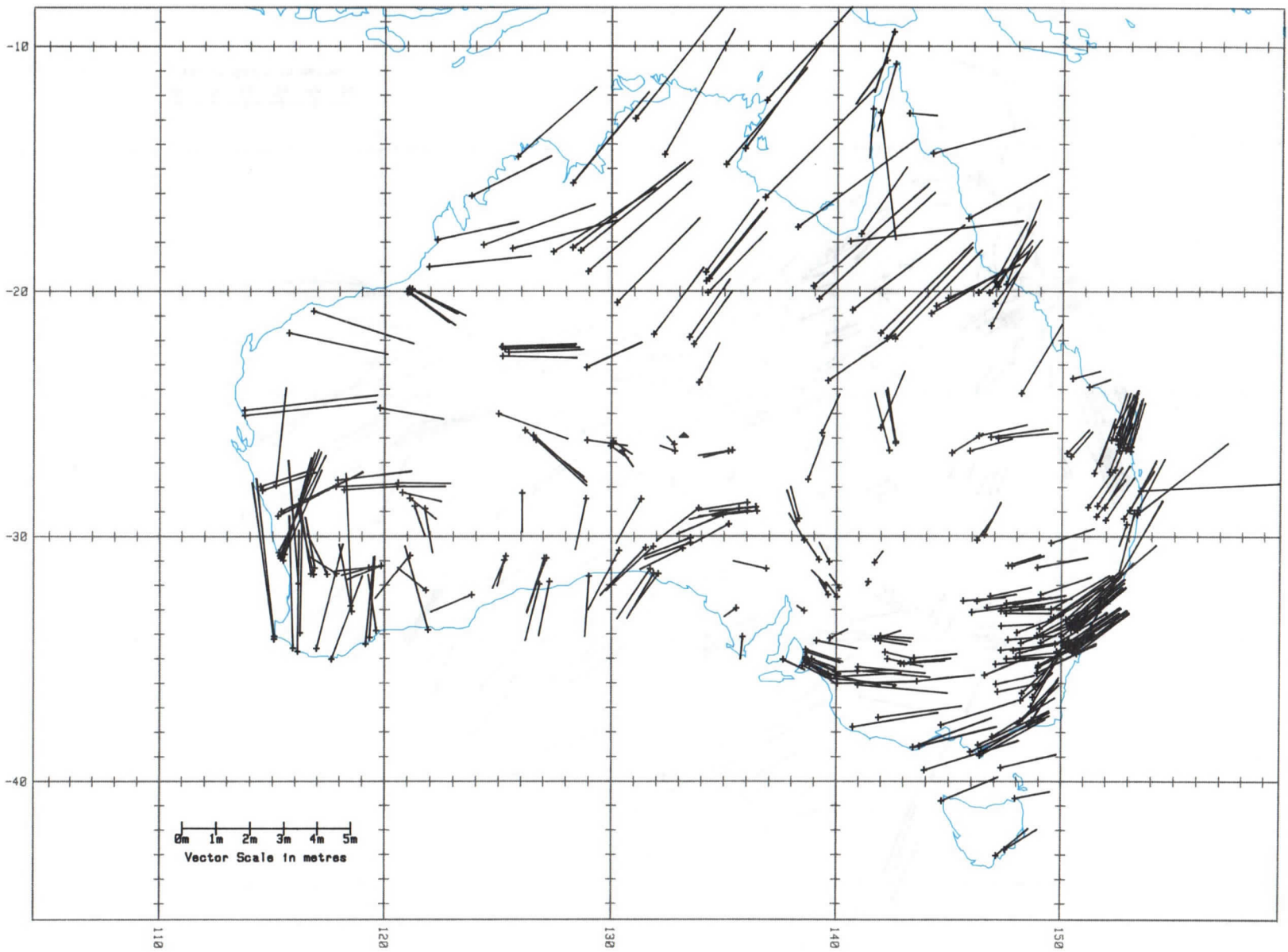


Figure 15. Displacement Vectors . . . GMA82 to GMA80

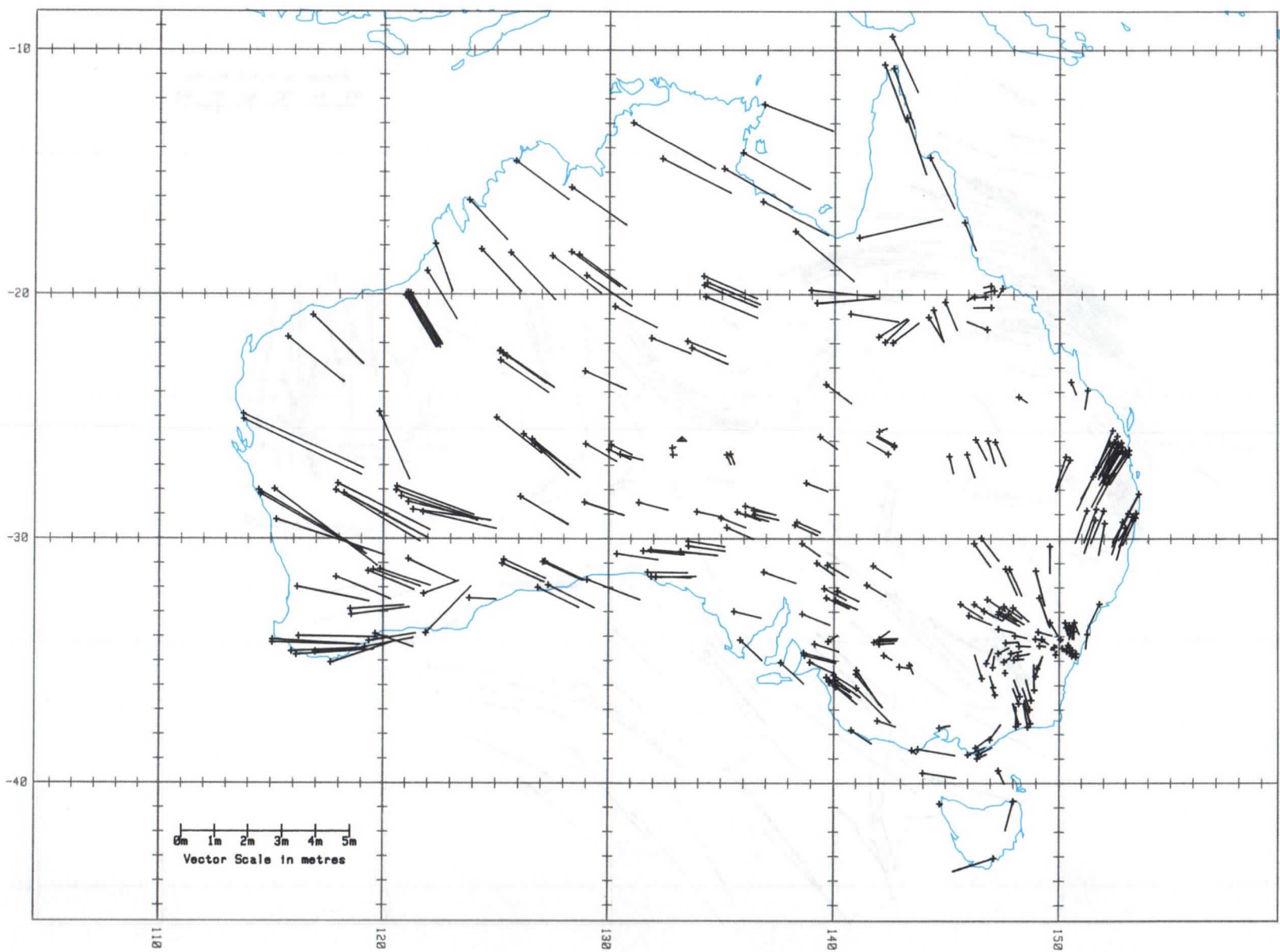


Figure 16. Displacement Vectors . . . GMA82 to Point Positions

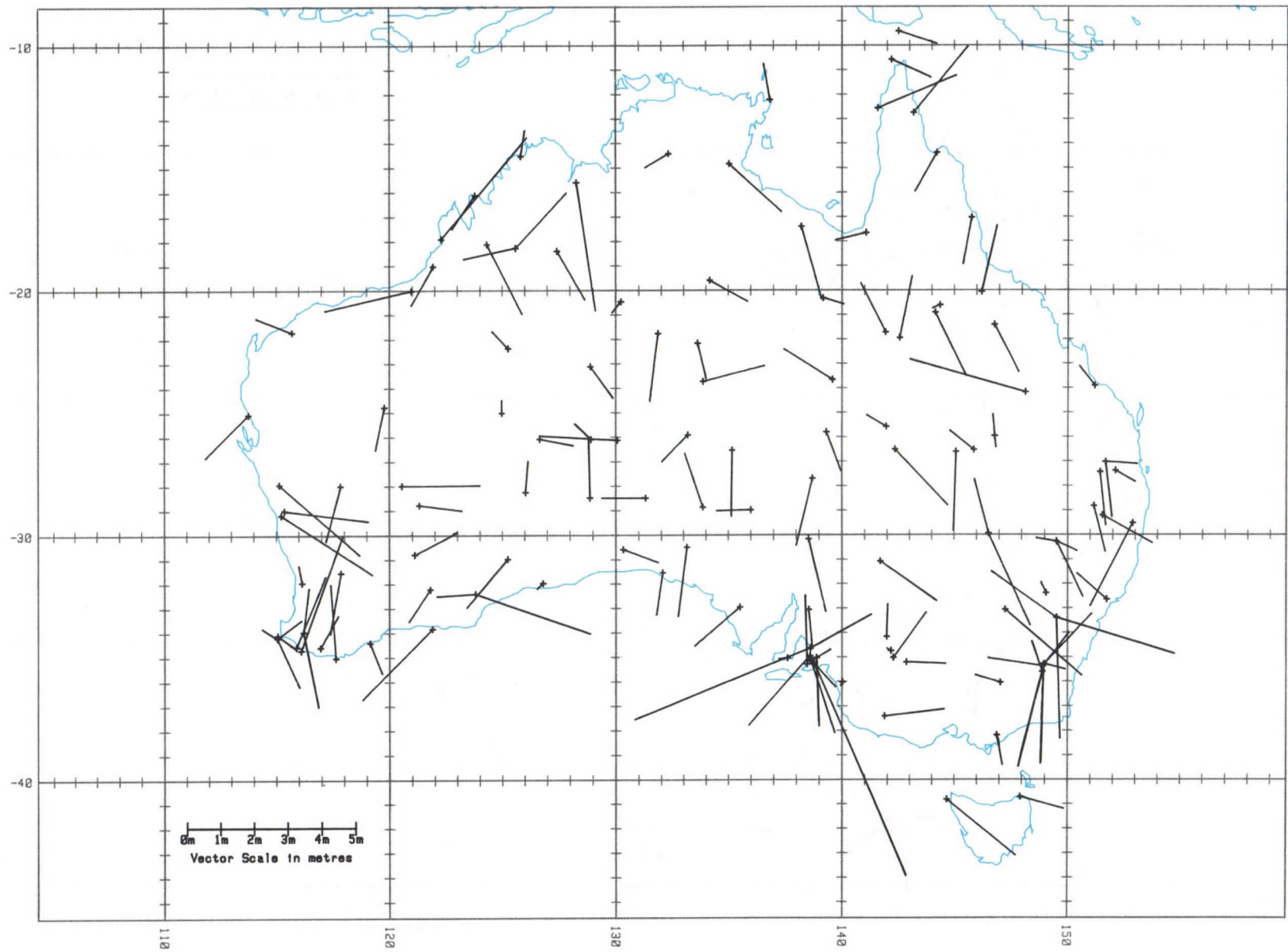
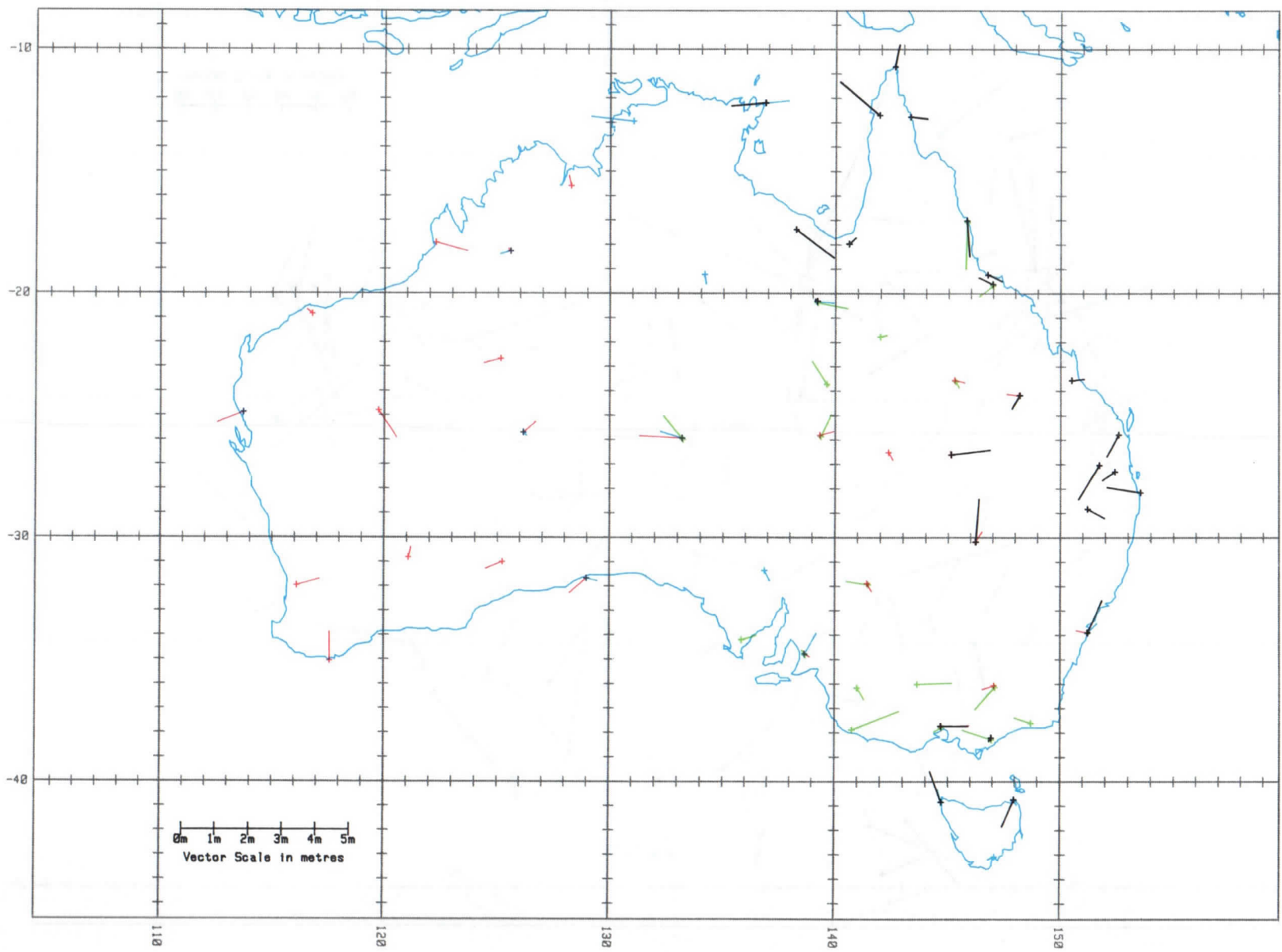


Figure 17. Displacement Vectors . . . . GMA82 to Multi-station Doppler



## 8. EFFECT OF THE VARIOUS DATA TYPES

In this section, the interaction of the dissimilar types of measurement is examined by comparing a number of Stage 2 type adjustments. For these adjustments, the data sets contained only selected types of observation or combinations of them. The results are evaluated by examining the displacement vectors formed by the coordinate shifts between the various adjustments.

### 8.1 The Effect of Adding the Point Positions to the Terrestrial Data

The transformed point positions were added to the terrestrial Stage 2 data and the combined data set was adjusted. The resulting coordinates were compared with those from the pure terrestrial adjustment and the vectors plotted in Figure 18.

Inspection of the plot shows that there are systematic block shifts in many regions of the continent. This is not surprising when it is remembered that the network consists mainly of dense networks around the major urban with weaker connections to the adjacent urban areas. This is particularly so in the northern and western parts of the continent where the connections are made through interlocking Tellurometer traverses. The addition of the point position Doppler measurements must strengthen these traverses and give more probable values for the coordinates of the stations. The magnitude of the vectors indicate that the traverses were in fact reasonably tight in that the displacement vectors are generally less than one part per million.

### 8.2 The Effect of Adding the Multi-station Doppler to the Point Position and the Terrestrial Data

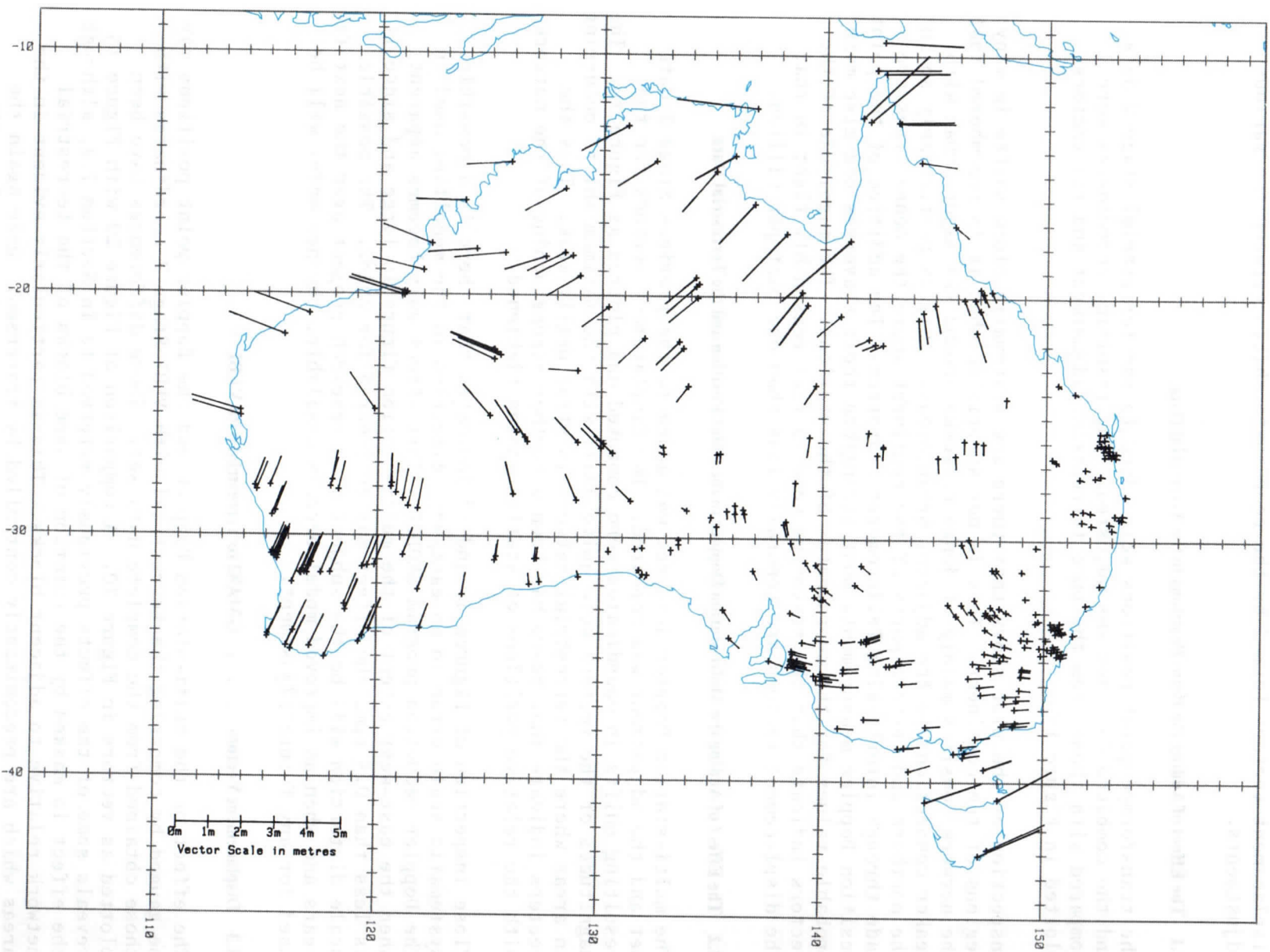
The multi-station Doppler data set was added to the previous Stage 2 data set and the adjustment was repeated. The displacement vectors for the resulting shifts in coordinates were computed and plotted as Figure 19. The magnitudes of the vectors were quite small with the maximum shifts occurring in areas where the terrestrial network is structurally weak. Thus the vectors indicate that there has been a further strengthening of the network with the relative positions of stations being tightened.

Close inspection of Figures 19 and 12 indicates that there is a possible systematic scale error in an east/west direction in the modelling used in the Doppler reduction program GEODOP. This effect only becomes apparent when the east-west extent of the multi-station figure is large and since it is less than 0.5 ppm, the effect may be ignored for GMA82. The possible scale distortion will be the subject of a research project over the next few years and when an improved model becomes available, the new model will be used for any future adjustment.

### 8.3 Displacement Vectors . . . GMA82 to Terrestrial with VLBI

The effect of the multi-station Doppler and the Doppler point positions may be gauged by comparing the terrestrial-with-VLBI Stage 2 coordinates with those obtained from the complete data set. These differences have been plotted as vectors in Figure 20. A comparison of Figure 20 with Figure 15 reveals some of the effects previously referred to in Section 7.4, although the effect is masked by the rotation of some blocks of the terrestrial network relative to adjacent blocks. This is particularly evident in the areas which are predominantly controlled by traverses. Once again the magnitudes of the shifts relative to the Johnston origin are less than 1 ppm.

Figure 18. Effect of Point Position Doppler



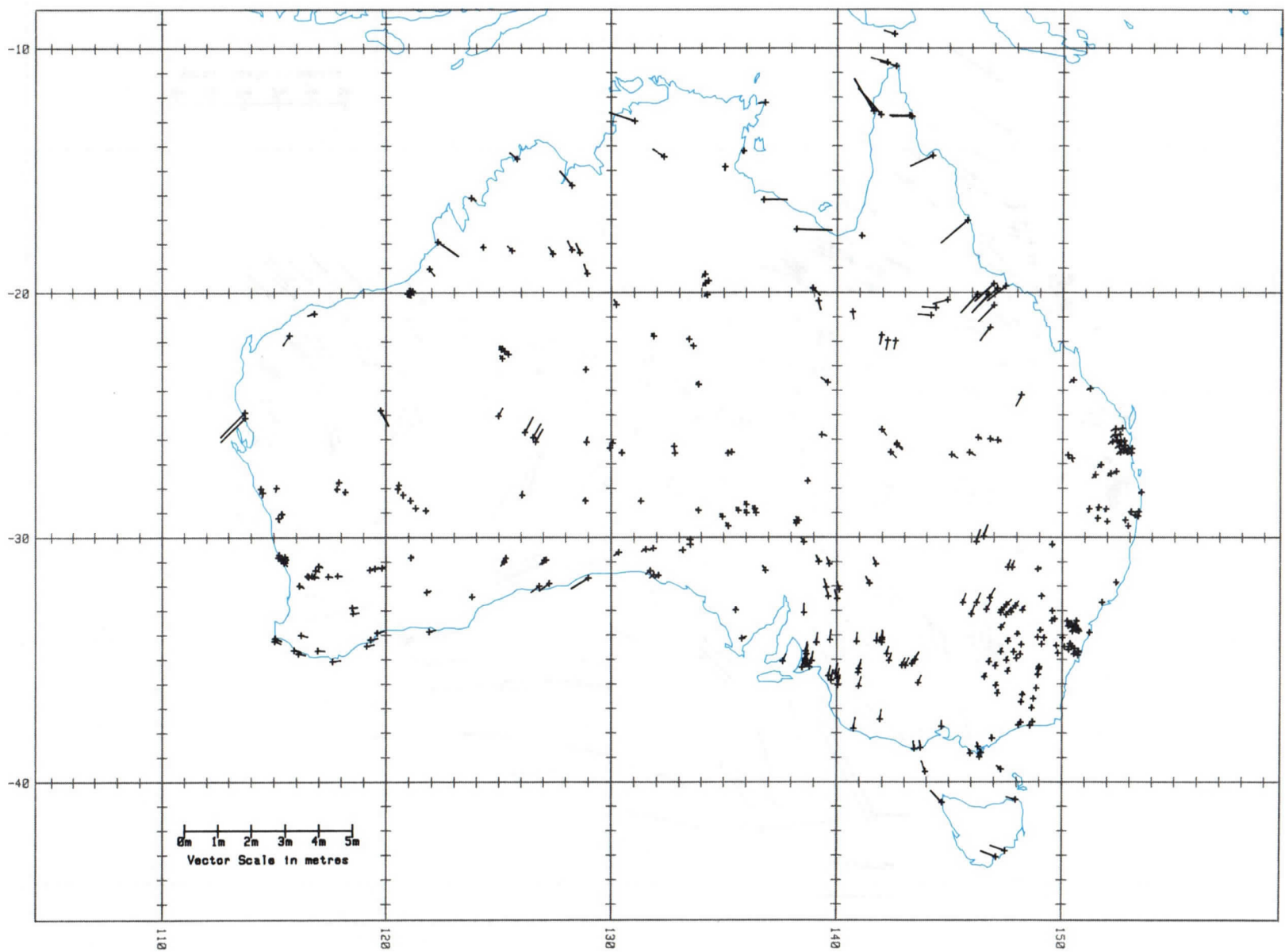
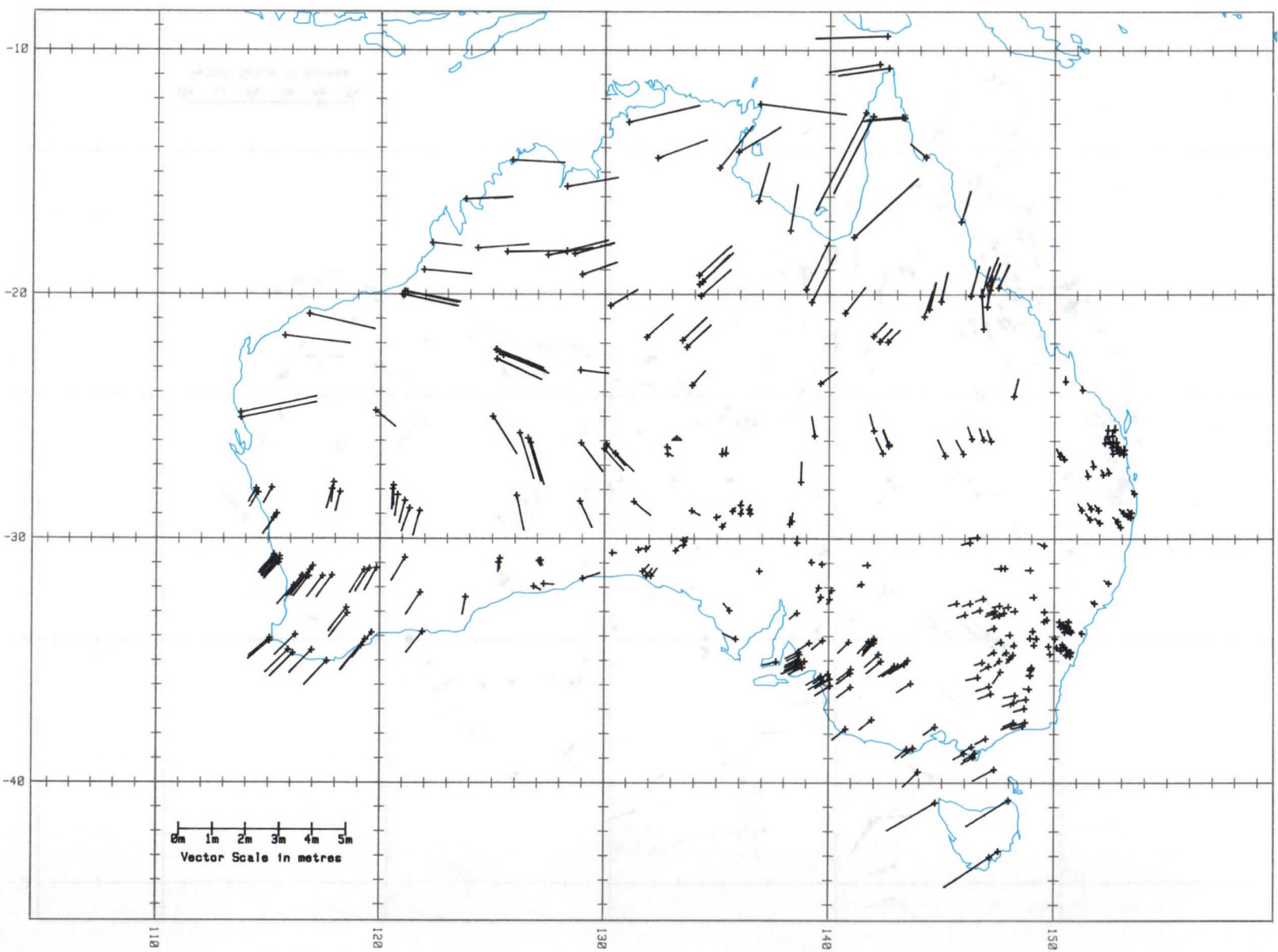


Figure 19. Effect of Multi-station Doppler

Figure 20. Displacement Vectors . . . GMA82 to Terrestrial + VLBI



#### 8.4 GMA82 without VLBI versus GMA82 with VLBI

In order to carry out this comparison, it was first necessary to readjust the data set using only the terrestrial data. These results were then used to recompute the seven parameters required to transform Doppler measurements from NSW-922 onto the AGD. This gave the values shown in Table 12 in which the final parameters are also given for comparison purposes.

Table 12.  
Transformation Parameters

<i>Parameter</i>	<i>Terrestrial Only</i>	<i>GMA82 Final</i>
Dx . . . . .	116.71m	116.00m
Dy . . . . .	50.89	50.47
Dz . . . . .	-139.73	-137.17
Rx . . . . .	0.19 sec	0.23 sec
Ry . . . . .	0.32	0.39
Rz . . . . .	-0.50	-0.47
Bs . . . . .	0.85 ppm	-0.70 ppm

The point position Doppler and the multi-station Doppler were then transformed and added to the terrestrial data set. The resulting data set was then adjusted and the displacement vectors between the model without the VLBI and SLR and the model with the VLBI and SLR included (GMA82) were computed and plotted as Figure 21.

The first impression is that including the SLR and VLBI measurements makes very little difference to the adjusted coordinates. Indeed, the maximum shift is 29 cm. The overall effect is thus a bracing of the network in the same manner that the addition of any other measurements would have a statistical bracing effect. For the small number (11) of measurements it is impossible to draw definite conclusions but there is no apparent evidence of a scale difference between the terrestrial and VLBI data.

#### 8.5 GMA82 without VLBI versus GMA80

Since the inclusion of the SLR and VLBI has very little effect other than localised bracing, the vectors for the GMA82 without SLR and VLBI versus GMA80 (Figure 22) are very similar to the vectors for the full GMA82 model versus GMA80 (Figure 15).

#### 8.6 GMA82 without VLBI versus Point Position Doppler

There are two separate data sets of point position Doppler measurements. The first comprises those measurements which were observed as point positions and have been included as such in the adjustment. The second data set comprises the point positions computed from the observations taken during the multi-station projects. Since these observations form part of the multi-station data set, the point positions have not been included in the adjustment. The second data set is included in the computation of the displacement vectors which give the difference between the transformed point positions and the model formed by GMA82 without SLR and VLBI. The vectors are plotted in Figure 23.

Figure 21. Displacement Vectors . . . GMA82 with and without VLBI

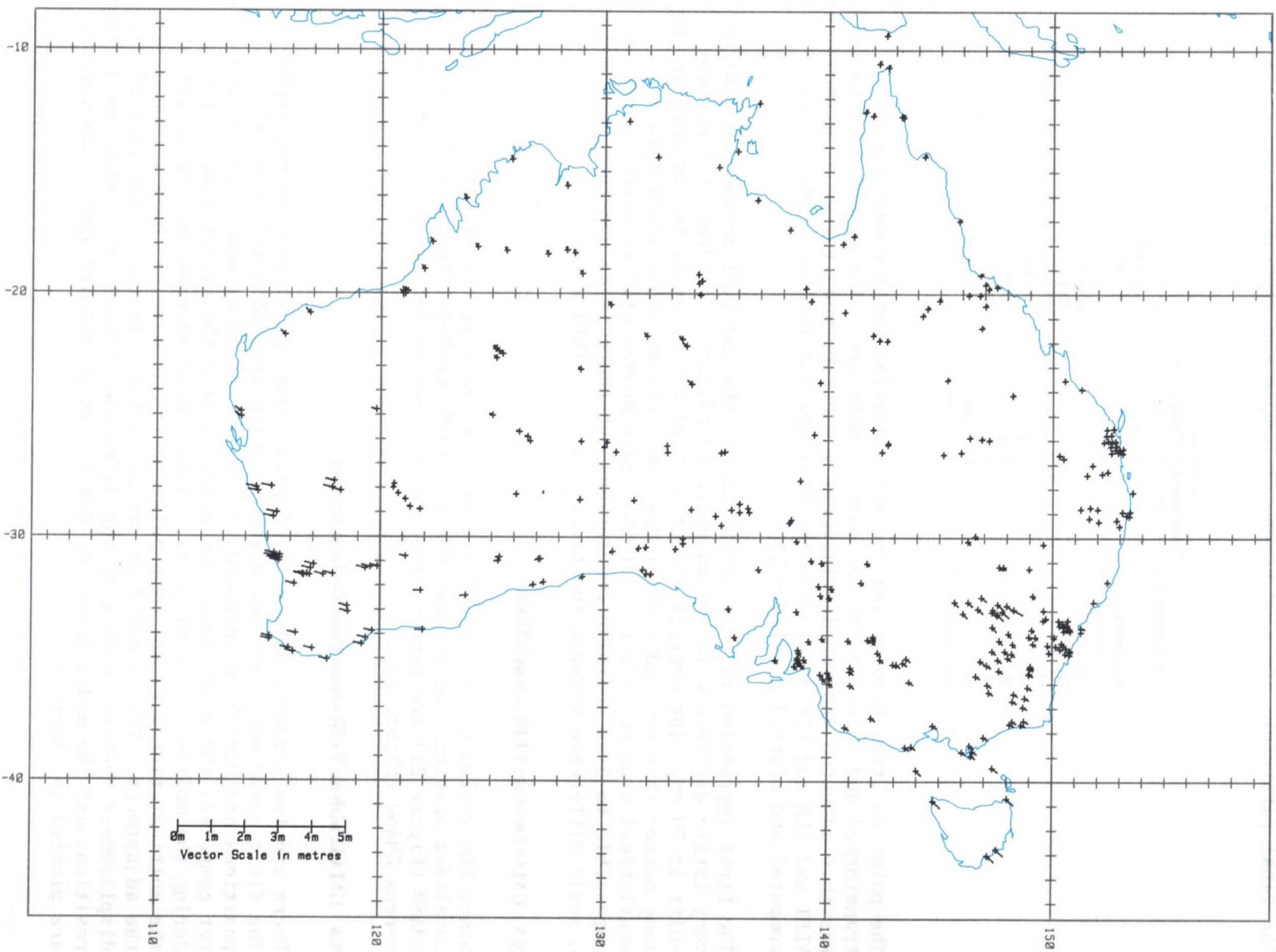


Figure 22. Displacement Vectors . . . GMA82 without VLBI to GMA80

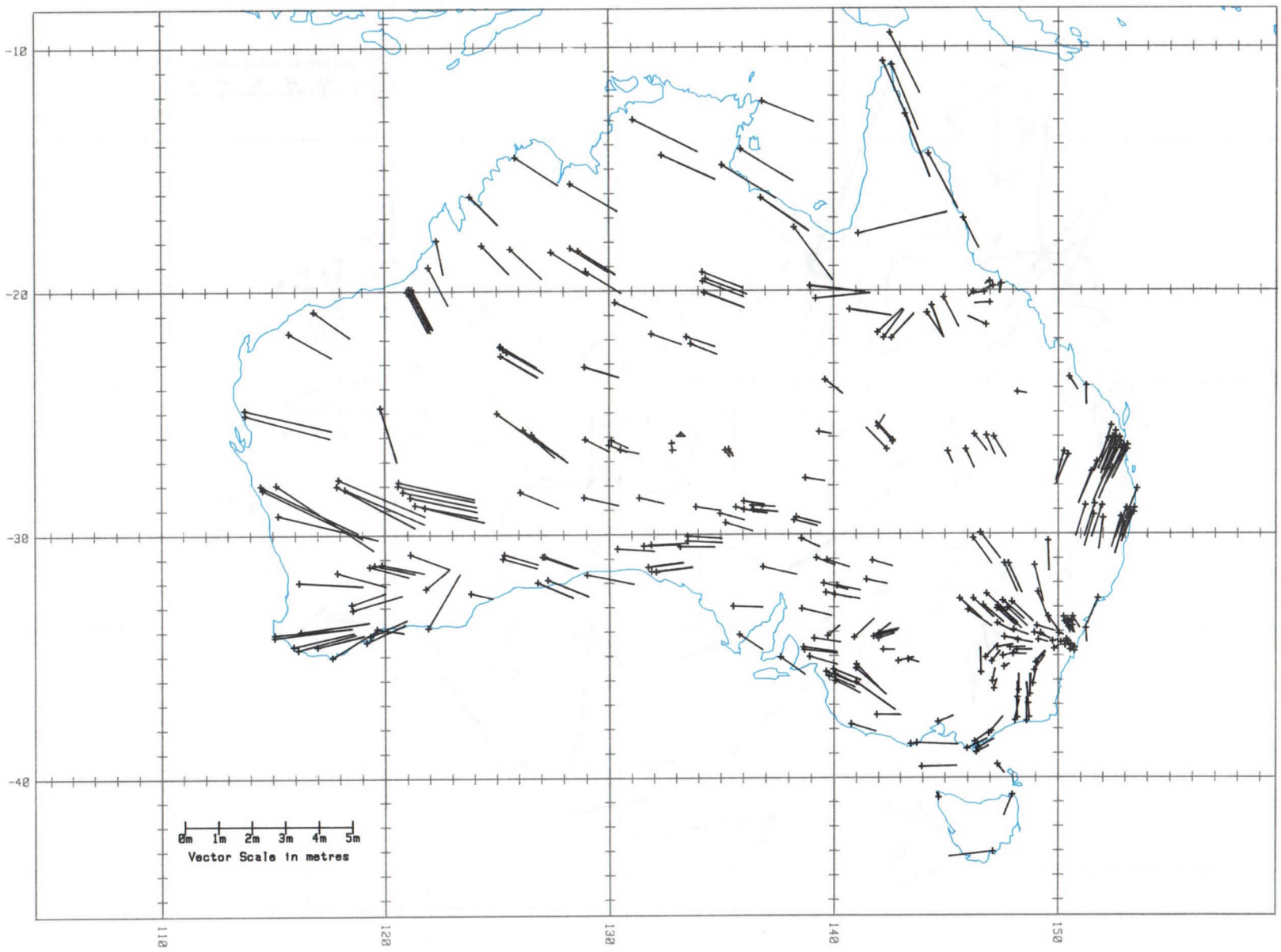


Figure 23. Displacement Vectors . . . GMA82 without VLBI to Point Positions

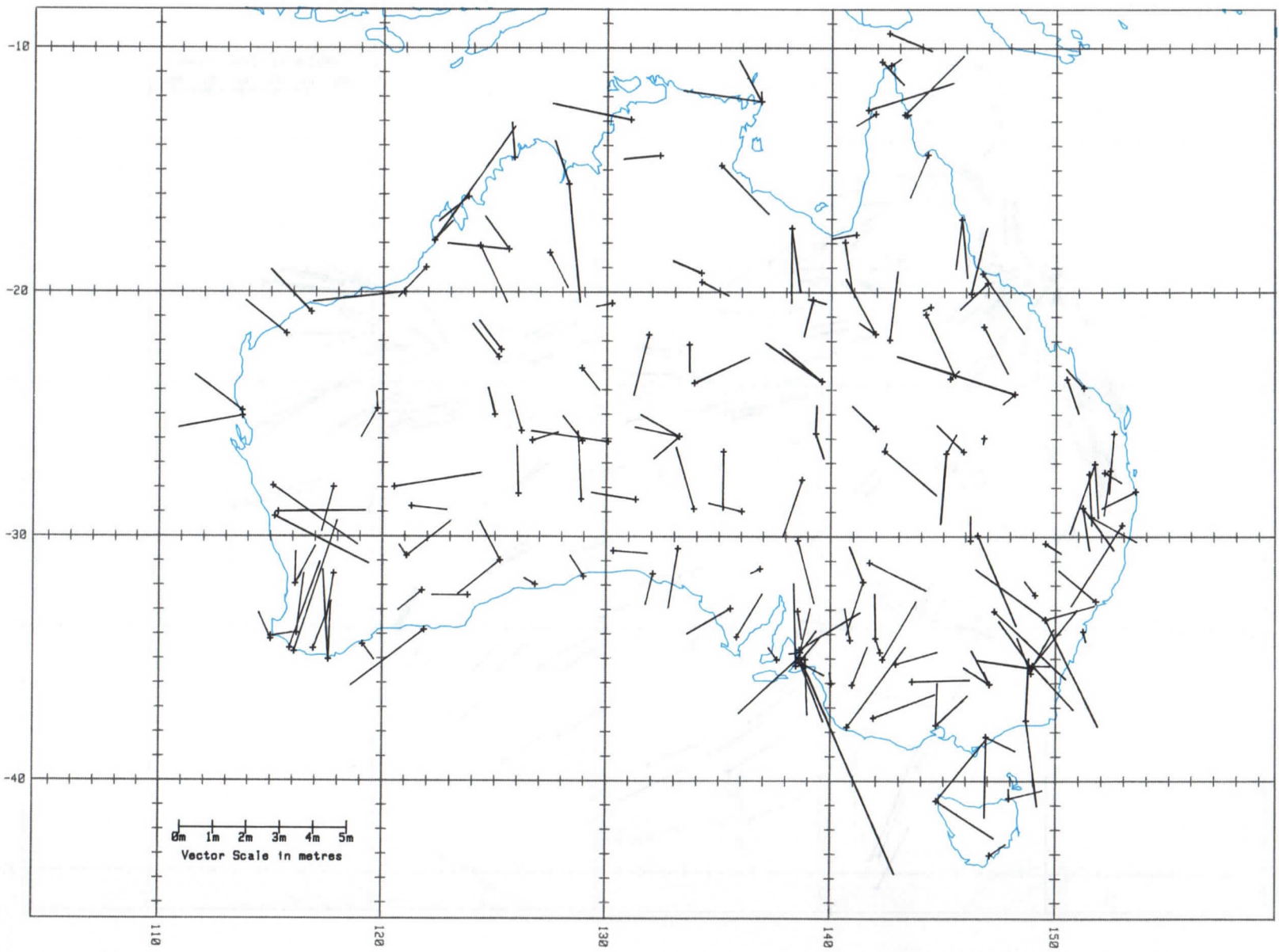
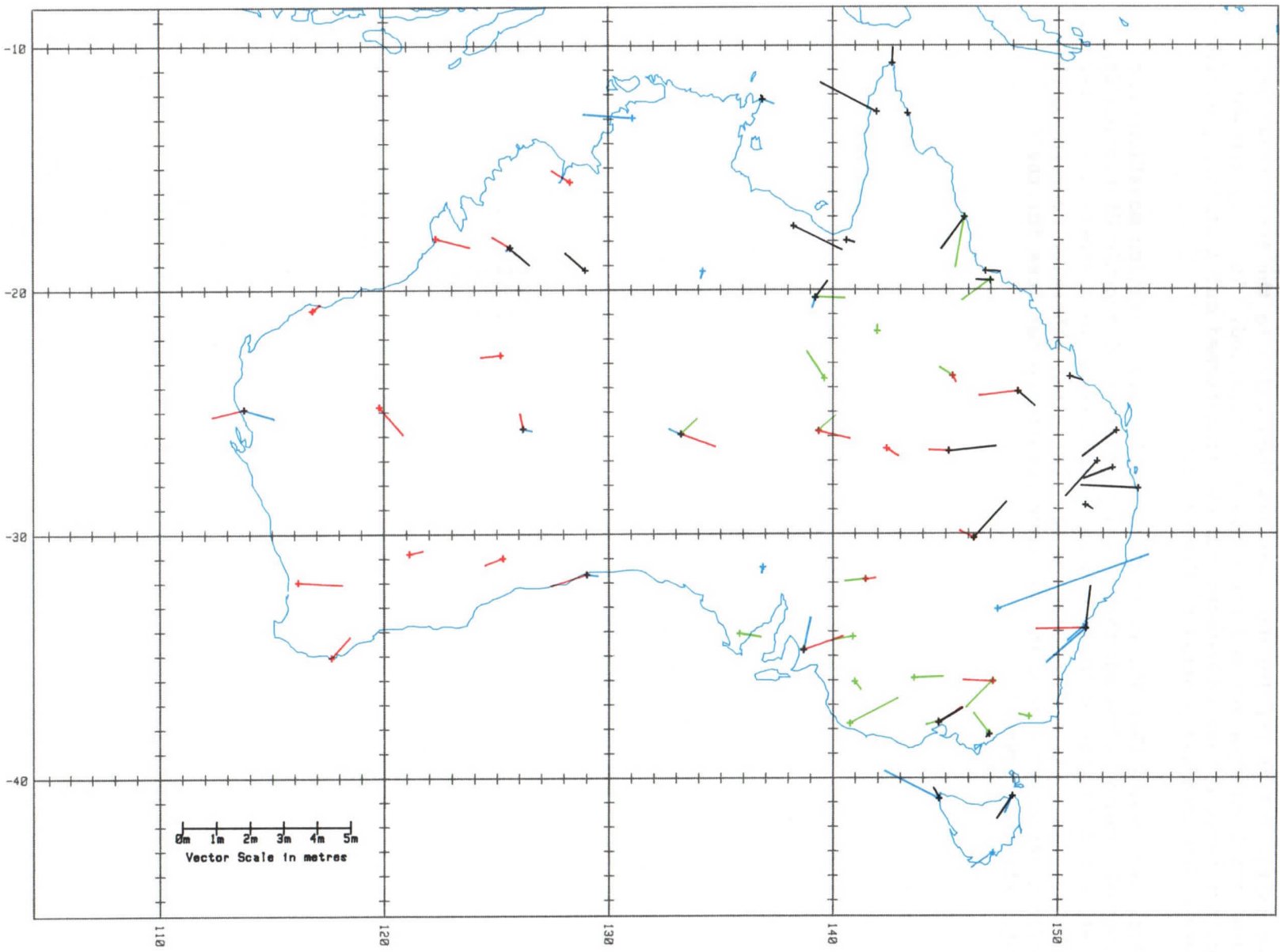


Figure 24. Displacement Vectors . . . GMA82 without VLBI to Multi-station Doppler



Inspection of the figure shows the generally random nature of the corrections to the Doppler measurements. The magnitude of the vectors is consistent with the precision of the point position measurements.

#### **8.7 GMA82 without VLBI versus Multi-station Doppler**

The multi-station Doppler data set was then added to the terrestrial and point position data set and the adjustment repeated. The displacement vectors between this adjustment and the transformed multi-station positions were calculated and plotted as Figure 24.

Inspection shows that the corrections to the multi-station positions are generally random although there is a suggestion of a scale distortion in an east/west direction of up to 0.5 ppm. This possible distortion was also discussed in Section 8.2. The vectors certainly otherwise conform to the pattern anticipated from the variance/covariance matrices for the multi-station figures.

## 9. CONCLUSION

The conclusion of this project is a significant milestone in that it is the first occasion when the diverse facets of geodetic theory and the multitude of geodetic measuring techniques have been brought together in a rigorous adjustment of a large geodetic network. The successful integration of:

Theodolite Directions	Geoid Model
Electronic Distance Measurements	Geoid/Spheroid Separations
Laplace Astronomic Observations	Deflections of the Vertical
Satellite Laser Ranging	Skew Normal Corrections
Very Long Baseline Interferometry	Transformation Parameters
Satellite Doppler Point Positioning	Multi-station Doppler Positioning
and Modern Error Theory	

means that the scientific, surveying and mapping communities can use a common framework for some time to come. The promised introduction of the Global Positioning System (GPS) will not alter this as most geodetic applications are likely to rely on differential mode techniques and GMA82 will provide the framework from which GPS will operate.

From the foregoing, it may be seen that the mathematical modelling of the different types of measurement has been more than satisfactory. From an academic point of view, some minor refinements could be made to the data set - notably in the observation variances - but this would not significantly affect the final coordinates. Thus, for practical purposes, the adjustment has been completed.

In the event of any future re-adjustment, it is unlikely that the vectors between the new adjustment and GMA82 would reach even 0.5 ppm unless there is either:

- (a) A further doubling of size of the terrestrial data set in areas where the current network is somewhat sparse.
- (b) New types of measurement with precisions at least one order of magnitude better than currently available and then only when sufficient data is available to provide a complete coverage of the continent.

There are always possible refinements that can be made to data sets and mathematical models. While there is academic merit in continually upgrading an adjustment, there are major difficulties in practice. One such difficulty is public acceptance of the necessity to change coordinates periodically. Thus the main consideration is really whether the current model in use is adequate for its purpose or whether it should be replaced by a new model.

The comparisons in Chapters 7 and 8 clearly indicate the strong internal consistency of GMA82. Thus, from a cost/benefit point of view, it is most unlikely that any organisation can justify the considerable amount of resources (computer time and manpower) necessary for a re-adjustment in the foreseeable future.

There are some areas in which the existing data set does not fully comply with Council specifications. In the event of major difficulties arising through these weaknesses, it is recommended that the additional data be added to the data set for the appropriate block and new coordinates be generated for that block by a Stage 3 adjustment.

## ACKNOWLEDGEMENTS

In carrying out this project, assistance has been received from many sources. The authors take this opportunity to express their gratitude to each and every one of them.

The major contributor was the University of New South Wales which by allowing the use of the computing facilities, support staff and other facilities made the whole project possible.

The authors are particularly indebted to Mr G. Jeffress, Professional Officer, who, when Dr G.J. Hoar unexpectedly left UNSW to take up a position with Magnavox Advanced Products and Systems Company, undertook the computation of all Doppler measurements. Without his dedication, it is doubtful whether the strengthening effect of the multi-station Doppler would have been ready in time for inclusion in GMA82.

Mr W. Milward provided sterling service in solving all the computer system problems and in general support.

The authors are indebted to the various members of the Working Group who generally responded so readily to the frequent enquiries regarding the data set. Mr I. Mattsson and Mr A. Jones of the South Australian Lands Department were particularly helpful in this regard.

The authors gratefully acknowledge the contributions made by so many organisations to the multi-station Doppler surveys (see Section 6).

The expertise required by this project has been developed under a program of research by Dr Allman which has been sponsored for the last decade by the Australian Research Grants Scheme. This sponsorship has provided computer hardware and a Professional Officer. Although the level of support was meagre when compared to that available overseas, when it is coupled with dedication and hard work it has proved to be more than adequate.

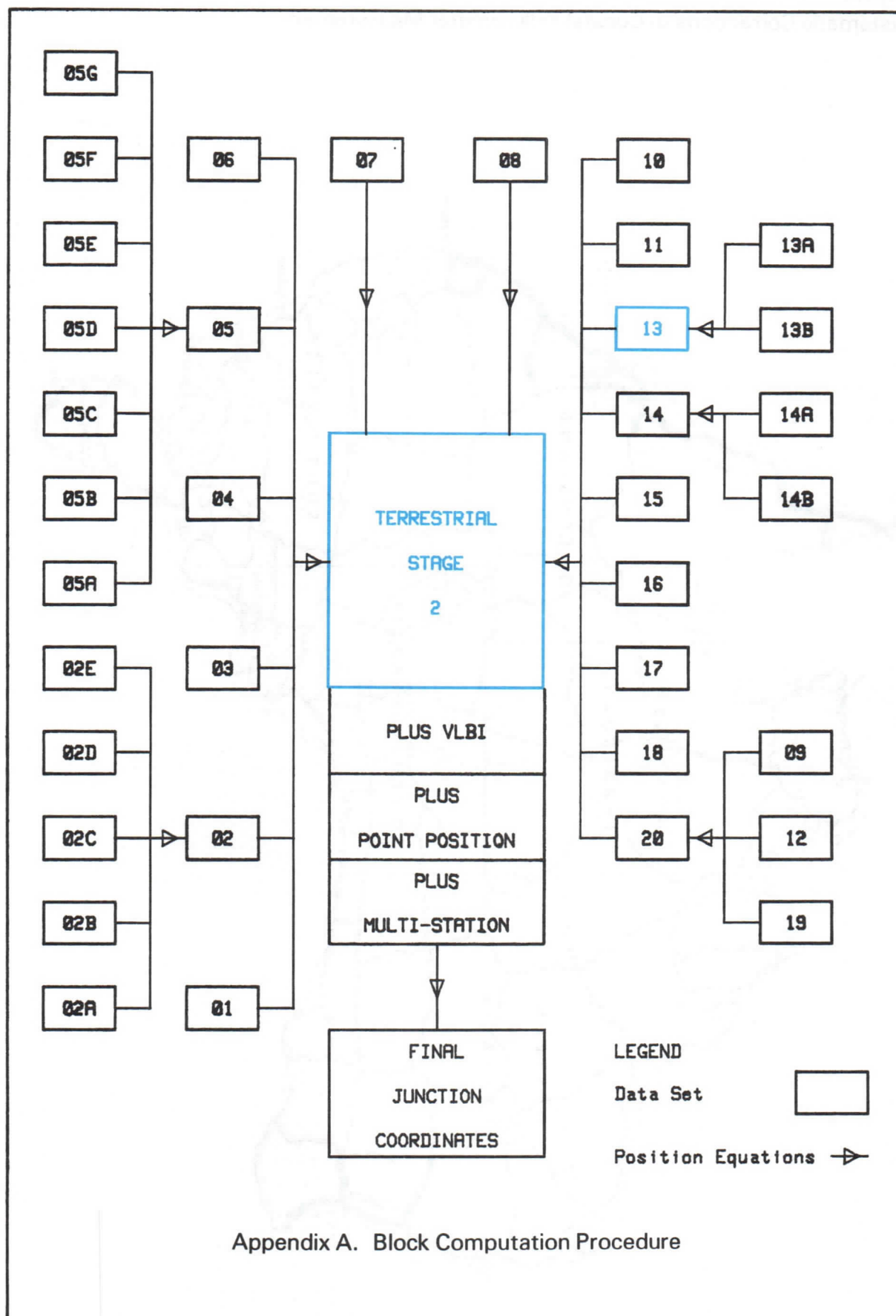
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Appendix B.  
Systematic Corrections to Coastal Tellurometer Measurement

